LECTURE #19 – SUMMARY

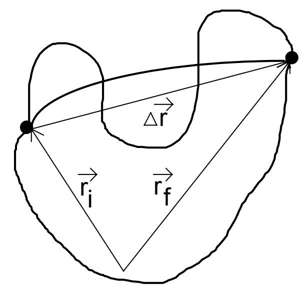
Power (defined as the rate at which work is done)

average power: $P_{avg} = \dfrac{\Delta W}{\Delta t}$ instantaneous power: $P = \lim\limits_{\Delta t \to 0} \dfrac{\Delta W}{\Delta t} = \dfrac{dW}{dt}$

- scalar quantity, units = Nm/s = J/s = Watt (W)

For the case of time-varying power output: $W = \int_{t_i}^{t_f} P\,dt$

For constant power output: $W = \int_{t_i}^{t_f} P\,dt = P\int_{t_i}^{t_f} dt = P(t_f - t_i) = P\Delta t$



Section III.7 Conservation of Energy
Conservative and Nonconservative Forces

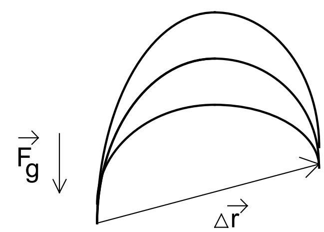
Consider a displacement $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.

How does the work depend on the path taken to go from $\vec{r}_i$ to $\vec{r}_f$?

Special case #1 - gravity

Work done by gravity: $W = \vec{F}_g \bullet \Delta\vec{r} = (0\hat{i} - g\hat{j}) \bullet (\Delta x\hat{i} + \Delta y\hat{j}) = -mg\Delta y$

The work is the same for all paths – all that matters are y_i and y_f.



Special case #2 - spring

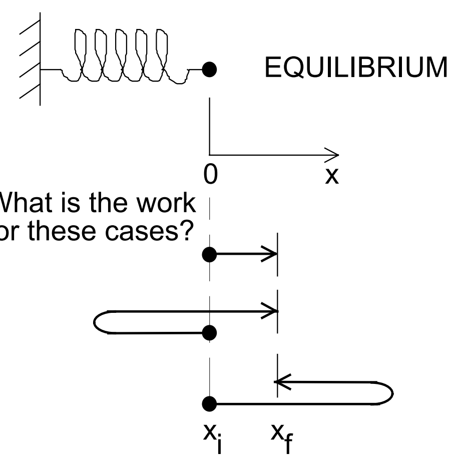
Work done by the spring on the object is

$$W_o^s = \int_{x_i}^{x_f} F_o^s\,dx = \int_{x_i}^{x_f} (-kx)\,dx = -\tfrac{1}{2}kx_f^2 + \tfrac{1}{2}kx_i^2$$

Work done by the object on the spring is

$W_s^o = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$ (using $F_s^o = +kx$)

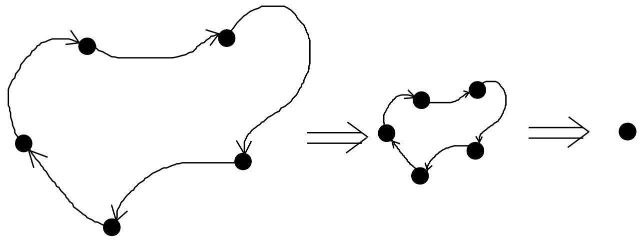
In both cases, the work is independent of the path - only depends on the initial and final positions of the spring.



When the work done by a force to move an object from point A to point B is independent of the path taken from A to B, then the force is a conservative force.

Define a closed path: a path for which start = end.

Let's say that we have only conservative forces acting. What is the work done in moving an object in a closed path?



Shrink the loop down to zero. The work must be the same since it is independent of the path. Finally, when the loop is infinitesimally small, the object doesn't move and so W = 0.

Thus, the work done by a conservative force in any closed path = 0.

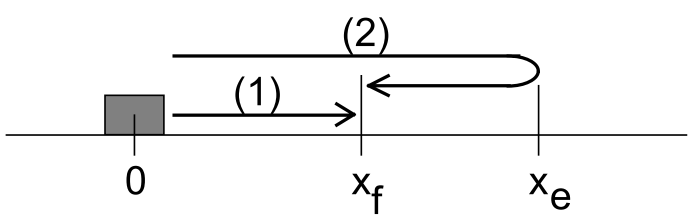
Mathematically, $\oint \vec{F}^c \bullet d\vec{r} = 0$

where $\vec{F}^c$ = conservative force, and the circle on the integral $\oint$ indicates that it is taken over a closed path.

Nonconservative forces

- work done by a nonconservative force <u>does</u> depend on the path
- a nonconservative force does not "give back" work

Friction - consider a sliding block on a table, with two paths (1) and (2):



What is the work done by friction in each case?

Path (1): $\vec{F}_k = -mg\mu_k \hat{i}$ and $W_k^{(1)} = \vec{F}_k \bullet \Delta\vec{x} = (-mg\mu_k)(x_f - 0) = -mg\mu_k x_f$

Path (2): $W_k^{(2)} = (-mg\mu_k x_e) + (mg\mu_k x_f - mg\mu_k x_e) = mg\mu_k x_f - 2mg\mu_k x_e$

$\therefore$

- work done by friction is dependent on the path since $W_k^{(1)} \neq W_k^{(2)}$
- if the block moved in a closed loop, so that x_f = 0, we would still have $W_k^{(2)} = -2mg\mu_k x_e \neq 0$

Conservative forces $\rightarrow$ no dissipation (gravity, elastic springs)
Nonconservative forces $\rightarrow$ dissipation (friction)