LECTURE #17 – SUMMARY

Work

The <u>work</u> done on an object by a constant force applied to it is: $\boxed{W \equiv \vec{F} \bullet \Delta\vec{r}}$

- units = N m = Joule (J), scalar quantity, can be positive or negative
- $W = F\Delta r\cos\theta$ where θ is the angle between $\vec{F}$ and $\Delta\vec{r}$

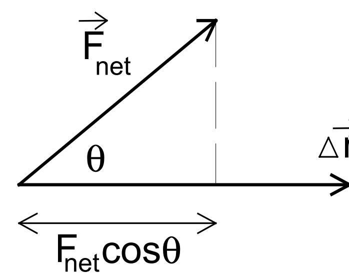
If more than one force is applied to an object, then the net force is: $\vec{F}_{net} = \sum_{i=1}^{N} \vec{F}_i$

The net work done is then (= sum of the work done by each force):

$$W_{net} = \vec{F}_{net} \bullet \Delta\vec{r} = \left(\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_N\right) \bullet \Delta\vec{r} = \vec{F}_1 \bullet \Delta\vec{r} + \vec{F}_2 \bullet \Delta\vec{r} + ... + \vec{F}_N \bullet \Delta\vec{r} = W_1 + W_2 + ... + W_N$$

The Work-Energy Theorem ($\vec{F}_{net} \bullet \Delta\vec{r} = \frac{1}{2}mv_f^{\ 2} - \frac{1}{2}mv_i^{\ 2}$) only deals with the NET FORCE. Thus $\boxed{W_{net} = K_f - K_i}$. This is a restatement of the Work-Energy Theorem.

"amount of work" ($\vec{F}_{net} \bullet \Delta\vec{r} = F_{net}\Delta r\cos\theta$) = magnitude of displacement $\times$ magnitude of force in the direction of displacement



i) W > 0 if displacement and projected force are in the same direction, i.e., <u>speed is increasing</u>

ii) W < 0 if displacement and projected force are in the opposite direction, i.e., <u>speed is decreasing</u>

Only the component of the force in the direction of the displacement does work.

The component of $\vec{F} \parallel \vec{v} \rightarrow$ does work (and $\therefore$ changes speed)

The component of $\vec{F} \perp \vec{v} \rightarrow$ does NO work (and $\therefore$ does not change speed)

The Generalized Work-Energy Theorem (for non-constant forces)

Consider a general trajectory and break it into little segments for each of which the force can be treated as constant.

For the j-th segment: $W_{net,j} = \vec{F}_{net,j} \bullet \Delta\vec{r}_j = K_j - K_{j-1}$

As the object moves from initial position $\vec{r}_o = \vec{r}_i$ to final position $\vec{r}_N = \vec{r}_f$:

$$W_{net} = \sum_{j=1}^{N}\left(\vec{F}_{net,j} \bullet \Delta\vec{r}_j\right) = \sum_{j=1}^{N}\left(K_j - K_{j-1}\right) = K_f - K_i$$

As $\Delta\vec{r}_j \rightarrow 0$: $\displaystyle \lim_{\Delta\vec{r}_j \rightarrow 0} \sum_{j=1}^{N} \vec{F}_{net,j} \bullet \Delta\vec{r}_j = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net} \bullet d\vec{r}$ (line integral along the trajectory)

<u>Generalized Work-Energy Theorem</u> $\boxed{W_{net} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net} \bullet d\vec{r} = K_f - K_i}$

Summary of the equations derived for <u>net work</u>:

(1) General case : $W_{net} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net} \bullet d\vec{r} = K_f - K_i$

(2) Special case - constant force: $W_{net} = \vec{F}_{net} \bullet \Delta\vec{r} = K_f - K_i$

(3) Special case - constant force in 1-D (x): $W_{net} = F_{net,x}\Delta x = K_f - K_i$

(4) Special case - non-constant force in 1-D (x): $W_{net} = \int_{x_i}^{x_f} F_{net,x} dx = K_f - K_i$

Summary of the equations derived for <u>work done by a specific force F</u> (not necessarily F_{net}):

(1) General case: $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \bullet d\vec{r}$

(2) Special case - constant force in 3-D: $W = \vec{F} \bullet \Delta\vec{r}$

(3) Special case - constant force in 1-D (x): $W = F_x \Delta x$

(4) Special case - non-constant force in 1-D (x): $W = \int_{x_i}^{x_f} F_x dx$

Work-Energy Theorem for net work:

3-D		1-D
$W_{net} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{net} \bullet d\vec{r} = K_f - K_i$	non-constant forces	$W_{net} = \int_{x_i}^{x_f} F_{net,x} dx = K_f - K_i$
$W_{net} = \vec{F}_{net} \bullet \Delta\vec{r} = K_f - K_i$	constant forces	$W_{net} = F_{net,x}\Delta x = K_f - K_i$

Work-Energy Theorem for work done by a specific force $\vec{F}$:

3-D		1-D
$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \bullet d\vec{r}$	non-constant force $\vec{F}$	$W = \int_{x_i}^{x_f} F_x dx$
$W = \vec{F} \bullet \Delta\vec{r}$	constant force $\vec{F}$	$W = F_x \Delta x$