

LECTURE #12 – SUMMARY

Section III.2 Inertial vs. Noninertial Reference Frames

Newton's Laws of Motion do not hold

- (1) when $v \approx c$, i.e., motion near the speed of light
 - (2) in non-inertial reference frames, i.e., in accelerating frames of reference
- References frames which move at constant velocity = inertial frames of reference.

Let's say that A is an inertial frame of reference.

$$\vec{v}_B^P = \vec{v}_A^P + \vec{v}_B^A \quad \text{and} \quad \vec{a}_B^P = \vec{a}_A^P + \vec{a}_B^A$$

First Case: \vec{v}_B^A is constant

\Rightarrow then B is also an inertial frame of reference

$$\vec{v}_B^P = \vec{v}_A^P + \vec{v}_B^A \quad \text{so A and B measure different velocities}$$

$$\vec{a}_B^P = \vec{a}_A^P \quad \text{so A and B measure the same}$$

acceleration

$$\therefore m\vec{a}_B^P = m\vec{a}_A^P \quad \text{and} \quad \vec{F}_A = \vec{F}_B$$

So observers in different inertial reference frames agree on the net force on P.

Special case: $\vec{a}_A^P = 0 \Rightarrow$ then $\vec{a}_B^P = 0$. Both observers say no net force acts on P.

This means that

- velocities can be different in different inertial frames of reference, but
 - accelerations and forces CANNOT differ in different inertial reference frames
- \therefore "rest" is relative: can only measure relative velocities of inertial reference frames

Second Case: \vec{v}_B^A is NOT constant \Rightarrow then B is a non-inertial reference frame

$$\vec{v}_B^P = \vec{v}_A^P + \vec{v}_B^A \quad \text{so A and B still measure different velocities}$$

$$\vec{a}_B^P = \vec{a}_A^P + \vec{a}_B^A \quad \text{so A and B now measure different accelerations}$$

$$\therefore m\vec{a}_B^P = m\vec{a}_A^P + m\vec{a}_B^A \quad \text{and} \quad \vec{F}_B = \vec{F}_A + \vec{F}_f$$

Observer B sees a different force applied to P than the force that observer A sees.

Observer B is mistaking his/her own acceleration for that of the object.

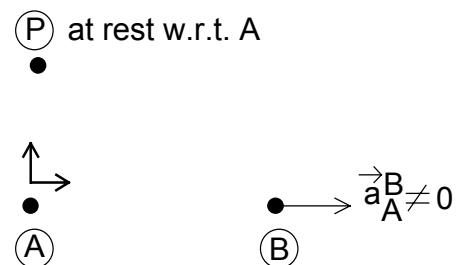
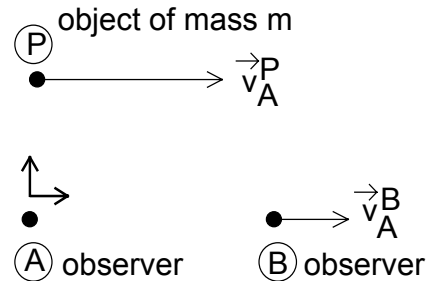
$$\text{error} = \vec{F}_f = m\vec{a}_B^A = \text{fictitious force}$$

$$\vec{a}_B^P = \vec{a}_A^P + \vec{a}_B^A = 0 + \vec{a}_B^A$$

$$\vec{F}_B = m\vec{a}_B^P \quad \therefore \vec{F}_B = m\vec{a}_B^A = -m\vec{a}_B^B$$

Two interpretations:

- (1) a force of unknown origin is accelerating P w.r.t. B
- (2) observer B is accelerating and object P only APPEARS to be accelerating



Therefore, we conclude that Newton's Laws of motion do not hold in non-inertial frames of reference. Do NOT apply " $\vec{F} = m\vec{a}$ " in accelerating frames of reference !