“Chaos is a friend of mine.”

Bob Dylan,
American folksinger
(1941-)

“Predictability: Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?”

Title of paper presented at the 139th Annual Meeting of the AAAS (29 Dec 1979).

Edward Lorenz, American mathematician and meteorologist (1917-2008)

PHY100 – One Last Time…

- Homepage

http://www.atmosp.physics.utoronto.ca/people/strong/phy100/phy100.html

- Portal/Blackboard
  - → I have activated the course on the Portal, but will be using the external homepage to provide course content
- Textbook
  - → *Physics: Concepts and Connections*, Fifth Edition, Art Hobson, Pearson Education (2010) – in the Bookstore
- Tutorials
  - → Second tutorial this week
  - → Quiz is on material covered <u>last week</u>
- Office hours
  - → 3-4 PM, Tuesdays and Thursdays, room MP710A

NOTE-TAKING SERVICES: ONLINE!

Volunteering will be more convenient than ever before – a new online repository of notes that can be accessed 24 hrs a day, 7 days a week!

Accessibility Services requires dependable volunteer note-takers in this course to assist students with disabilities. Benefits of volunteering:

- The quality and consistency of your notes will improve
- Gain valuable (and useful!) experience
- Receive a certificate of recognition

Want to volunteer as a note-taker?
Volunteer with the service through the Accessibility Services website (http://www.accessibility.utoronto.ca/)
or review which courses have requests for note-takers at https://www.studentlife.utoronto.ca/accessibility/pcourselist.aspx
(You can also add your courses if they are not on the list, and you will be notified if your services are required.)
Currently a volunteer?
Log in to the new system at https://www.studentlife.utoronto.ca/accessibility/vollogin.aspx

Contact as.notetaking@utoronto.ca if you have any questions, concerns, or require assistance.

Current Assignments ...

For today:

- Read notes on Chaos, posted on homepage

For Lecture 7:

- Read Sections 6.1 - 6.6 (Energy)

Suggested Conceptual Exercises:

- None on Chaos

Attend your second tutorial this week

Homework #1

- Due 11:00 AM, Thursday, January 31 in the Drop Box labelled for your tutorial

Writing Assignment #1

- Writing Assignment #1 is now available
  - → Posted under Homework on the homepage
- Due **11:00 AM, Thursday, February 28** in the Drop Boxes (late penalty = 5%/day for 1 week)
- Topic: Write a report on a “physics for the layman” book. Some suggestions are given, but you may suggest your own book, as long as you obtain approval from me or your tutor.
- The essay should be ~2 pages or 500 words
- Detailed guidelines are given – read them!
- **Avoid plagiarism** (and its penalties)!

Writing and Plagiarism

From the UofT “**ACADEMIC INTEGRITY HANDBOOK”**

“Emphasize that not knowing the rules is not an excuse and that students are expected to know and follow the rules of the University.”

“Plagiarism: Most students know that plagiarism is an academic offence. However, many students don’t appear to understand what constitutes plagiarism. Remind them that plagiarism includes:

- Taking an idea from a source and not acknowledging it with a citation.
- Copying material word for word and not indicating that the words are copied with quotation marks or indentation. Providing a citation is not enough! Failure to use quotation marks is an academic offence.
- Changing a few words in a sentence or reversing the order of clause. This is not paraphrasing!
- Including translated material from a source in another language without acknowledgement.”

Ensure that any work which you submit is entirely your own.

Writing Assignment #1

“The White Rabbit put on his spectacles. ‘Where shall I begin, please your Majesty?’ he asked.

‘Begin at the beginning’, the King said very gravely, ‘and go on till you come to the end: then stop.’ ”

Lewis Carroll, *Alice in Wonderland* (1865)

Review of Lecture 5

Textbook, Sections 5.2, 5.5, 5.6, and 2.6

- Newton's Law of Gravity
- "Weightlessness" and free fall
- The Newtonian worldview
- Beyond Newton

Plan for Lecture 6

Notes on Chaos

- Introduction to chaos
- The three-body gravitational problem
- The pendulum as an attractor
- Lorentz attractors
- Fractals

Chaos

From: http://webecoist.com/2008/09/07/17-amazing-examples-of-fractals-in-nature/

Chaos Theory

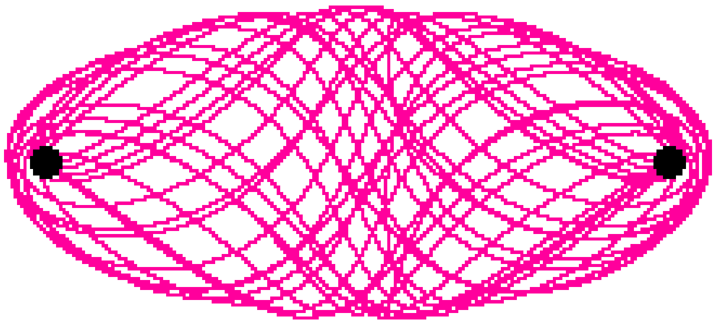
- Deals with systems that are very sensitive to initial conditions
  - → Looks for the underlying order in apparently random systems
- This sensitivity results in exponential growth of error that makes chaotic systems seem to behave in a random way
  - → Very small differences in the initial state of a system can cause very large differences in its final state
- But chaotic systems are actually deterministic, not random.

Three-Body Gravitational Problem

- Newton used his Laws of Motion and the Universal Law of Gravitation to analyze the orbit of a planet around the Sun
  - → He found that the solution is an ellipse
  - → Simple problem to solve
- The Sun is an <u>attractor</u> of the planet
- Newton later tried to solve the problem of two Suns and a planet
  - → He failed!
  - → There is no analytic solution to this second-simplest gravitational system.

The Three-Body Problem

- An analytic solution to this problem does not exist, but we can obtain an approximate solution using a computer.
- We know: (1) position and velocity of Earth at some time, and (2) forces acting on Earth, so we can calculate the acceleration of Earth using Newton's Second Law
- Then calculate the new position and velocity of Earth at VERY small time later.
- Then calculate the new acceleration.
- Then calculate the new position and velocity of Earth at another VERY small time later.
- Repeat. Repeat. Repeat...
- Smaller time intervals give a better approximation.

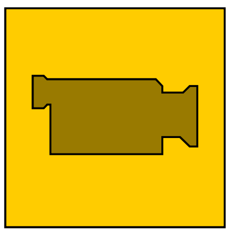


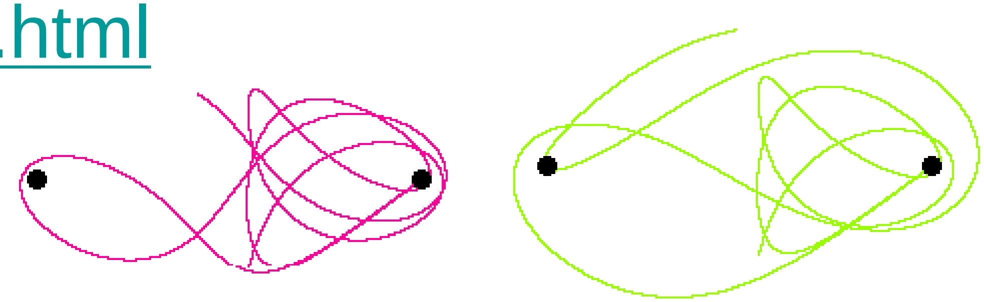
Three-Body Gravitational Problem

Two solutions to the three-body problem

(their initial horizontal speeds differ by one percent)

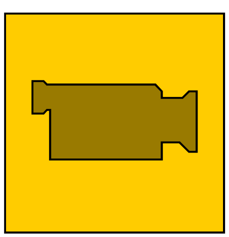
- http://www.upscale.utoronto.ca/PVB/Harrison/Chaos/anim3body2.html





Flash animation: two fixed suns, one planet

- http://www.upscale.utoronto.ca/GeneralInterest/Harrison/Flash/Chaos/ThreeBody/ThreeBody.html



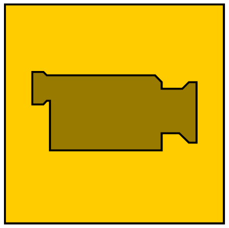
Try this one at home.

The Butterfly Effect

- Small differences in initial conditions may produce large differences in the long-term behavior of a system.
  - → The flapping of a butterfly's wings might create tiny changes in atmospheric motion that are amplified and ultimately cause a tornado.
- Coined by Edward Lorenz (1960s)
  - → Ran a computer program that modelled weather
  - → Restarted a model run in the middle, and found that it gave a very different result from original.
  - → Realized this was due to rounding off the input to the second run from 0.506127 to 0.506.

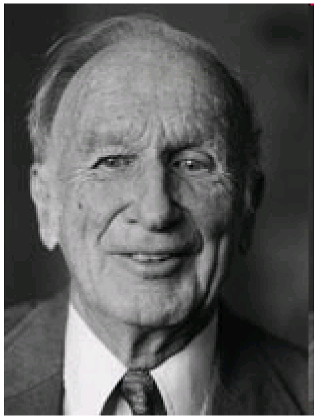
Illustration of the Butterfly Effect

- http://www.exploratorium.edu/complexity/java/lorenz.html



http://www.ams.org/mathmedia/mathdigest/md-200804-toc.html

Edward Lorenz



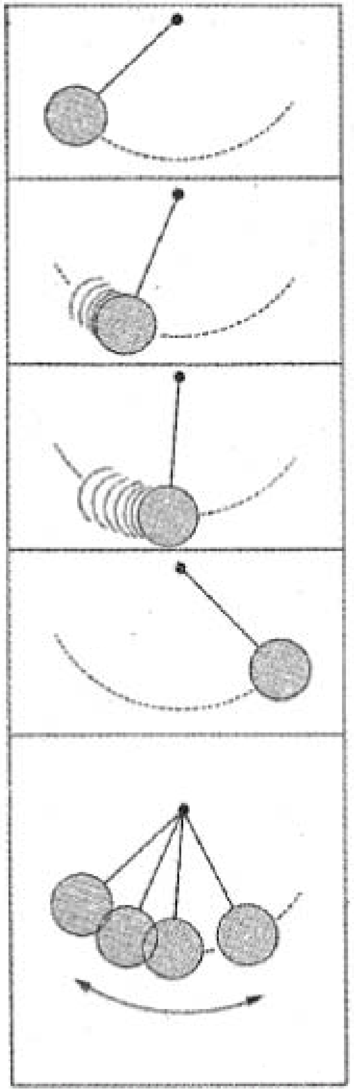
http://web.mit.edu/newsoffice/2008/obit-lorenz-0416.html

- Awarded Kyoto Prize in 1991.
- Cited for establishing "the theoretical basis of weather and climate predictability, as well as the basis for computer-aided atmospheric physics and meteorology."
- The committee added that Lorenz "made his boldest scientific achievement in discovering 'deterministic chaos,' a principle which has profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind's view of nature since Sir Isaac Newton."
- Some claim the 20th century had three scientific revolutions: relativity, quantum mechanics, & chaos.

Attractors

- An attractor is a set of properties toward which a system tends to evolve, regardless of the initial conditions of the system.
  - → Can be a point, a curve, or a fractal structure.
  - → Even a complicated set with a fractal structure
- A strange attractor is one which exhibits a chaotic approach to its final set of properties.
- Describing the attractors of chaotic dynamical systems is one of the achievements of chaos theory.

The Pendulum as an Attractor

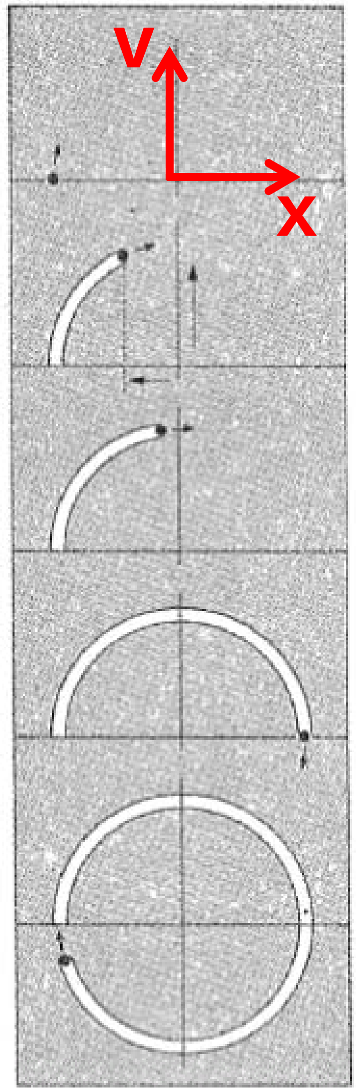


Velocity is zero as the pendulum starts its swing. Position is a negative number, the distance to the left of the center.

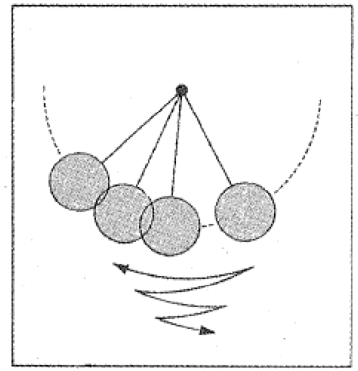
The two numbers specify a single point in two-dimensional phase space.

Velocity reaches its maximum as the pendulum's position passes through zero.

Velocity declines again to zero, and then becomes negative to represent leftward motion.

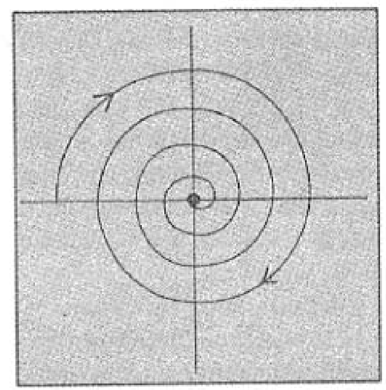


ANOTHER WAY TO SEE A PENDULUM. One point in phase space (*right*) contains all the information about the state of a dynamical system at any instant (*left*). For a simple pendulum, two numbers—velocity and position—are all you need to know.



The points trace a trajectory that provides a way of visualizing the continuous long-term behavior of a dynamical system. A repeating loop represents a system that repeats itself at regular intervals forever.

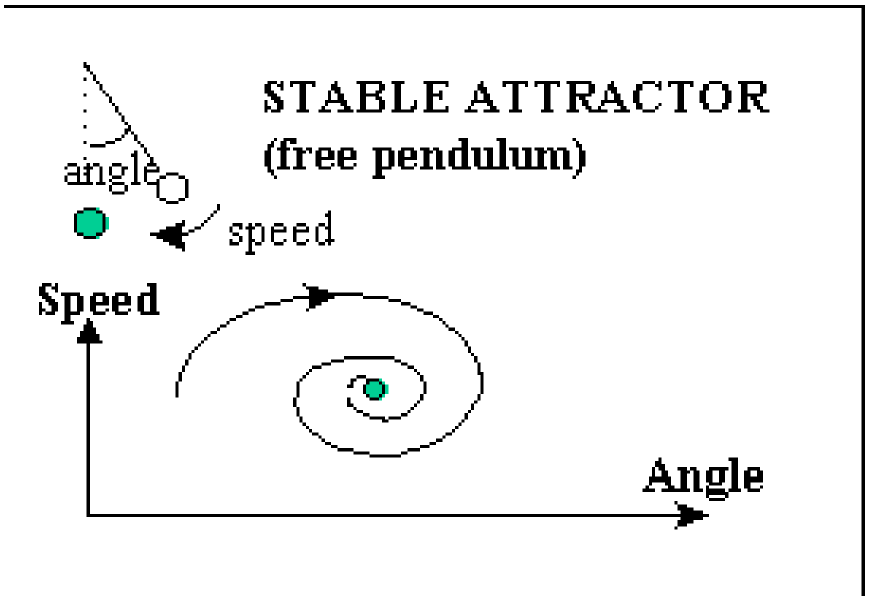
If the repeating behavior is stable, as in a pendulum clock, then the system returns to this orbit after small perturbations. In phase space, trajectories near the orbit are drawn into it; the orbit is an attractor.

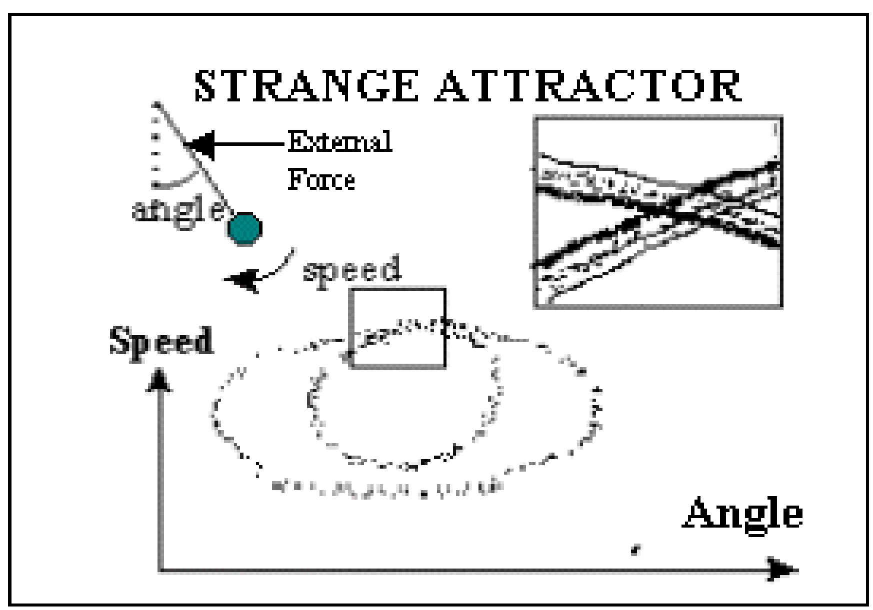


An attractor can be a single point. For a pendulum steadily losing energy to friction, all trajectories spiral inward toward a point that represents a steady state—in this case, the steady state of no motion at all.

From *Chaos*, by James Gleick, pages 136-137 (1987)

Attractors

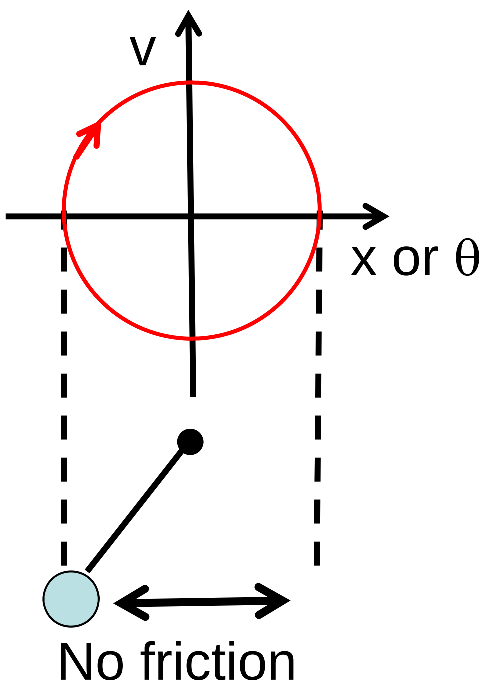

STABLE ATTRACTOR
(free pendulum)
angle
speed
Speed
Angle

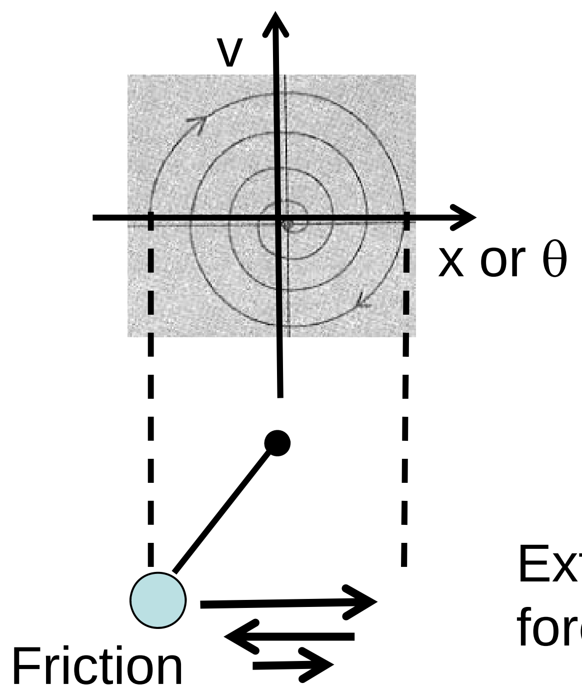

STRANGE ATTRACTOR
External
Force
angle
speed
Speed
Angle

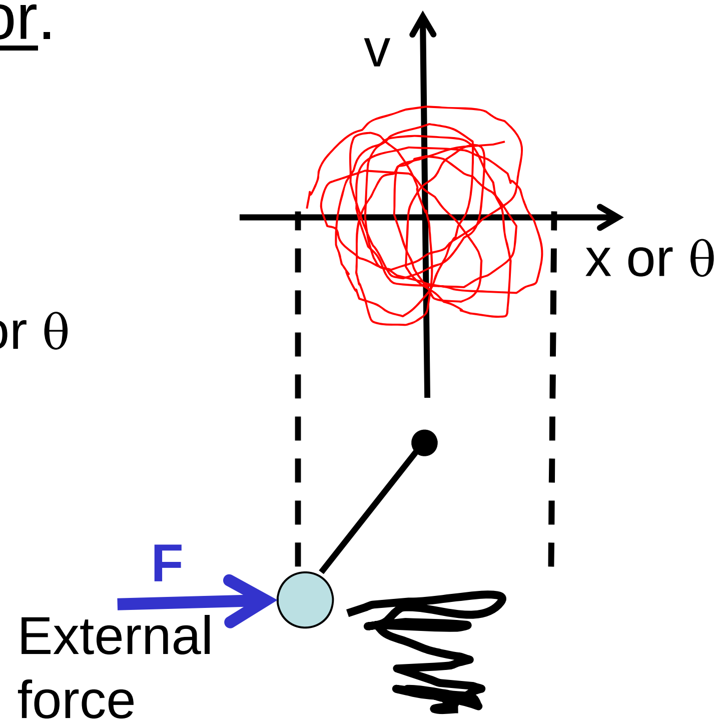
Pendulum as Attractor

Sketch velocity vs. position for three cases:

1) No friction – trajectory is a circle
2) Friction – trajectory is a spiral.
   The (0,0) point is the stable attractor.
3) External force – apparently random trajectory.
   The trajectory is a strange attractor.



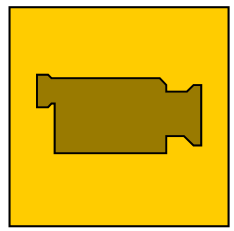




The Pendulum as an Attractor

Java applet of pendulum motion

- http://www.physics.orst.edu/~rubin/nacphy/JAVA_pend/COMP/



Double Pendulum Demo

- A double pendulum is a pendulum with another pendulum attached to its end.
  - → It is governed by a set of coupled ordinary differential equations.
  - → For certain energies its motion is chaotic.
- Compared to real world systems (weather; molecular vibrations; fluid dynamics; solar systems, galaxies), it is a simple system which can be mathematically modelled.
- However, unlike a simple single pendulum, it is impossible to predict the long-term behaviour of the double pendulum.

The Lorenz Attractor

- Starting with a set of equations related to turbulence in the atmosphere, Lorenz came up with a set of 3 equations that were very sensitive to initial conditions.
  - → Used a computer to solve them approximately.
  - → They seemed to produce random behavior.
- But, when plotted in 3 dimensions (x-y-z), the trajectory always followed a double spiral.
  - → Never repeated or settled on a single point.
  - → Has two points that attract the trajectory - similar to the three-body gravitational problem.

The Lorenz Attractor

3D movie of the Lorenz Attractor

- http://paulbourke.net/fractals/lorenz/lorenz.m4v

More on the Lorenz Attractor - movie, images, computer codes

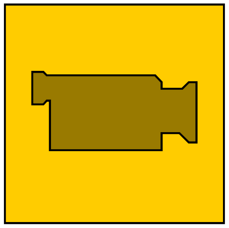
- http://paulbourke.net/fractals/lorenz/

Try these at home.

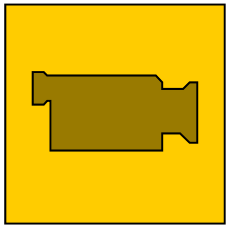
The Lorentz Attractor

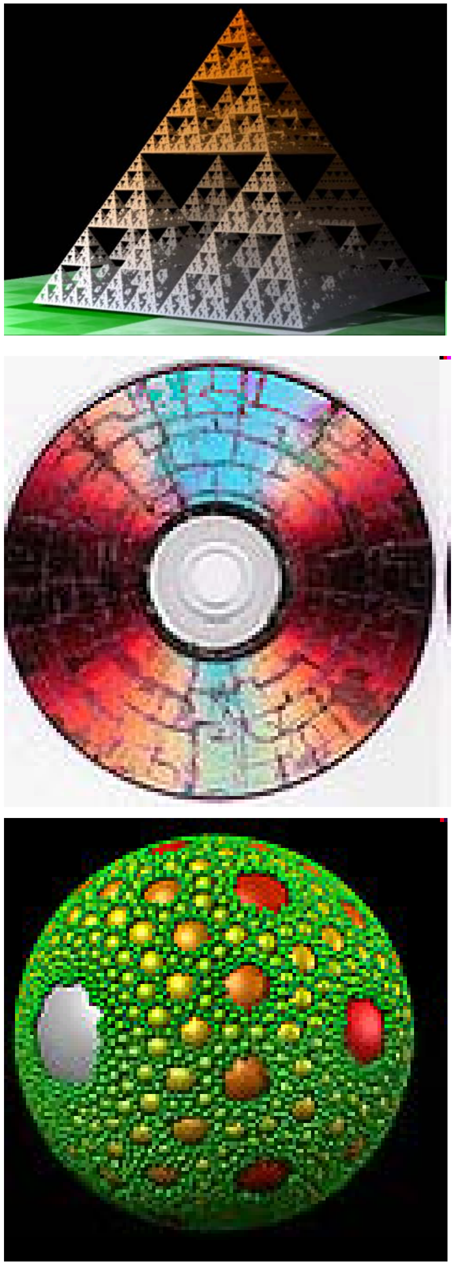
David Harrison's animations

- http://www.upscale.utoronto.ca/PVB/Harrison/Chaos/animlorenz.html



- http://www.upscale.utoronto.ca/PVB/Harrison/Flash/Chaos/Lorenz/Lorenz.html





"Fractal geometry will make you see everything differently. There is a danger in reading further. You risk the loss of your childhood vision of clouds, forests, flowers, galaxies, leaves, feathers, rocks, mountains, torrents of water, carpet, bricks, and much else besides. Never again will your interpretation of these things be quite the same."

Michael F. Barnsley, mathematician,
Fractals Everywhere (2000)

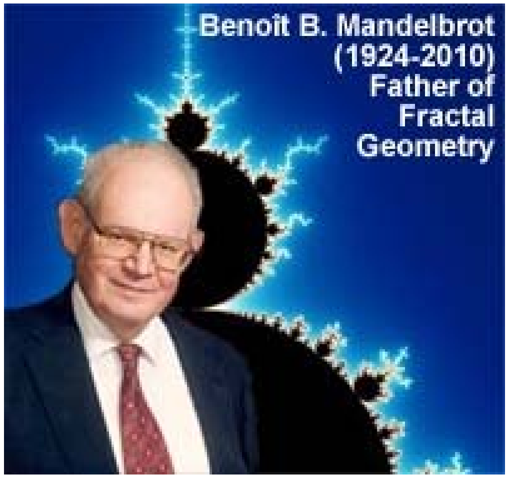
Fractals

- Def'n: "A geometric pattern that is repeated at ever smaller scales to produce irregular shapes and surfaces that cannot be represented by classical geometry. Fractals are used especially in computer modeling of irregular patterns and structures in nature."

The American Heritage® Dictionary of the English Language, Fourth Edition
Copyright ©2000 by Houghton Mifflin Company. Updated in 2009.

- Fractal = a contraction of "fractional dimension" (Mandelbrot, 1975)
- A fractal is <u>self-similar</u> - it resembles itself on all scales.

Benoit Mandelbrot

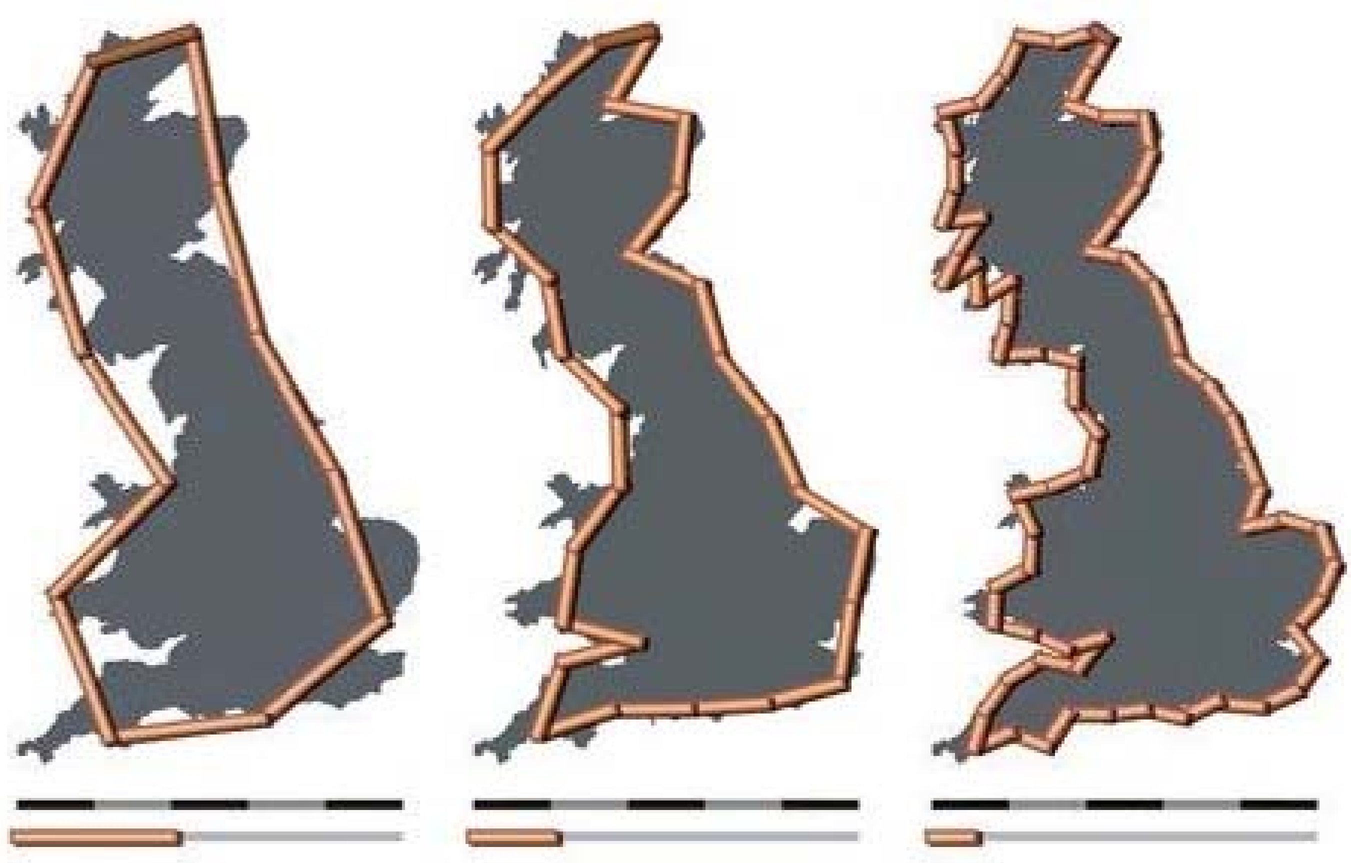


http://www.miqel.com/fractals_math_patterns/visual-math-mandelbrot-magic.html

- Father of fractal geometry
- Coined the term “fractal”
- Recognized that many phenomena shared the common feature of power-law, or self-similar, scaling
- This was demonstrated for the length of coastlines by Richardson
- Many examples in nature: topography, lakes, islands, river networks, earthquakes, precipitation patterns, turbulence, faults, flow

Eos, 93(4), 44, 24 January 2012

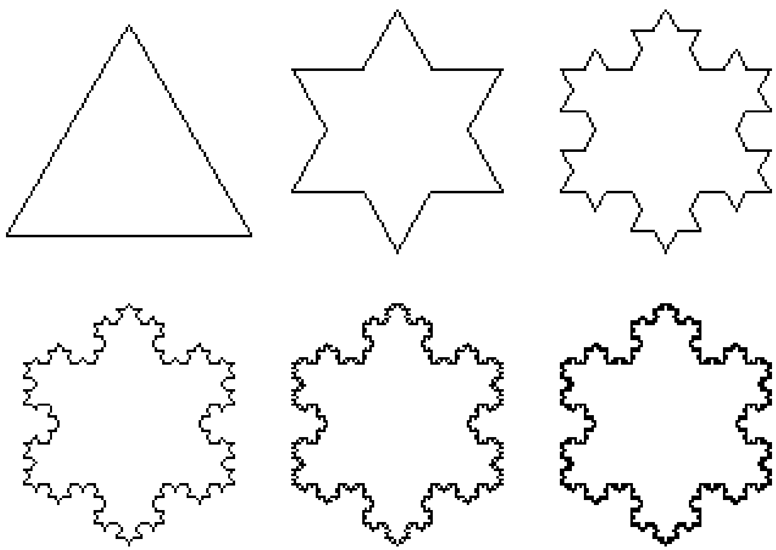
How Long Is the Coast of Britain?



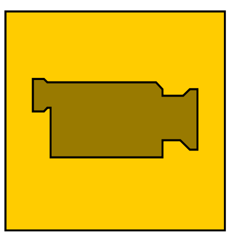
"The coastline of Britain as measured with measuring rods of 200 km, 100 km and 50 km in length.   The resulting coastline is about 2350 km, 2775 km and 3425 km; the shorter the scale, the longer the measured length of the coast."

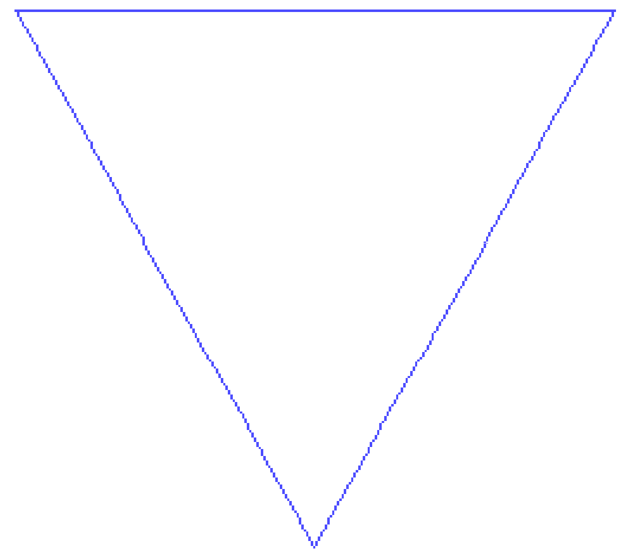
http://upload.wikimedia.org/wikipedia/commons/2/20/Britain-fractal-coastline-combined.jpg

Koch Snowflakes



http://www.afractal.com/fractalex.htm

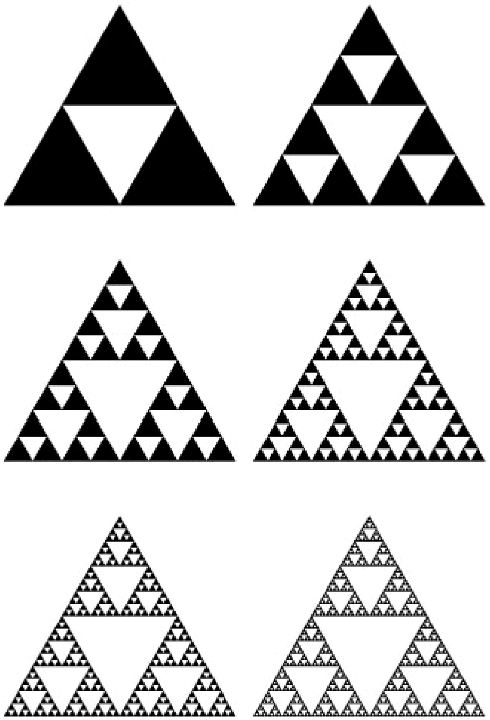




António Miguel de Campos
http://upload.wikimedia.org/wikipedia
/commons/f/fd/Von_Koch_curve.gif

http://www.absorblearning.com/media/
attachment.action?quick=mf&att=1606

Sierpinski Triangles



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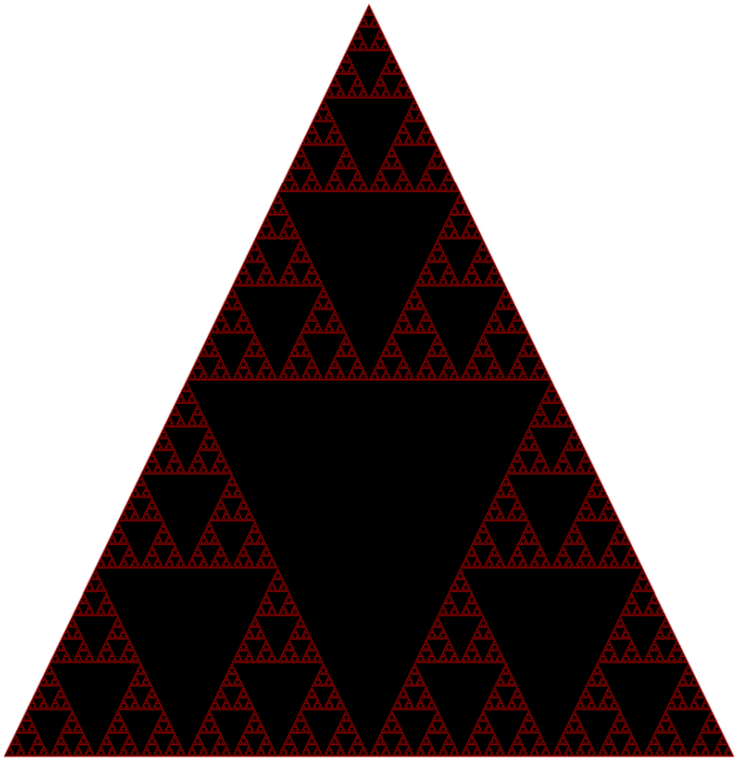


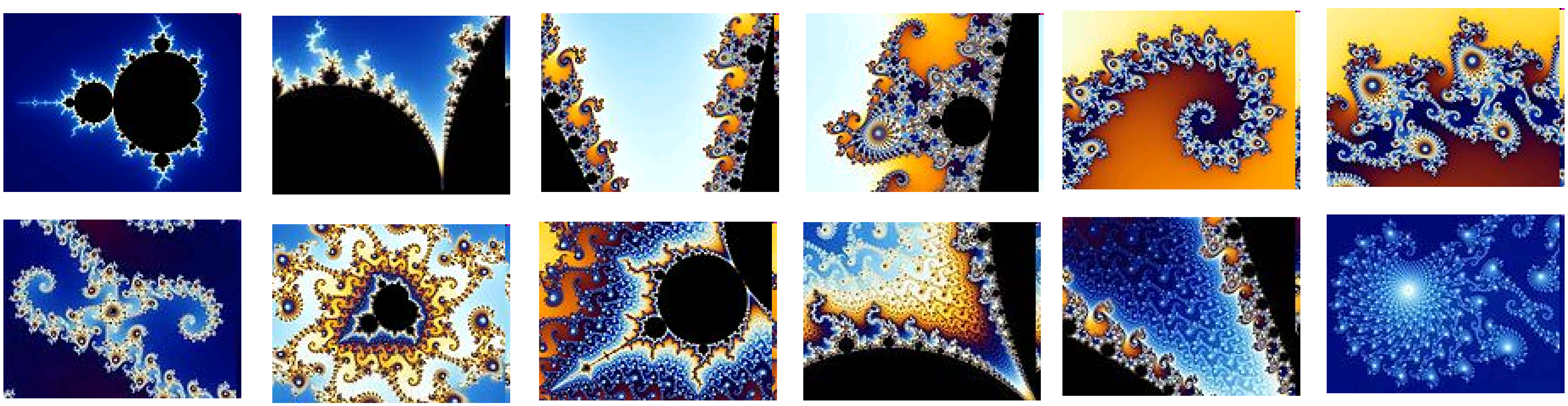
Dino at en.wikipedia,
http://upload.wikimedia.org/wikipedia/commons/7/74
/Animated_construction_of_Sierpinski_Triangle.gif

http://math.bu.edu/DYSYS/animations/sierp-zoom.html

The Mandelbrot Set

"The most complex object in mathematics, the Mandelbrot Set ... is so complex as to be uncontrollable by mankind and describable as 'chaos'."

Benoit Mandelbrot (1924 -)



Created by Wolfgang Beyer, http://en.wikipedia.org/wiki/Mandelbrot_set

The Mandelbrot Set

- The Mandelbrot set is an example of a fractal that is not created by an iterative process, but which exhibits similar complexity.

- This generates a figure with a complex boundary that never simplifies at any magnification.
  - → This boundary is a fractal.
  - → Self-similar, but not identical at each scale.

The Mandelbrot Set

Movie

- http://math.bu.edu/DYSYS/animations/Farey.qt.mov

Interactive animation

- http://www.ph.biu.ac.il/~rapaport/java-apps/mandel.html

Summary: Properties of Chaotic Systems

- Deterministic in a mathematical sense.
- Sensitive to initial conditions.
- Unpredictable and unrepeatable.
- Nonlinear.
- Transition to chaos is preceded by infinite levels of bifurcation.
- Fractional dimensionality.

Hidden in the apparent disorganization of chaos is a great deal of structure.