



$E = mc^2$ on the Taipei 101 building



Albert Einstein Memorial, Washington, DC
Sculptor: Robert Berks



$E = mc^2$ on the Walk of Ideas in Berlin

Images from www.wikipedia.com

$E = mc^2$: Energy equals milk chocolate squared

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Current Assignments ...

For today

- Read Sections 10.8

For Lecture 17

- Read Sections 11.1 - 11.7

Homework #3 (new due date)

- Handed out February 14, due **FRIDAY, MARCH 8**

Homework #4

- Handed out today. Due 11:00 AM, Friday, March 22

Writing Assignment #2

- Posted Feb. 28. Due 11:00 AM, Thursday, April 4

Suggested Conceptual Exercises

- Chapter 10: 21,25,27,31,35,37,39,41,43,47,49,51

Tutorial #7

Office hours:
2-3 Tuesday &
3-4 Thursday

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Review of Lecture 15

Textbook, Sections 10.6 - 10.7

- Time travel
- The relativity of space - length contraction
- The relativity of mass

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Plan for Lecture 16

Textbook, Sections 10.8

- Space-time diagrams and worldlines
- $E = mc^2$

“If a body releases the energy L in the form of radiation, its mass is decreased by L / V^2 .”

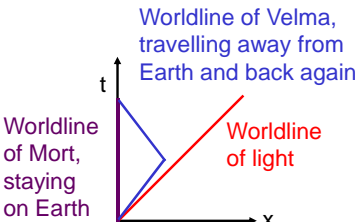
Albert Einstein (1879-1955)

L = energy
 V = lightspeed from *Annalen der Physik*, 18, 639-641, 1905.

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Space-Time Diagrams

- Provide a visual illustration of the path of an object through space and time - its worldline
- Plot distance on x-axis and time on y-axis



Worldline of Velma, travelling away from Earth and back again

Worldline of Mort, staying on Earth

Worldline of light

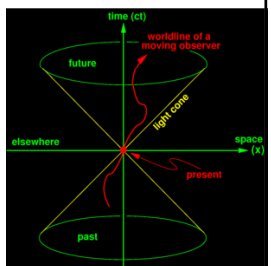
Space-time diagram for the twin “paradox”

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More on Space-Time Diagrams: Light Cones

- This light cone diagram shows a 2D representation of 4D spacetime.

→ The red line is the worldline of an observer moving in 2D.



<http://einstein.stanford.edu/SPACETIME/spacetime2.html>

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What about two approaching spacecraft?



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What about two approaching spacecraft?



- Is their relative speed $1.6c$?
- Galilean relativity says yes, and would be wrong! Special relativity says no.
- Both time and space are transformed at high speeds - this results in the "Lorentz velocity transformation".

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{v_B - v_A}{1 - v_B v_A / c^2} = \frac{-0.85c - 0.75c}{1 - (-0.85c)(0.75c)/c^2} = -0.98c$$

where u' = speed of B with respect to A

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Mass and Energy

- By combining Special Relativity and Conservation of Energy, Einstein found that any increase in the energy of an object results in an increase to its mass.
- The change in mass is equal to the change in energy, divided by the square of the speed of light: $\Delta m = \Delta E / c^2$
- The speed of light is very large, so the change in mass is undetectable in ordinary situations.

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Nuclear Reactions

- This change in mass due to changes in energy becomes large in nuclear reactions.
- The energies are extremely large when nuclei are involved.
- The fission (splitting) of 1 kg of uranium decreases its mass by 1 gram, which is easily measured.
- This contradicts the conservation of matter – mass has actually been converted to energy.

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$E = mc^2$

- Thus rest mass (= matter) is not conserved.
- Einstein extended the energy-mass relationship to all masses.
- In words: The total mass of a system is equal to the total energy of that system, divided by the square of the speed of light.
 $m = E/c^2$ or $E = mc^2$
- This gives the energy content of any mass.
- Rest mass alone is not conserved, but energy – including mass energy – is.

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The Principle of Mass-Energy Equivalence

Energy has mass; that is, energy has inertia.
And mass has energy; that is, mass has the ability to do work.

The quantitative relation between the energy of any system and the mass of that system is $E = mc^2$.

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$E = mc^2$ Again

- More strictly, all objects have an intrinsic, or rest, energy $E_o = m_o c^2$ where m_o is the rest mass of the object.
- This is true even if the object is stationary.
- If the object is moving at speed v , then the total energy = rest energy + kinetic energy:
 $E = mc^2 = \gamma m_o c^2 = m_o c^2 + K$.
- “ $E=mc^2$ ” either means $E=m_o c^2$ for an object at rest, or $E=\gamma m_o c^2$ when the object is moving.

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$E = mc^2$: In which of the following processes does the system's total mass change?

- (A) An automobile speeds up
- (B) A rubber ball is squeezed
- (C) The total mass changes in both of the above processes
- (D) The total mass changed in neither of the above processes.

An increase in energy causes an increase in total mass ($m = E/c^2$). Both A and B increase the total energy (kinetic and elastic) of the system, and so increase the mass.

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$E = mc^2$: In which of the following processes does the system's rest mass change?

- (A) An automobile speeds up
- (B) A rubber ball is squeezed
- (C) The total mass changes in both of the above processes
- (D) The total mass changed in neither of the above processes.

The rest mass of the automobile is unchanged. The rubber ball is at rest, but its energy increases (work is done), so its rest mass increases.

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The Ultimate Speed is c .

- Objects cannot be accelerated to velocities equal to or greater than the speed of light.
- This has been verified by experiments on electrons in particle accelerators.
→ The graph of speed vs. kinetic energy of the electrons approaches, but does not reach, c .
- According to $E = \gamma m_o c^2$, as v approaches c , γ , and therefore E , approach infinity.
- It would take an infinite amount of energy for an object to reach a speed of c .

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The Ultimate Speed is c .

Or is it?

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Can Neutrinos Move Faster Than Light?

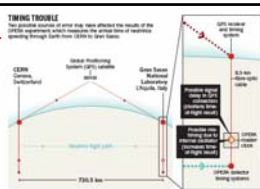
- September 2011: The OPERA experiment announced detection of the **faster-than-light neutrino anomaly**.
- They created muon neutrinos (subatomic particles) at the CERN lab and detected them in Italy, using GPS (the global positioning system) to measure the locations and times of their creation and detection.
- They found that the neutrinos appeared to travel faster than light, arriving 60.7 ns sooner than light would have, thus violating Special Relativity.
- The initial press release stated that further scrutiny and independent tests were necessary to confirm or disprove the results.

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Nature, 1 March 2012
"Is it an epic blunder or a textbook demonstration of how science should work?"

The OPERA team confirmed two possible errors:

- (1) a faulty connection from a fibre-optic cable bringing the GPS signal into the master clock, causing this clock to run slow and the travel time to seem shorter than it was;
- (2) an oscillator in the master clock was running fast; this would cause the travel time seem longer.



Timing glitches dog neutrino claim

Team admits to possible errors in faster-than-light finding.

BY FREDERIC BARNIER, PARIS

It is an epic blunder or a textbook demonstration of how science should work? To some physicists, the OPERA Oscillation Project with Emulsion tracking Apparatus collaboration does not credit for disclosing possible errors in its paradigm-challenging measurement of neutrinos travelling faster than light. "I think we did the right thing to continue to investigate," says Christian Andreu of the Institute of Theoretical Physics of Geneva in France, who presented the original results and notes that the collaboration had spent six months checking its result before its announcement last September.

To others, the revelation shows that the OPERA team went public too soon with its claim that neutrinos from CERN's European particle physics laboratory near Geneva in Switzerland were beating Albert Einstein's absolute limit on the speed of light as they traversed the 730 kilometres to the OPERA detector at the underground Gran Sasso National Laboratory near Aosta, Italy. "I think we oversteered," says a spokesman of the National Institute of Nuclear Physics in Padova, Italy, an OPERA member who initially refused to sign a paper about the result. "Maybe we should have been more cautious and done more checks."

On 22 February, OPERA team members reported two possible sources of error in the

experiment. The initial result suggested that the neutrinos were reaching the detector 60 nanoseconds faster than the speed of light would allow. Both potential errors would affect the neutrino's arrival time, as measured by OPERA's master clock (see "Timing trouble"). The first is a faulty connection at the point at which the light from a fibre-optic cable brings a worldwide Global Positioning System (GPS) signal into the master clock. The fault could have skewed the GPS signal, causing the master clock to run slow and thus causing the neutrino's travel time to appear shorter than it actually was.

"It is a subtle effect," says Andreu, and one that was evident only when the team examined master measurements of signals passing through the connection. Tests of the timing system showed up a second, opposing effect: an oscillator within the master clock that kept time between the arrival of non-synchronous signals was running fast. That would have made the neutrino's travel time seem longer.

The collaboration says that it has not yet worked out the magnitude of these effects. Andreu says that the most serious might be at the result and the possibility of systematic and hidden, the collaboration might not discover the potential error promptly. The OPERA team plans to correct the data and repeat the experiment after CERN's neutrino beam is switched on again in March, following a winter break.

Two independent checks of the measurements are also being considered. One, at Spain's Teide to El Teide (TET) neutrino experiment, would offer valuable insight into the results of the OPERA data, but may now prove harder to find, as international co-operation, says Chang Ren-Bang, a physicist at Tsinghua University in New York, that involves the Italian Institute of Nuclear Physics (INFN) experiment, which has neutrinos from Fermilab in Batavia, Illinois, to an underground detector in northern Minnesota, will proceed, at a cost of about \$100 million. "We never said also to have multiple measurements," says INFN's co-speechman Bob Phillips.

Large theories of physics at the highest level of the Institute in Lausanne, says that the admission by OPERA is a good one to have made, about one that should have been avoided. "The positive message of the episode is that we should have been checked and double-checked any public announcement," he says. ■