Interpreting Stationary Wave Nonlinearity

Lei Wang, Paul J. Kushner   Department of Physics, University of Toronto   lei@atmosp.physics.utoronto.ca

Introduction

The stationary wave field, the zonally asymmetric part of the climatological mean flow, is an important part of the atmospheric general circulation, plays a significant role in the eddy driven zonal mean circulation, and is key to understanding climate variability and change on regional scales (e.g. Held et al. 2002 and references therein). Stationary wave theory has progressed from a focus on the simple linear response to thermal and orographic forcing to a quantitative framework that accounts for nonlinear stationary wave effects, transient eddy effects, and sensitivity to the zonal mean.

"Stationary wave nonlinearity", also known as "stationary nonlinearity" (SNL) or "nonlinear self-interaction" (e.g. Ting et al. 2001 and references therein), which arises primarily through advective terms in the equations of motion and becomes more important for larger amplitude stationary waves. Classically, linear stationary wave models have diagnosed the importance of stationary wave nonlinearity by imposing the stationary wave nonlinearity as an external forcing (e.g. Valdes-Hoskins 1991, other refs). But weakly nonlinear techniques have also been developed that predict the stationary wave nonlinearity as part of a stationary wave calculation (Ringler and Cook 1997, Ting and Yu 1998, Held et al. 2002).

Motivation and Method

Our aim in this study is to improve our dynamical understanding of stationary wave nonlinearity and to evaluate the weakly nonlinear stationary wave modelling technique of Ting and Yu (1998) and Held et al. (2002). We do so in the classical setting of barotropic QG dynamics on the sphere, in which we will see that stationary wave nonlinear effects primarily involve stationary Rossby wave reflection at critical latitudes (e.g. Nigam and Held 1983, other refs).

Models and Wave Activity Analysis

The barotropic QG equation on the sphere is solved to obtain the linear and nonlinear stationary waves using a pseudospectral model (T85 or 1.4° resolution) from the NOAA/GFDL Flexible Modelling System;

The nonlinear equation: $\bar{\ }$: time mean, ': deviation from time mean, [·]: zonal mean, *: zonally asymmetric component

$$\frac{\partial\zeta}{\partial t}+\nabla\left\{\left(f+\zeta+\frac{f_0 h}{H}\right)\vec{u}\right\}+\frac{1}{\tau_Z}\left([\zeta]-\zeta_{ref}\right)+\frac{1}{\tau_E}\zeta^*+\nu\nabla^8\zeta=0$$

where $h=h_0\exp\left(-\frac{(\lambda-\lambda_0)^2}{(\Delta\lambda)^2}-\frac{(\theta-\theta_0)^2}{(\Delta\theta)^2}\right)$, $\lambda_0=90°, \theta_0=30°$, $\Delta\lambda=\Delta\theta=22.5°$, h_0=2000m, and the reference state ζ_{ref} corresponds to $U_{ref}=25\cos\theta-30\cos^3\theta+300\sin^2\theta\cos^6\theta$, $V_{ref}=0$. The damping time scale for zonal mean τ_Z= 5 days, for waves τ_E= 5 days in the strongly damped (**SD**) case and $\tau_E=\infty$ in the weakly damped (**WD**) case.

The linear equation:

$$\frac{\partial\zeta^*}{\partial t}+\nabla\left\{\left(f+Z+\frac{f_0[h]}{H}\right)\vec{v}^*+\left(\zeta^*+\frac{f_0 h^*}{H}\right)U\right\}+\frac{1}{\tau_E}\zeta^*=F^*$$

where Z and U are prescribed time-independent and longitude-independent vorticity and zonal wind fields. τ_E= 5 days in SD case and τ_E= 40 days in WD case. The right-hand-side term F^* is a prescribed "forcing" that represents the zonally asymmetric component of the eddy vorticity flux convergence (from either or both of the stationary and transient waves).

$$F^*=-\nabla\left(\left(\bar{\zeta}^*+\frac{f_0 h^*}{H}\right)\vec{\bar{u}}^*+\overline{\zeta'\vec{u}'}\right)^*$$

Takaya-Nakamura (1997) wave activity flux, which for barotropic flow linearized about a zonally symmetric basic state reduces to

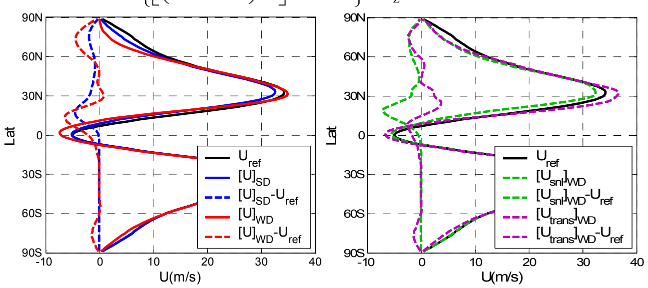
$$\mathbf{W}=\frac{[U]\cos\phi}{2|\mathbf{U}|}\left(v^{*2}-\frac{\psi^*}{a\cos\phi}\frac{\partial v^*}{\partial\lambda},\quad -u^*v^*-\frac{\psi^*}{a}\frac{\partial v^*}{\partial\phi}\right)$$

This wave activity flux is parallel to the local group velocity of stationary Rossby wave in the WKB limit.

Results

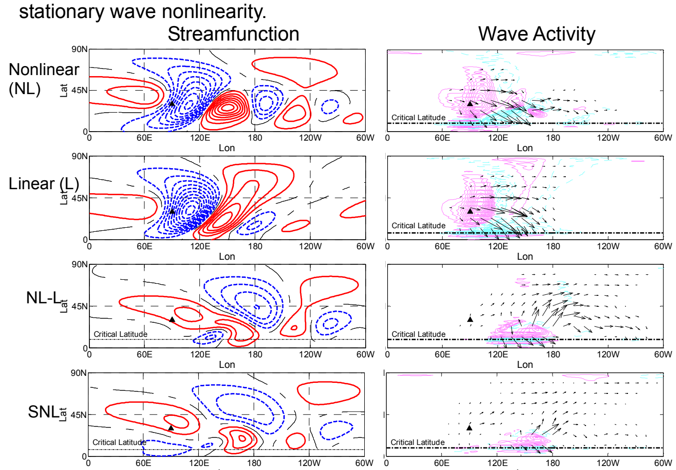
The zonal mean zonal wind responses to a 2 km high topography in strongly / weakly damped cases and the further decomposition according to the zonal mean balance equation:

$$\nabla\left\{\left[\left(\bar{\zeta}^*+\frac{f_0 h^*}{H}\right)\vec{\bar{u}}^*\right]+\left[\overline{\zeta'\vec{u}'}\right]\right\}+\frac{1}{\tau_Z}\left(\left[\bar{\zeta}\right]-\zeta_{ref}\right)=0$$



Strongly Damped Case

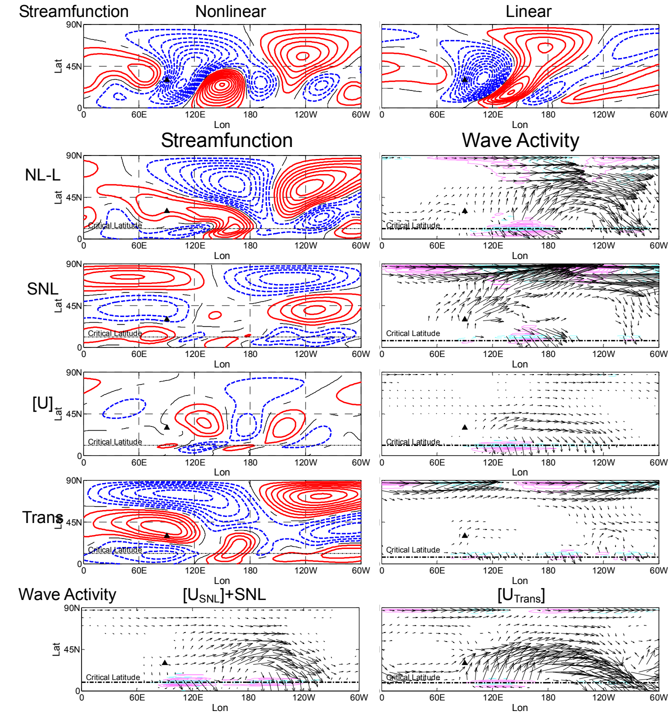
Nonlinear stationary wave can be reproduced by the linear model with prescribed nonlinear zonal mean basic state and zonally asymmetric stationary wave nonlinearity.



- Stationary wave nonlinearity explains most of the difference between nonlinear and classic linear solution;
- Only stationary wave nonlinearity is responsible for the wave reflection near the critical latitude (around 150°E).

Weakly Damped Case

A weak damping (τ_E = 40 days) on waves is necessary in the linear model to obtain the linear solution because there would be resonant modes if transients were allowed to freely evolve in the linear model.



- Stationary wave nonlinearity, transients, zonal mean responses all contribute to the difference between nonlinear and classic linear solution;
- None of the three factors above **alone** results in wave reflection near the critical latitude (around 150°E);
- The zonally asymmetric stationary wave nonlinearity plus its zonal mean component induced zonal mean response leads to significant wave reflection;
- The zonal mean response to transients, which sharpens the jet, also generates pronounced wave reflection.

Summary

Stationary wave nonlinearity is important in explaining the difference between linear and full nonlinear stationary wave responses to topographic forcing, although there are relatively smaller but still significant impacts from transients and the changes in the zonal mean basic state. Only stationary wave nonlinearity leads to substantial reflection according to the wave activity analysis in the strongly damped case. While the presence of transients increases the complexity of mechanism in the weakly damped case, stationary wave nonlinearity is still essential to the wave reflection. Our investigation also reveals that weakly nonlinear techniques work fairly well to capture stationary wave nonlinearity (not shown here).

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