The Stationary Wave Response to Topography in a Barotropic Model

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1. Introduction

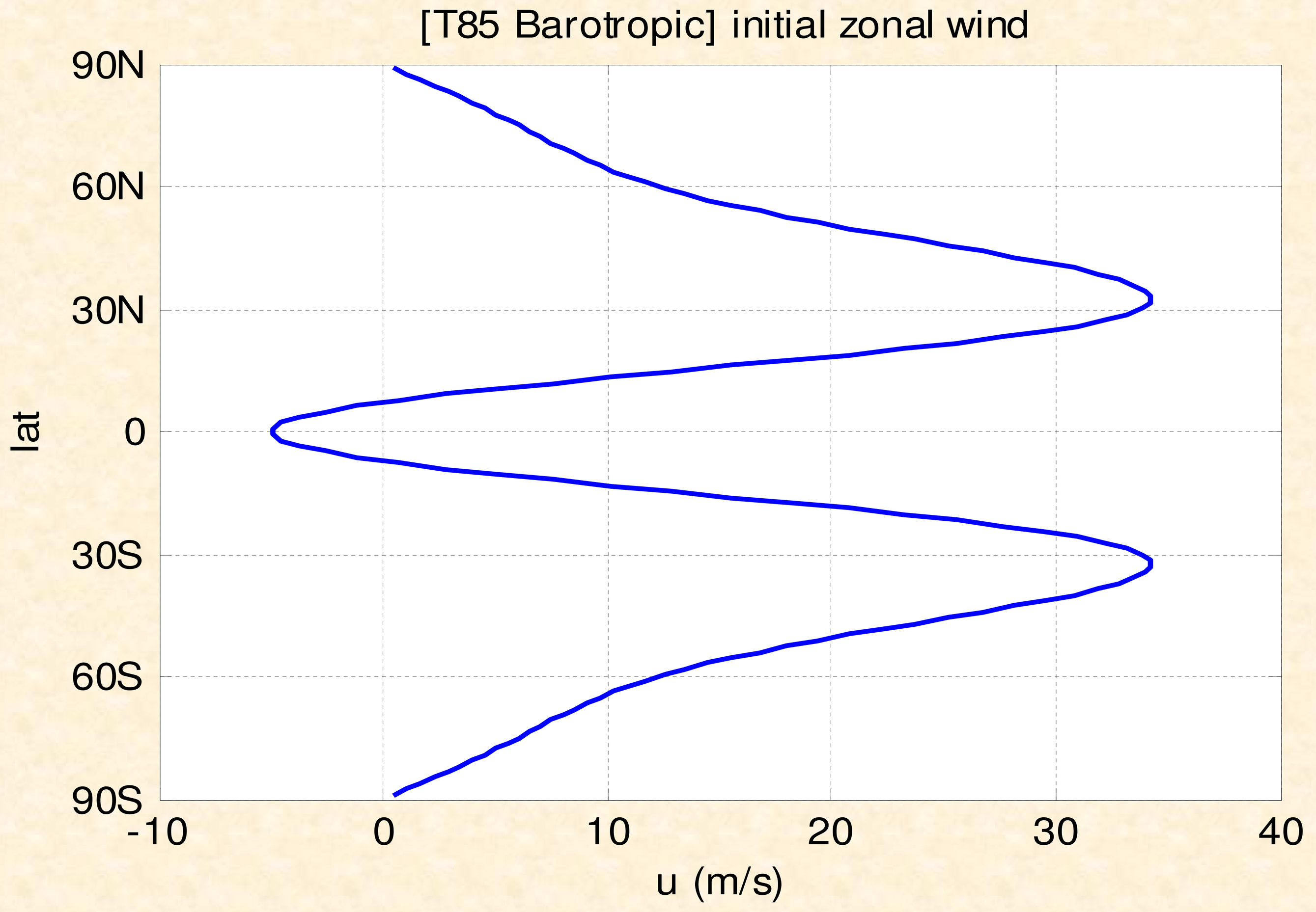
The stationary wave, defined as the zonally asymmetric component of the atmospheric climatology, is generated by the zonal varying lower boundary or other forcings. The classical approach to study the stationary wave is to build a theory for linearizing perturbations to a zonally symmetric basic state (e.g., Charney and Eliassen, 1949 and Hoskins and Karoly, 1981). More recent investigations consider zonally varying basic states (e.g., Valdes and Hoskins 1991; Ting and Yu 1998). We take advantage of the simplicity of a barotropic stationary wave model to clarify which factors are important to eliminate the difference between the linearized and nonlinear models.

2. The barotropic model

- T85 barotropic model on the surface of the sphere base on GFDL dynamical core.
- Rayleigh friction on time-scale of 5 days.
- Prescribed zonal mean basic state for full nonlinear model.

$$u = 25\cos\theta - 30\cos^3\theta + 300\sin^2\theta\cos^6\theta, \quad v = 0$$

[T85 Barotropic] initial zonal wind



- Topography is taken as in Hoskins and Karoly (1981):

$$h_B = 2000\exp\left(-\frac{(\theta-\theta_0)^2 + (\lambda-\lambda_0)^2}{(22.5°)^2}\right)\cos^2\theta$$

3. Results

a) Zonally Symmetric Basic State

- The full nonlinear model solves the potential vorticity equation on the surface of the sphere:

$$\frac{\partial \zeta_{nl}}{\partial t} = -\nabla\cdot\left\{\left(f + \zeta_{nl} + \frac{f_0}{H}h_B\right)\underline{v}_{nl}\right\} - \frac{1}{\tau}(\zeta_{nl} - \zeta_e) \qquad (1)$$

where τ=5 days, λ_0=90°E, θ_0=30°N in h_B, and ζ_e is the prescribed zonal mean field.

- The zonal mean flow of the above equation satisfies:

$$0 = -\nabla\cdot\left\{\left[\left(\bar{\zeta}_{nl}^* + \frac{f_0}{H}h_B^*\right)\bar{\underline{v}}_{nl}^*\right] + \left[\overline{\zeta_{nl}'\underline{v}_{nl}'}\right]\right\} - \frac{1}{\tau}\left(\left[\bar{\zeta}_{nl}\right] - \zeta_e\right) \qquad (2)$$

where overbar " ¯ " represents the time mean state, prime " ' " the deviations from the time mean state, " [·] " the zonal mean, and " * " the zonal asymmetric part (following Peixoto and Oort, 1992), the subscript "*nl*" represents nonlinear run.

- The stationary wave field of equation (1) solves:

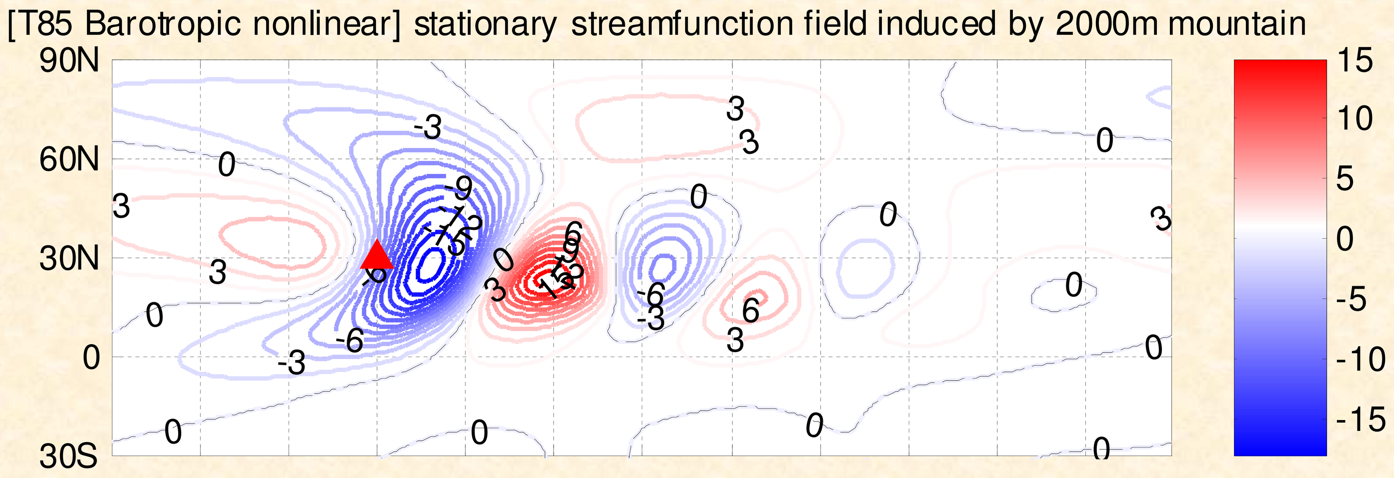
$$0 = -\nabla\cdot\left\{\left(f + \left[\bar{\zeta}_{nl}\right] + \frac{f_0}{H}[h_B]\right)\bar{\underline{v}}_{nl}^* + \left(\bar{\zeta}_{nl}^* + \frac{f_0}{H}h_B^*\right)\left[\bar{\underline{v}}_{nl}\right] + \left(\left(\bar{\zeta}_{nl}^* + \frac{f_0}{H}h_B^*\right)\bar{\underline{v}}_{nl}^*\right)^* + \overline{\zeta_{nl}'\underline{v}_{nl}'}^*\right\} - \frac{1}{\tau}\bar{\zeta}_{nl}^* \qquad (3)$$

- Aiming at obtaining the stationary wave response to the topographic forcing, we linearize equation (1) about the prescribed zonal mean equilibrium state of full nonlinear model.

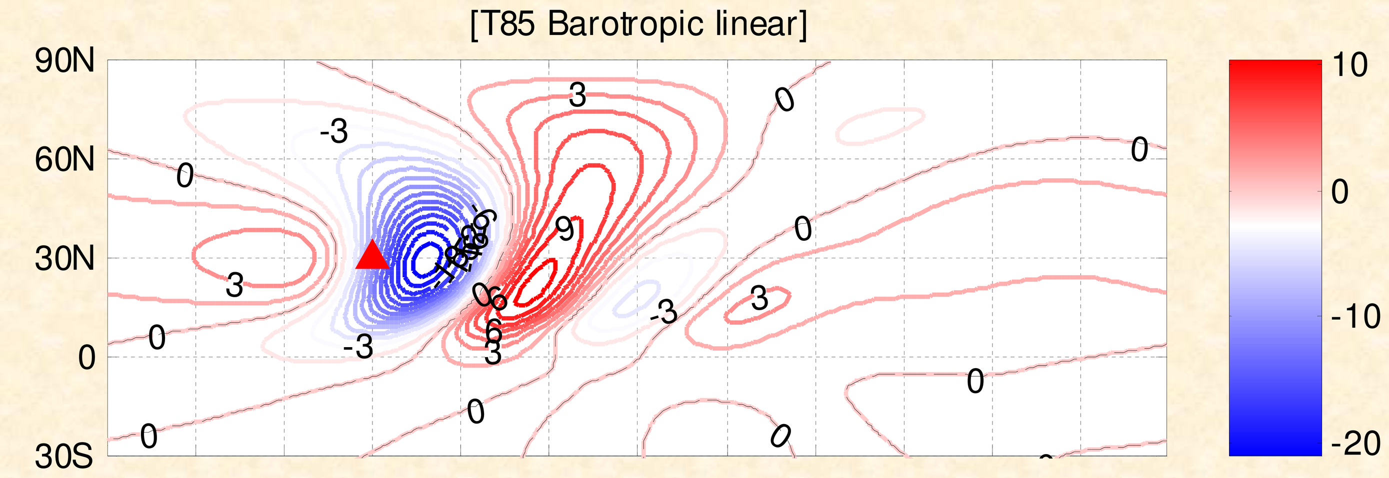
$$\frac{\partial \zeta_l}{\partial t} = -U_e\frac{\partial}{\partial x}\left(\zeta_l + \frac{f_0}{H}h_B^*\right) - v_l\frac{\partial}{\partial y}\left(f + [\zeta_e] + \frac{f_0}{H}[h_B]\right) - \frac{1}{\tau}\zeta_l \qquad (4)$$

- The streamfunction responses to the orography in linear and nonlinear models and their difference are shown below (in $10^6 m^2 s^{-1}$):

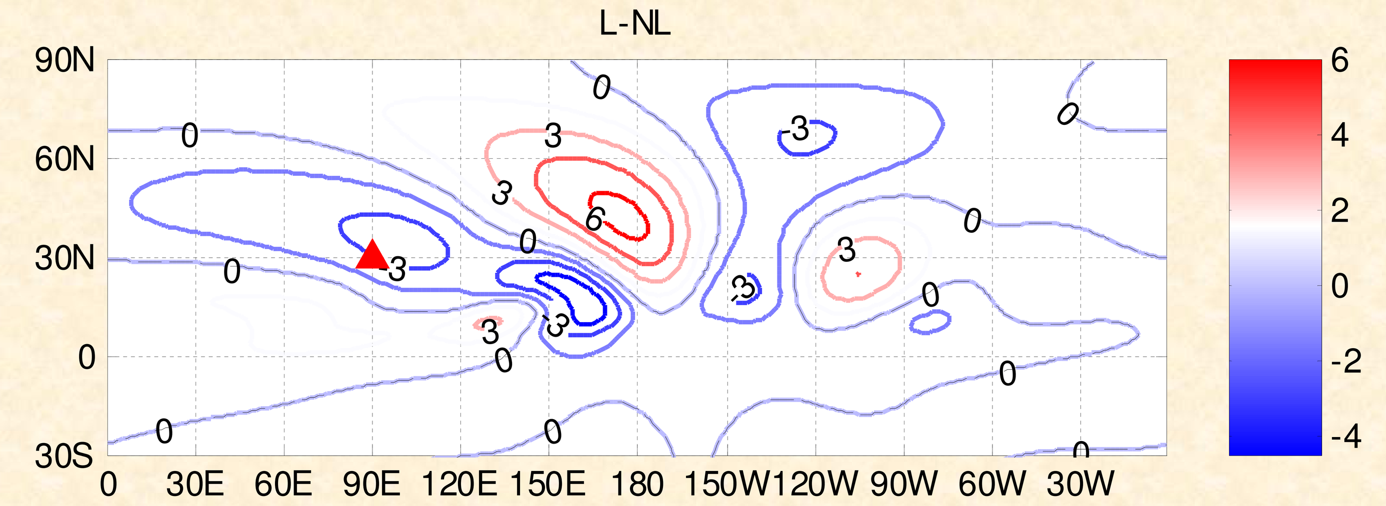
[T85 Barotropic nonlinear] stationary streamfunction field induced by 2000m mountain



[T85 Barotropic linear]



L-NL

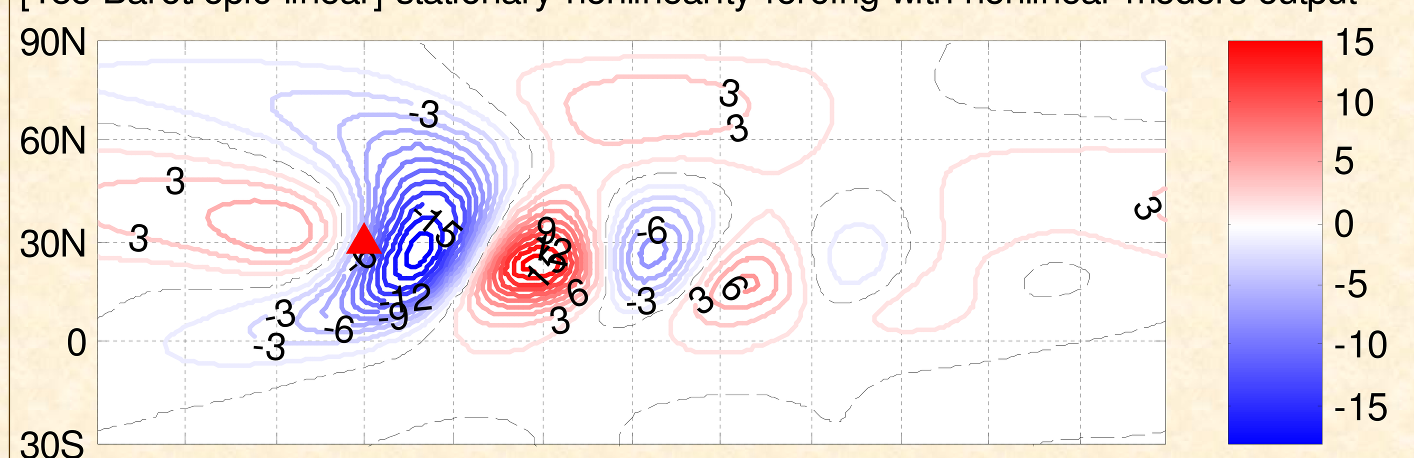


- The linear response is different from the nonlinear one primarily due to the lack of nonlinear interaction of the stationary wave with itself, that is, the *stationary nonlinearity* (Ting, 1994).

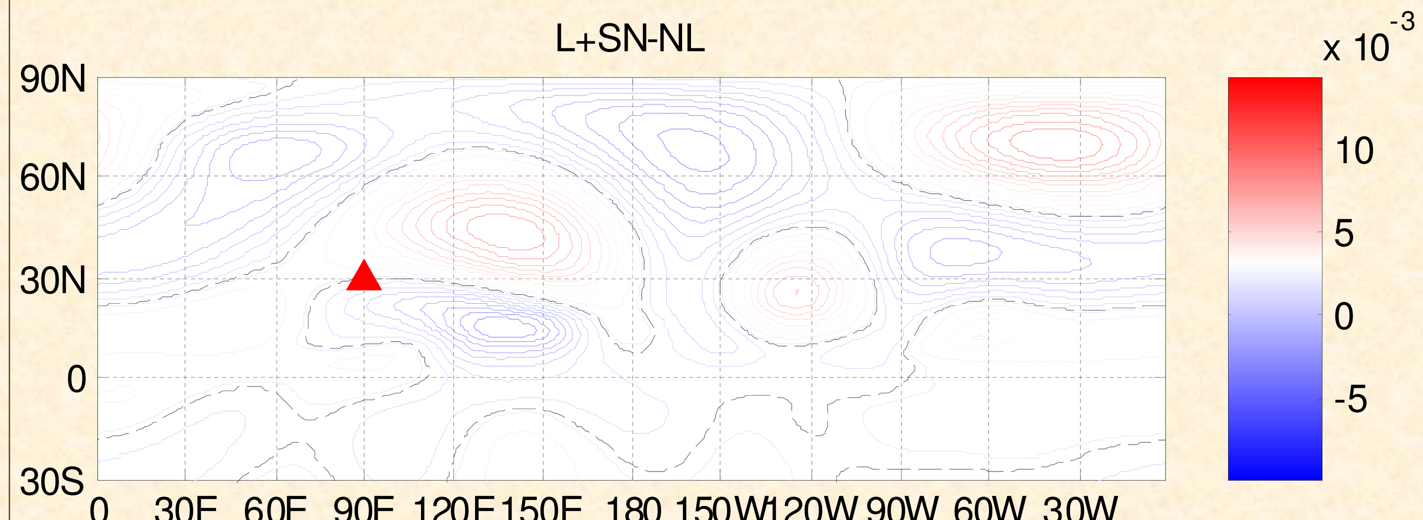
$$\nabla\cdot\left\{\left(\bar{\zeta}_{nl}^* + \frac{f_0}{H}h_B^*\right)\bar{\underline{v}}_{nl}^*\right\}^*$$

- $\nabla\cdot\left\{\overline{\zeta_{nl}'\underline{v}_{nl}'}\right\}^*$, the *transient momentum flux divergence*, describes the impact of time-varying waves on the stationary wave. It is strongly damped and several order less than the stationary nonlinear, thus negligible in the barotropic stationary wave model.
- Another factor causing the difference is zonal mean response to the orography, which can be counted in by taking the zonal mean fields of the steady state of nonlinear model as the basic state of (4).
- Therefore, the linear stationary wave model response to the orography can be made consistent with that in the nonlinear model by adding stationary nonlinearity forcing and including zonal mean response in winds and vorticity.

[T85 Barotropic linear] stationary nonlinearity forcing with nonlinear model's output



L+SN-NL



b) Zonally Asymmetric Basic State

- Furthermore, we consider the linearization about a zonally varying basic state, which is the steady state of the nonlinear model with the mountain at (90°E, 30°N).
- The second mountain is added to the topography 30° east to the first one, with the same height and shape. Then the time evolution of the vorticity corresponding to the second mountain is given by:

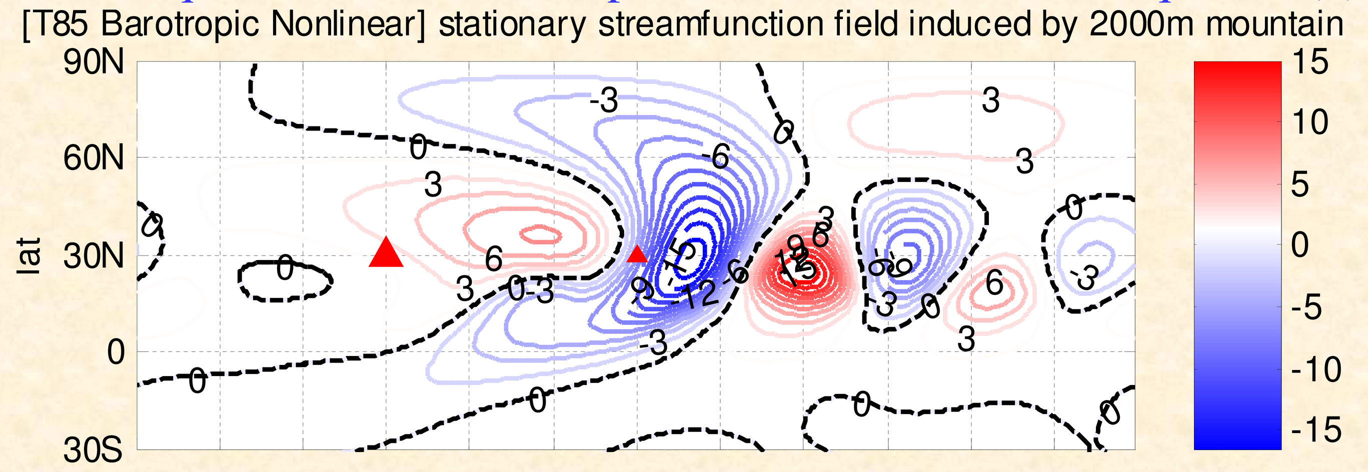
$$\frac{\partial \zeta_2}{\partial t} = -\nabla\cdot\left\{\left(f + \zeta_1 + \frac{f_0}{H}h_{B1}\right)\underline{v}_2 + \left(\zeta_2 + \frac{f_0}{H}h_{B2}\right)\underline{v}_1 + \left(\zeta_2 + \frac{f_0}{H}h_{B2}\right)\underline{v}_2\right\} - \frac{1}{\tau}\zeta_2 \quad (5)$$

- The above equation is linearized as

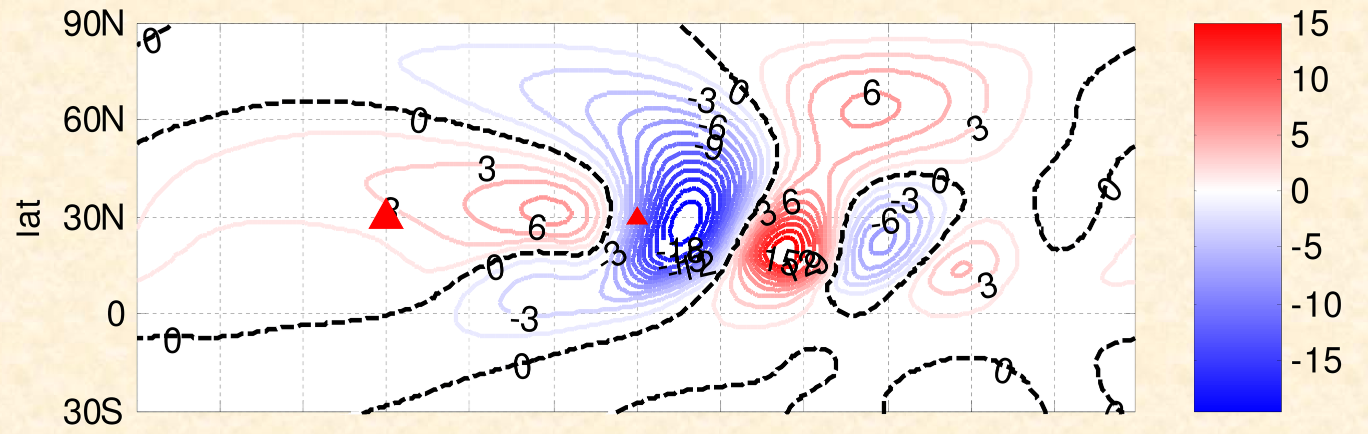
$$\frac{\partial \zeta_2}{\partial t} = -\nabla\cdot\left\{\left(f + \zeta_1 + \frac{f_0}{H}h_{B1}\right)\underline{v}_2 + \left(\zeta_2 + \frac{f_0}{H}h_{B2}\right)\underline{v}_1\right\} - \frac{1}{\tau}\zeta_2 \qquad (6)$$

where quantities with subscript "1" are calculated from equation (1).

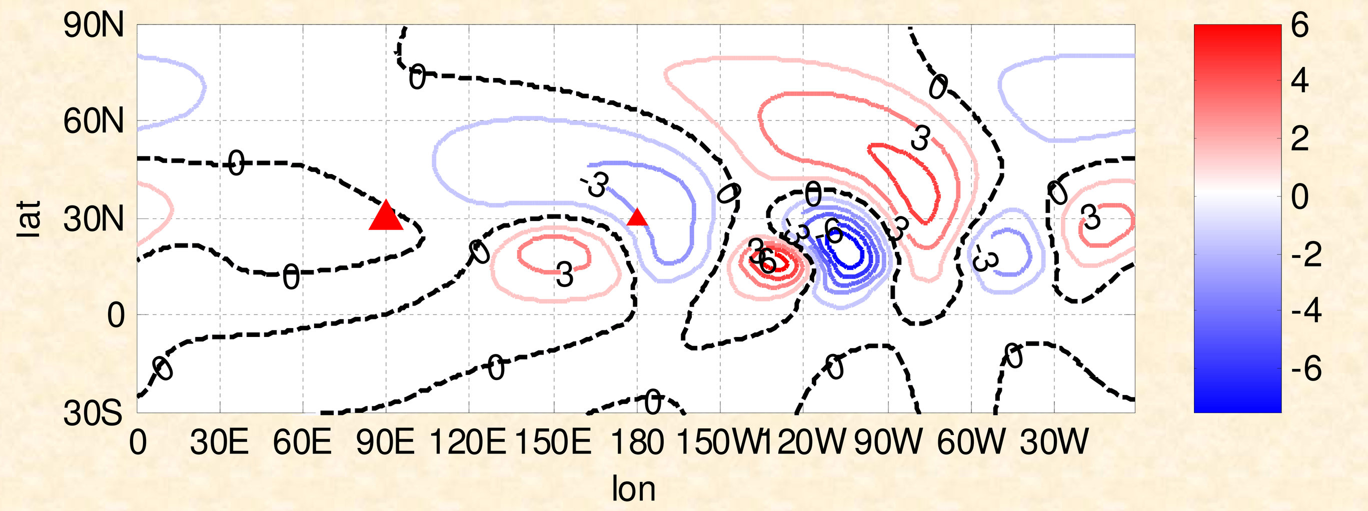
[T85 Barotropic Nonlinear] stationary streamfunction field induced by 2000m mountain



Linear

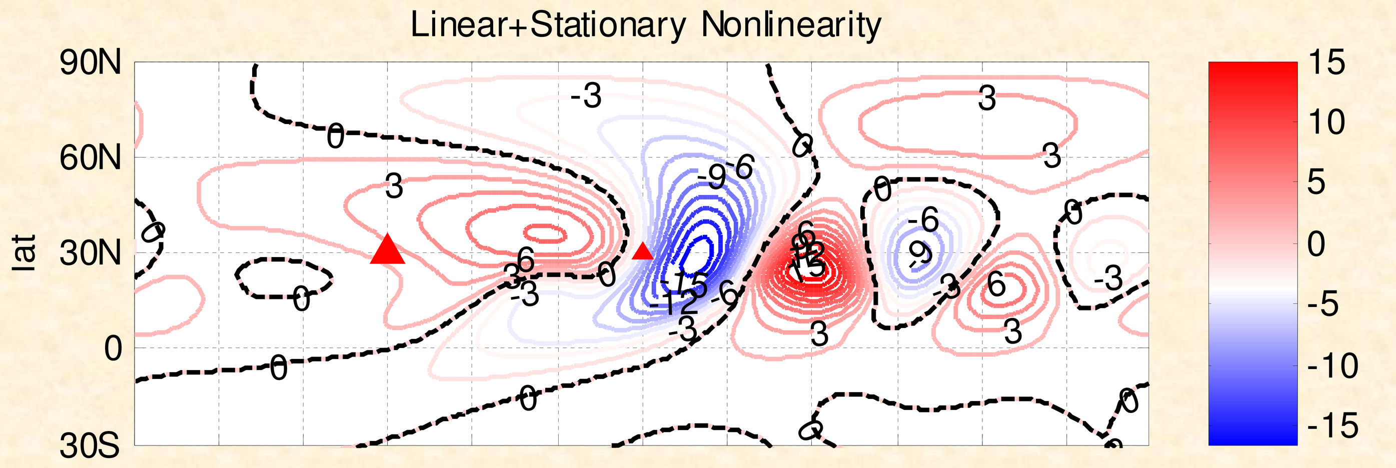


L-NL



- The term with subscript "2" in equation (5) can be calculated from the difference of one/two-mountain runs, and then treated as a forcing on the RHS of equation (6). As a result, the nonlinear stationary wave response of the zonally varying flow to the topography can be reproduced by the model linearized about the zonally asymmetric state forced by the stationary nonlinearity.

Linear+Stationary Nonlinearity



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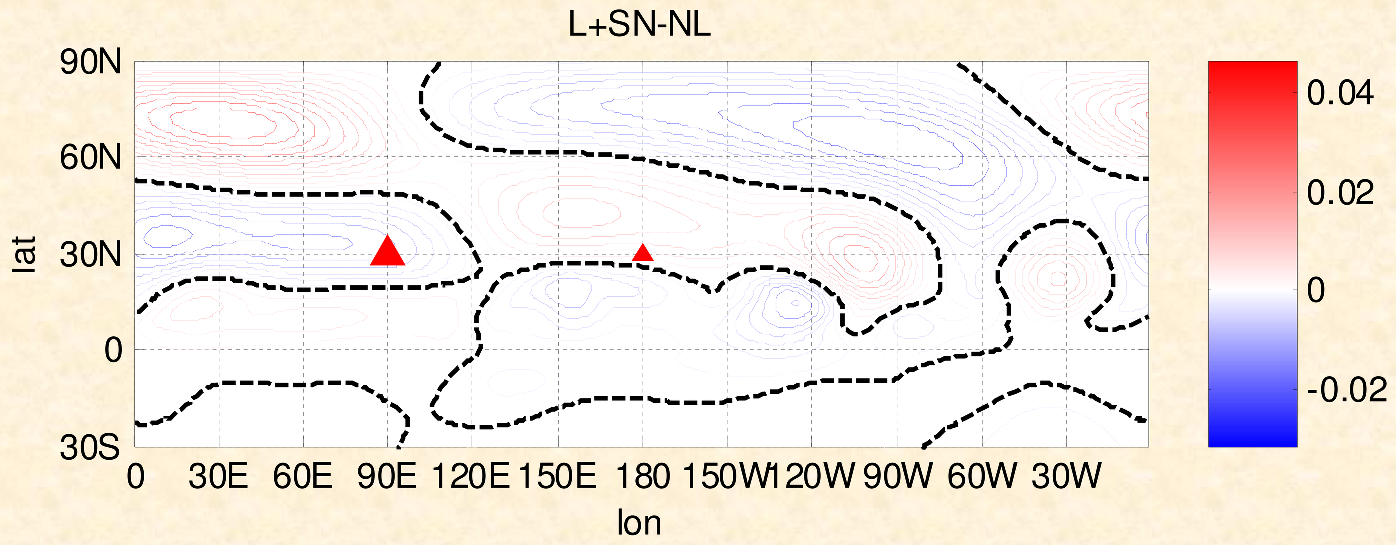

4. Conclusion

- The nonlinear stationary wave response to the topographic forcing with a zonally symmetric or asymmetric basic state can be reproduced by the linearized model with the *stationary nonlinearity* forcing.
- This method will be used to study the stationary wave responses to the climate change in the stratosphere/troposphere and possible interaction between them.