A Discussion of Plausible Solar Irradiance Variations, 1700–1992

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From satellite observations the solar total irradiance is known to vary. Sunspot blocking, facular emission, and network emission are three identified causes for the variations. In this paper we examine several different solar indices measured over the past century that are potential proxy measures for the Sun’s irradiance. These indices are (1) the equatorial solar rotation rate, (2) the sunspot structure, the decay rate of individual sunspots, and the number of sunspots without umbrae, and (3) the length and decay rate of the sunspot cycle. Each index can be used to develop a model for the Sun’s total irradiance as seen at the Earth. Three solar indices allow the irradiance to be modeled back to the mid-1700s. The indices are (1) the length of the solar cycle, (2) the normalized decay rate of the solar cycle, and (3) the mean level of solar activity. All the indices are well correlated, and one possible explanation for their nearly simultaneous variations is changes in the Sun’s convective energy transport. Although changes in the Sun’s convective energy transport are outside the realm of normal stellar structure theory (e.g., mixing length theory), one can imagine variations arising from even the simplest view of sunspots as vertical tubes of magnetic flux, which would serve as rigid pillars affecting the energy flow patterns by ensuring larger-scale eddies. A composite solar irradiance model, based upon these proxies, is compared to the northern hemisphere temperature departures for 1700–1992. Approximately 71% of the decadal variance in the last century can be modeled with these solar indices, although this analysis does not include anthropogenic or other variations which would affect the results. Over the entire three centuries, ~50% of the variance is modeled. Both this analysis and previous similar analyses have correlations of model solar irradiances and measured Earth surface temperatures that are significant at better than the 95% confidence level. To understand our present climate variations, we must place the anthropogenic variations in the context of natural variability from solar, volcanic, oceanic, and other sources.

1. INTRODUCTION

In the past two centuries, many people have hypothesized that the solar irradiance at the top of the Earth’s atmosphere varies. Recent satellite measurements confirm that such variations exist, at least on the timescale of the 11-year solar cycle [e.g., Willson and Hudson, 1991]. Most of the modeling undertaken to date, to understand these secular variations in the solar constant, have been phenomenological, offering proxies which enable the solar constant data to be fit (see, for example, Lean [1991] for a review). Although the models do not answer questions concerning the basic cause of secular solar constant variations, they do allow us to examine the photospheric manifestations of these variations.

To date, most of the solar constant secular variations observed (which only includes a timescale on the order of a decade) have been associated with photospheric blemishes (dark sunspots, bright faculæ, and bright network). At present, there seem to be very few attempts to understand potential secular trends. Phenomena in the Sun which could lead to irradiance variations on the timescale of decades to centuries were explored by Endal et al. [1985]. One view of active region physics [Schatten and Mayr, 1985, p. 1060] did suggest a positive correlation of solar constant with solar activity when they stated “Thus the (active region) process effectively transfers heat outward, aiding the Sun to shed its luminosity. If active regions have any effect on the solar luminosity, it should be a weak positive correlation.” The view undertaken by these authors was that active regions are distinguished from the background photosphere by the influence of the magnetic field, which allowed a larger-scale flow pattern to develop (larger than granulation and supergranulation), and this pattern manifested itself in sunspots, where downflows were present, and faculæ where upflows occurred. The zeroth-order effect was thought to be small as the two energies balance to zeroth order; however the first-order effect allowed heat (and energy) to be transferred outward. These effects were thought to be the origin of the positive correlation of solar activity with solar irradiance variations.

Even if we can understand the solar cycle correlation with activity, these features may be merely “photospheric blemishes” and may not have a great influence on the longer timescale “river of solar luminosity” flowing outward from the Sun’s interior, but rather primarily serve only to divert the flow and/or temporarily store and release minor amounts of this vast energy flow. The perturbations from active regions may not necessarily extend to the deep interior to influence very long timescale solar luminosity and solar constant variations. To understand the long-term secular variations, the Sun might need to be viewed on a larger, more global scale, with global observations (e.g., solar rotation, solar diameter, etc.).

On the timescale of decades to centuries, four classes of models exist which postulate different variations of the Sun’s
output. These models can be called the "constant quiet Sun model," the "solar diameter model," the "activity envelope model," and the "umbra/penumbra variations model." The constant quiet Sun model postulates that the solar irradiance has only an 11-year cycle and all radiation changes can be explained by active features. Since all solar minima are the same in these models, it is called the constant quiet Sun model. Foukal and Lean [1990] and Schatten and Orozco [1990] present models of this type. The solar diameter model uses the solar diameter or its time rate of change as a proxy for solar irradiance variations. Some controversy still exists about the history of the solar diameter variations so this model will not be considered further here. The activity envelope model postulates that long-term solar irradiance variations follow the envelope of solar activity such as the Gleissberg cycle, so that solar minima irradiance varies over time (see, for example, Eddy [1976] or Reid [1991]). The umbra/penumbra (U/P) variations models is so called because early models of this class by Nords [1955] and Hoyt [1979a] used sunspot structure expressed as the ratio of umbral areas to penumbral areas as a proxy measure of solar irradiance. Subsequent studies have used the solar equatorial rotation rate and the sunspot cycle length to derive very similar models. The U/P variations model and activity envelope model are similar except they are out of phase with each other with variations occurring ~20 years earlier in the U/P variations model. This paper presents evidence in support of the U/P variations model. We argue that the solar indices used in the U/P variations model are proxy indicators of long-term secular changes in convective energy transport. Although changes in the Sun’s convective energy transport are outside the realm of normal stellar structure theory (e.g., mixing length theory), one can imagine variations arising from even the simplest view of sunspots as vertical tubes of magnetic flux, which would serve as rigid pillars affecting the flow patterns by ensuring larger-scale eddies. Additional proxies for the U/P variations model are introduced here for the first time, namely the sunspot decay rate, the fraction of penumbral sunspots, the decay rate of the solar cycle, and the mean level of solar activity.

There is a variety of experimental evidence that indicates that there may be long-term irradiance variations which are not correlated with solar activity. For example, measurements of the last couple of decades have revealed trends in the equivalent widths of lines and the bisectors of lines. These observations can be interpreted as changes in the temperature gradient in the photosphere which says the convective energy transport is secularly changing in a manner not correlated with solar activity. These convective flux changes imply there may be an underlying secular change in irradiance in addition to the already identified 11-year cycle.

Evidence for changes in solar convective flux on longer timescales requires the use and interpretation of other solar proxies. In the past 13 years, several authors have postulated secular solar irradiance variations which are not correlated with solar activity. Hoyt [1979a, 1990], Gilliland [1982], Frisk-Christensen and Lassen [1991], and others all independently arrived at the conclusion the Sun’s irradiance increased from the late 1800s to a peak in the 1930s or 1940s. In the ideal case a full physical theory, starting from the basic equations for magnetohydrodynamics, could be developed to explain the observations of the above authors. Such a theory, however, is not yet available. Therefore, in section 2, plausible reasons for the relationship of sunspot decay rates, sunspot structure, solar cycle lengths, and other proxy indices will be developed. The basic approach is to show that many of these indices can be related mathematically to sunspot decay rates. Once a relationship to sunspot decay rates is developed, it is deduced that there are changes in convective velocities, convective energy transport, and hence solar irradiance. Five irradiance models will be combined to form a composite solar irradiance model for 1700-1992.

In section 3 we review various experiments to see if any independent data exist which would support the modeled secular changes in solar irradiance.

Because the model irradiance variations may be of interest to climatologists who are seeking explanations for climatic change, section 4 is devoted to examining the temperature changes on the Earth which may be induced by the changes in solar output. In this regard, we will examine the issue of whether the Earth can be used as a radiometer, which requires that we pay attention to the stability of climate in the absence of external forcing. In the final section we will comment upon the uncertainties in our understanding of the Sun and climate and discuss directions for future research.

2. SECULAR IRRADIANCE VARIATIONS CAUSED BY CHANGES IN CONVECTIVE ENERGY TRANSPORT

Variations in the Sun's spectral irradiance (S₀) can be written [Oster et al., 1981] as

\[ \Delta S(t) = S_0(t) - S = \int \int \Delta I_{ph}(\lambda, A, t) \, d\lambda \, dA \]  

(1)

where \( \lambda \) is the wavelength, \( A \) is a unit area on the solar disk, \( \Delta I_{ph} \) represents variations in the photospheric intensity, and \( t \) is time. Most solar irradiance variations are assumed to arise from active features on the Sun such as sunspots, faculae, and the active network. Equation (1) can be expanded in terms of solar limb darkening as follows:

\[ \Delta S_0(t) = \int \int \left[ a_p(\lambda) + b_p(\lambda) \mu + c_p(\lambda) \mu^2 \right] C_{act} \, d\lambda \, dA \]  

(2)

where \( a_p, b_p, \) and \( c_p \) are the limb darkening constants [e.g., Allen, 1976]. \( C_{act} \) is the relative contrast and equals zero in the absence of contrast features such as sunspots and faculae. Expressions for \( C_{act} \) are given by Schatten [1988] and will not be considered further in this paper. Instead, we are concerned with processes which could lead to changes in \( a_p, b_p, \) or \( c_p \) and, in particular, are interested in proxy solar indices which would allow long-term secular changes in convective energy transport to be deduced.

A hypothetical change in convective energy transport may manifest itself by variations in the solar limb darkening, the equivalent widths of lines, and the bisectors of lines. These diagnostic measurements are limited to the last two decades and discussed in more detail in section 3.2. It is desirable to search for parameters which provide similar information over decades and centuries. Several candidate parameters are available and will be examined here. Since convection and rotation are strongly coupled in the lower convection
zone through the Coriolis force, an increase in convection may manifest itself by a change in solar rotation. The theory of solar rotation is not well developed but is discussed in section 2.1. The rate at which sunspots decay can plausibly be argued to be proportional to convective velocities. A change in sunspot decay rate would therefore be a plausible proxy for a change in convective energy transport and solar irradiance. Such changes can be shown mathematically to manifest themselves as changes in sunspot structure since penumbrae of sunspots are more readily destroyed than are their umbra. In addition, the fraction of sunspots consisting of only penumbra will also change as a result of a change in the sunspot decay rate. Section 2.2 discusses these three effects. An increased convective energy transport may cause more sunspots to appear because of the increased upward transport of magnetic flux tubes. This same increased convection may cause more rapid sunspot destruction and lead to a more rapid decay of the envelope of solar activity and hence shorter solar cycles. Section 2.3 discusses these effects.

2.1. Changes in the Equatorial Solar Rotation Rate

Theoretically, a strong coupling between rotation and convection should exist [e.g., Rudiger, 1989], with rotation generally viewed as being driven by convection, so we start our discussion with a brief look at solar rotation. If a change in solar rotation is observed, a change in convective energy transport can be expected. Rudiger indicates it is safe to make the following three comments: (1) The interaction between convection and rotation is nonlinear; (2) the interactions are strongest in the lower portion of the convection zone; and (3) the rotation tends to occur in disk-shaped isoplanes rather than cylinders. Can a change in solar rotation occur which is not accompanied by a change in convection? Any change in rotation is a persuasive indicator that the deeper levels of convection are varying and hence a variation in luminosity and irradiance is occurring.

Several authors have noted that changes in solar rotation rate occur. Because of the likely strong coupling between solar rotation and convection through the Coriolis force, these authors have argued that solar rotation can be used as a proxy measure of solar irradiance. Sakurai [1977] notes there was an increasing rate of equatorial solar rotation over solar cycles 18 to 20. Eddy et al. [1976] claim that the solar rotation rate during the Maunder minimum was 4% faster than modern values. Hoyt [1990] shows that equatorial solar rotation is high in the late 1800s and decreases to a minimum in the second quarter of the 1900s before increasing in recent years (see Figure 1).

2.2. Sunspot Structure and Sunspot Decay Rate Variations

Sunspots consist of a dark central region called the umbra which are nearly always surrounded by a less dark penumbra. The sum of the corrected umbral area (U) and the corrected penumbral area (P) gives the corrected whole spot area (W) measured in millionths of the solar hemisphere (MSH). The ratio of the umbral to whole spot areas (U/W) or umbral to penumbral areas (U/P) may be a monitor of conditions in the Sun's convective zone. Previous work on this topic is given by Hoyt [1979a, b, 1990], Brown and Price [1984], and Nordo [1955]. In the following paragraphs we will show that changes in sunspot structure can be explained by changes in the rate of decay of sunspots. Because sunspot decay rates change over time, the fraction of sunspots consisting only of penumbrae (i.e., penumbral spots) also changes. Unlike sunspot structure for which measurements stop in 1977, the fraction of penumbral spots can be updated to 1989. This proxy index will be the one used in our solar irradiance models.

Moreno-Insertis and Vazquez [1988] have measured sunspot decay rates (Dspot) using the Greenwich Royal Observatory record for 1874–1939. They find that (1) the decay rate is linear in time for 95% of the sunspots, (2) it is independent of the maximum sunspot area, and (3) it is proportional to the perimeter of the spot. Their last conclusion suggests that some property of the photosphere is controlling the decay rate of sunspots. It appears as if the photosphere is dispersing magnetic elements of the sunspot into the surrounding photosphere. Meyer et al.'s [1974] dispersal theory for sunspot decay has a decay rate proportional to a mean convective velocity. This suggests the convective velocities and convective energy fluxes are secularly changing. Alternative theories based upon subduction of sunspots or reconnection of magnetic fields also have decay rates proportional to convective velocities. Because of the complexity of the decay process, it is not yet possible to relate quantitatively a change in sunspot decay rate with a change in the convective velocity spectrum and hence with solar irradiance.

Moreno-Insertis and Vazquez (abbreviated M-V in the next few paragraphs) find that the decay rate has extreme variations of as much as 25% over several cycles. For solar cycle 13 (1899–1901), complex groups decayed linearly at the slow rate of 36 ± 2 millionths of the solar hemisphere per day (MSH/day). For cycle 16 (1923–1933) the fast decay rate was 44 ± 2 MSH/d. For isolated sunspots the decay rate varied from 16 to 20 MSH/d for these two cycles. For both types of groups, the mean variation is from 26 to 32 MSH/d. The decay rates vary over a range of ~25%. For these two cycles, umbras in complex groups decayed at a mean rate of 8.3 ± 0.5 MSH/d, but for isolated sunspots, it is 3.9 ± 0.4 MSH/d. In Figure 2 we illustrate the measured decay of sunspots in cycles 13 and 16.

If, in each cycle, one starts with identical sunspots with
Slowly Decaying Sunspot (U/W = 0.163) showing spot and umbral radii

Rapidly Decaying Sunspot (U/W = 0.202) showing spot and umbral radii

Fig. 2. Moreno-Insertis and Vazquez’s [1988] measured sunspot decay rates for cycle 13 in the 1890s (upper curve) and for cycle 16 in the 1930s (lower curve) are illustrated. The hatched areas are penumbrae, and the black areas are umbrae. Note that more penumbral sunspots are predicted for cycle 13 than for cycle 16.

Sunspot Structure for Sunspots with slow and fast decay rates

Identical structure, then over time both the areas and structure of two sunspots will diverge. Figure 3 provides numerical simulations of sunspot structure using M-V’s measured decay rates for these two cycles. The final point on each curve is equivalent to the cumulative mean sunspot structure measured over many days and corresponds to the time when the sunspot disappears. The simulated values of U/W using M-V’s decay rates are 0.150 and 0.185 for these cycles compared to measured values of 0.165 and 0.182. Comparisons of measured and simulated values of U/W are complicated by several factors:

1. Because of solar rotation, sunspots are not measured throughout their life but may enter or leave the solar disk at any stage in their lives. Using many years of data to derive a value for U/W, as we do in this study, appears to minimize this sampling problem.

2. The growth phase of sunspots is neglected in the above analysis. Since sunspots grow rapidly and decay slowly, most of their life is spent in the decay phase. This helps minimize the contribution of the growth phase to the measured U/W values and is another contributor to the successful agreement in the last paragraph.

3. The simulations assume the umbral decay rates are constant from cycle to cycle. M-V’s umbral decay rates tend to support this conclusion since 4 out of 5 cycles have umbral decay rates within one standard deviation of each other for both isolated sunspots and for complex groups. A time variation in umbral decay rates would complicate, but not invalidate, our modeling since it can either improve or degrade the match between model and measurements.

U/W can be written in terms of umbral decay rate (D_umbra) and spot decay rate (D_spot) for a sunspot lasting N days:

$$\frac{U}{W} = \frac{\sum_{t=1}^{N} U_t}{\sum_{t=1}^{N} W_t} = \frac{\sum_{t=1}^{N} (U_0 - D_{umbra}t)}{\sum_{t=1}^{N} (W_0 - D_{spot})}$$

$$= \frac{2U_0 - D_{umbra} - ND_{umbra}}{2W_0 - D_{spot} - ND_{spot}}$$

where t is the time in days and one measurement per day is made. U_0 and W_0 are the initial umbra and spot sizes. For cycles 13 and 16 for isolated sunspots, (3) gives a mean U/W values of 0.154 and 0.185 compared to the numerical simulations of 0.150 and 0.185. Not only can secular sunspot structure variations be explained by secular variations in the decay rate of sunspots, but the process can be reversed to derive sunspot decay rates from the more extensive sunspot structure measurements. Using the approximate daily total mean properties for sunspots over the last century based
upon Allen’s tables and M-V’s observations (i.e., $D_{\text{umbra}} = 6.0 \pm 0.4$ MSH/d, $U_0 = 138$ MSH, $W_0 = 728$ MSH, and $N = 22$ days), the sunspot decay rate in MSH/day can be expressed as

$$D_{\text{spot}} = \left[ 63.3 - \frac{(6.0 \pm 0.4)}{U/W} \right]$$

This equation gives a mean value of $D_{\text{spot}}$ over the previous century of 28.8 MSH/day. Derived values of $D_{\text{spot}}$ are plotted in Figure 4 for those active years when the annual mean umbral areas exceed 100 MSH. The derived values of $D_{\text{spot}}$ range from ~15 to 34 MSH/d and follow the same temporal form as the measured values, also shown in Figure 4.

A change in the rate of sunspot decay has additional consequences, which are apparent in Figure 2. Near the end of the life of a sunspot, it often appears only as a penumbral spot, the umbra having already vanished. For slowly decaying sunspots the penumbral spots last longer than for rapidly decaying sunspots. Penumbral spots are more stable and common when sunspot decay rates are low. Therefore the fraction of sunspots consisting only of penumbrae should vary as a function of time. Figure 5 shows the time variation of the fraction of penumbral spots plotted inversely and overlaid on the sunspot structure values. Both curves are similar. Sunspot structure shows several discontinuities in the early years that almost vanish in the fraction of penumbral spot curve. Using the Rome Observatory measurements, this curve can be extended to 1989, while the $U/W$ measurements stop in 1977. Thus the fraction of penumbral spots will be used in our irradiance reconstruction. Sunspot structure values are heavily influenced by the larger sunspots, but the fraction of penumbral sunspots is dominated by the smaller sunspots. The two measurements are nearly independent. The self-consistency between the measured sunspot decay rates, sunspot structure, and fraction of penumbral spots increases one’s confidence that the measured secular variations are generally reliable.

2.3. Solar Cycle Length and Sunspot and Cycle Decay Rate Variations

Friis-Christensen and Lassen [1991] have recently argued that changes in the smoothed solar cycle length ($L$) may provide a measure of the Sun’s irradiance. A correlation between the Earth’s temperature and solar cycle length exists which Friis-Christensen and Lassen attribute to variations in the Sun’s output. Using arguments based upon rocket and balloon measurements of the solar irradiance in the late 1960s and early 1970s, they estimate that the peak-to-peak amplitude variation over the past century in solar irradiance is 1%. Friis-Christensen and Lassen used a 1-2-1 filter to smooth the cycle lengths. We used a differencing technique. Each year has a level of activity which may be expressed as a percent of the maximum level of activity for the cycle it belongs to. For each year one may find the cycle lengths by measuring the elapsed time between equal percentage levels of activity. Two cycle length determinations are made each year in this approach, so nearly all the data are used rather than selected extremum points in the cycle. A 23-year running mean was then applied to obtain the final results. These values, along with Friis-Christensen and Lassen’s values, are shown in Figure 6. In this section we examine these cycle length variations and relate them to changes in the decay rate of individual sunspots, the decay rate of sunspot cycles, and the mean level of solar activity. Earlier work relating variations in solar cycle length to climatic variations is given by Clough [1905, 1933, 1943] and Muller [1926].

The sunspot cycle length in months ($L_{\text{mon}}$) can be linearly fit in terms of the mean sunspot decay rate ($D_{\text{spot}}$) for cycles 12 to 16 as follows:
Inverse Solar Cycle Length

Fig. 6. Solar cycle lengths based upon a 1-2-1 filter technique using solar maxima (dashed line) and upon a differencing technique followed by a 23-year smoothing. The differencing method uses all the years of data rather than just the years of maxima and/or minima.

\[ L_{\text{mon}} = (251.1 \pm 5.4) - (3.98 \pm 1.02)d_{\text{spot}} \] (5)

Cycle lengths and sunspot decay rates have 84% of their variance in common. Using this equation and M-V’s unweighted mean spot decay rate of 30 MSH/d, \( L_{\text{mon}} = 131.7 \) months or 10.98 years. If we use Stewart and Panoofsky’s [1938] measurements of cycle properties and Gieisberg’s [1949] cycle model, cycle length and sunspot decay rate are related by the following equation:

\[ L_{\text{mon}} = (246.5 \pm 0.7) - (4.73 \pm 0.07)d_{\text{spot}} \] (6)

The differences between these two results can be explained, in part, by noting that (6) is based upon sunspots of all types and (5) is based upon the mean of isolated and complex sunspots. From (6) a mean cycle length of 10.7 years for the twentieth century gives a mean sunspot decay rate of 24.9 MSH/d. This seemingly slow decay rate, compared to M-V’s results, simply tells us that the average sunspot is more like an isolated sunspot than it is like a complex group. Since minimum to minimum cycle lengths have varied from ~9.9 to 12.1 years over the last century, (6) implies sunspot decay rates have varied from ~21.6 to 26.8 MSH/d.

Using Gieisberg’s sunspot cycle model and Monte Carlo techniques, the solar cycle can be simulated as a function of sunspot decay rates. In any one solar cycle, many sunspots are being generated more or less randomly in time followed by their destruction, 95% of which exhibit a linear decay. The sum of the areas of individual sunspots \( (A_{\text{total}}) \), generated at times \( t_{0,i} \) and decaying with an average rate \( d_{\text{spot}} \), can be expressed as follows:

\[ A_{\text{total}}(t) = \sum_{j}^{N} \sum_{i}^{T_j} \left[ A_{0,j} - D_{\text{spot}}(t_{i,j} - t_{0,j}) \right] \] (7)

where \( j \) is the number for each sunspot for \( N \) total spots, \( A_{0,j} \) is the initial spot area at time \( t_{0,j} \), and \( t_{i,j} \) is the time since \( t_{0,j} \) for the \( j \)th sunspot. The rate of decay of the total sunspot area is proportional to mean rate of decay of individual sunspots. Using (7) and Gleisberg’s model to simulate the probability for sunspot generation, a Monte Carlo simulation of two different types of solar cycles was produced (Figure 7). Both cycles were normalized to have the same sunspot generation at sunspot maximum. Figure 7 shows a plot of the mean of 30 simulations of each cycle. This simulation shows that the rapid destruction of individual sunspots leads to fewer sunspot groups being present during the cycle decay and eventually leads to a shorter solar cycle. The synthesized decay curve shows the appearance of an exponential decay for the sunspot cycle as a whole. On the basis of these simulations, cycle 16 should be shorter than cycle 13 by 2.2 years. In fact, cycle 16 was 1.9 years shorter [e.g., Allen, 1976].

Solar cycle lengths can be split into a rise time from sunspot minimum to sunspot maximum and a decay or fall off time from the maximum to the next minimum. Cycle lengths vary mainly because of changes in the length of the rise time while the risetimes are much more nearly constant at \( 4.30 \pm 1.10 \) years based upon cycles 1 to 21. For cycles 1 to 20 the variance in cycle lengths explained by the risetimes is 12% compared to 36% explained by the fall off times. For cycles 8 to 20, when better measurements are available, 65% of the variance is explained by the cycle decay times.

From the above discussion we expect a more rapid decay of the activity cycle to be associated with more rapid decay of individual sunspots and with shorter solar cycle lengths. Hence the change in downward slope can be used as another proxy to monitor long-term secular changes in the Sun. Dividing the mean cycle decay rate of the Wolf sunspot numbers by the maximum Wolf number of the cycle gives a normalized cycle decay rate. This normalization simply removes the variations arising from variations in the rate of sunspot generation. An example of how normalized sunspot cycles appear is given in Figure 7. The decay rate of sunspots is related to the normalized cycle decay rate by the following regression equation:

\[ d_{\text{spot}} = -(3.93 \pm 1.58) + (192.3 \pm 66.7) \frac{(dR_{z}/dt)_{\max}}{R_{z,\max}} \] (8)

Effect of Changes in Sunspot Decay Rate
with identical sunspot generation

Fig. 7. A Monte Carlo simulation of sunspot cycles in which sunspots decay slowly (22 MSH/d) or rapidly (28 MSH/d). The rate of generation of sunspots at solar maximum are set equal for the two cycles, but the results are not significantly affected by having differing rates of sunspot generation. The mean of 30 simulated cycles are shown. Both the length of the cycle and the decay rate of the cycle change as functions of individual sunspot decay rates.
where \( R_{z,\text{max}} \) is the smoothed sunspot maximum published by Waldmeier [1961] used to normalize the sunspot decay rate to a decay rate per group, and \( [dR_{z}/d\text{max}] \) is absolute value of the mean cycle decay rate per year averaged over 5 years. The constant 192.3 provides a conversion factor and is theoretically expected to be 365.2 or 182.5. The measured individual sunspot decay rates and the normalized cycle decay rates have 86\% of their variance in common. The maximum and minimum decay rates derived from (8) are 31.3 ± 3.5 and 25.9 ± 2.9 MSH/d respectively. These results indicate a peak value between 1920 and 1931, or a few years earlier than other irradiance proxies, and indicate that recently the irradiance may have leveled off at a value higher than it was at the turn of the century.

Previous authors have noted that the length of solar cycles may have a frequency modulation. For example, Groedler [1957] points out that the solar cycle length is not a constant over time and a frequency modulation is probable. He indicates that the solar cycle length in months measured from minimum to minimum \( (L_{\text{non}}) \) is not an independent variable but is a function of the mean Wolf sunspot number for the cycle \( (\bar{R}_{z}) \). An equation relating these two indices is

\[
L_{\text{non}} = \frac{12}{0.074086 + 0.00347 \bar{R}_{z}} \tag{9}
\]

Approximately 66\% of the cycle length variance can be explained by (9). In effect, the mean level of activity for each cycle can be used to derive a model for cycle lengths. Thus the mean level of solar activity can be used to derive a solar irradiance model, if one grants that the mean level of solar activity is a leading indicator of changes in solar convection. Dicke [1979] and Brown and Price [1984] point out that magnetic flux tubes may take many years to rise from the base of the convection zone to the photosphere. In this study we take the delay time to be 11 years to place this index in phase with the other indices. Dicke suggested 13 years as the time for flux tubes to rise from the base of the convection zone to the photosphere.

If the above discussion about cycle lengths and sunspot decay rates is correct, some unusual cycle lengths and decay rates would be expected during the Maunder Minimum in the late 1600s. At this time, the Wolf sunspot number was near zero for many years. Using (9) above and setting \( R_{z} \) to zero, one anticipates cycle lengths would average 13.5 years. From examination of Kochanov's [1987] carbon 14 observations, it appears there were five solar maxima at around 1646, 1660, 1674, 1692, and 1705 compared to the usually accepted sequence of six maxima at 1649, 1660, 1675, 1685, 1693, and 1705. With five solar maxima, one obtains an average length of 14.75 years from 1646 to 1705. During the Dalton Minimum around 1800, two solar cycles lasted 14 years each [Hoyt and Schatten, 1992]. If convection was weak in the Maunder Minimum, then it follows that sunspots lived longer on average than present-day sunspots. Using (5) and (6), the sunspot decay rate can be estimated to be ~16.6 MSH/d in the late 1600s. Observational evidence for slow sunspot decay rates comes from Spoerer [1889] where 2 out 23 sunspots observed from 1672 to 1700 lasted for four solar rotations. In the past century, one finds ~1 out of 769 sunspots survive through four solar rotations [e.g., Allen, 1976]. The probability of seeing two long-lived sunspots out of a sample of 23 spots is 1 in ~1100 if the Sun is same in the Maunder Minimum as it is now. The observations suggest that the Sun was indeed different in the late 1600s.

In summary, there are reasons to think that changes in solar cycle length and the normalized decay rate of solar activity reflect changes in solar convective strength and hence in solar irradiance. Cycle lengths, the normalized cycle decay rate, and the mean level of solar activity allow models of solar irradiance to be extended back to the mid-1700s. All three models share many similarities. If further research finds that solar cycles lasted ~14 years during the Maunder Minimum, this would provide more support for changes in solar irradiance and would allow the models to be extended back to the 1600s.

2.4. A Composite Model for Irradiance Variations

In each of the above sections we have discussed how a change in convective energy transport may manifest itself by changes in five solar indices: (1) the fraction of penumbral spots, (2) solar cycle length, (3) equatorial rotation rate, (4) decay rate of the solar cycle, and (5) mean level of solar activity.

All the solar indices which we are proposing as solar irradiance proxies rise from a minimum around 1880 to a maximum in the 1930s. These extremes represent the peak-to-peak irradiance variation for the last century which can have only one value. There are several approaches that could be taken to derive the amplitude of the variations. In an earlier study of this problem by Hoyt [1979a], a peak-to-peak amplitude of 0.38\% was deduced based upon sunspot structure, sunspot decay, and the mixing length theory for convection. Using the Nimbus 7 observations and Spencer and Christy's [1990] temperature record for 1979 to 1990, the Earth climate sensitivity can be estimated as 1.67 °K change for each 1% change in the solar irradiance. For the 0.5 °K rise in temperature from 1880 to 1940 a 0.30% peak-to-peak sensitivity is implied. A 0.30% amplitude also gives the best correlation with climate. Nonetheless, both these numbers seem high, since such a large upward trend would probably manifest itself in the satellite measurements. Lean et al. [1992] estimate that the Maunder Minimum may have had an irradiance ~2.7 W/m² lower than the 1986 minimum, but Nesmes-Ribes and Mangeney [1992] estimate a decrease of 0.5% or 6.8 W/m². If the Dalton minimum and the Maunder minimum both had cycle lengths of ~14 years and therefore similar levels of irradiance, a peak-to-peak variation over the last century of 0.14% to 0.35% is found. The value 0.14% is used in this paper since it is based solely on known solar properties, so no recourse to a climate response needs to be invoked. On the basis of our present understanding of the sensitivity of the Earth to fluctuations in solar irradiance, there are no known mechanisms that allow such a low amplitude of variation to explain the observed climate fluctuations.

The five models are illustrated in Figure 8. The fraction of penumbral spots model has more year to year variability than the other models and is probably picking up real solar variations which the other models cannot resolve. The solar rotation model has two peaks which arises, in part, from the difficulty of obtaining a good measure of solar rotation with the few observations available. Each model is taken to be a different and somewhat imperfect measurement of an underlying “true” variations. There is relatively good phase...
agreement between the solar indices, as summarized by Table 1. The solar rotation appears to be \(-11\) years out of phase with the other indices. Perhaps solar rotation is responding to convection near the base of the convection zone while the other indices are responding to convection changes near the top of the convection zone.

To approximate an 11-year solar activity component which includes contributions from facular emission and sunspot blocking, we use the measurements of the Wolf sunspot number and the Nimbus 7 solar irradiances for 1978–1992 [Hoyt et al., 1992]. During this period the annual mean Wolf number \((R_z)\) has varied from \(-0\) to \(150\) and the solar irradiances \((\Delta S)\) have varied by \(-1.5\) W/m\(^2\). Thus the activity component of the solar irradiances can be approximated as

\[
\Delta S_{\text{activity}} = 0.01R_z
\]

(10)

A composite solar irradiance model based upon five solar indices plus an added activity component is shown in Figure 9. The one standard deviation uncertainty provides a measure of the agreement among the different techniques used to derive the irradiance variations. For 1700–1874, three indices exist for solar irradiance reconstruction, namely cycle length, cycle decay rate, and mean level of solar activity. For 1875–1978, up to five solar indices are used, namely the three just mentioned plus solar rotation and the fraction of penumbral sunspots. Two solar indices (sunspot structure and the time rate of change of the solar diameter) give similar time variations, but because of uncertainties in their values are not used here. For 1979–1992 the irradiances are scaled to the mean of the Nimbus 7 measurements. In the composite model, adjacent solar minima may differ by only a few hundredths of a percent and would be difficult to detect experimentally, a subject to which we now turn.

3. Experimental Evidence for Irradiance
   or Convection Changes

Is there any direct observational evidence to support the hypothesis that the Sun has long-term variations in irradiance like the composite model? In this section we examine two groups of experimental evidence which bear on this question. The first line of evidence is based directly upon radiometric measurements of solar irradiance either from satellites or the ground. The second line of evidence concerns further indirect measures of solar irradiance or diagnostic measurements of the photosphere that may indicate changes in solar convection or irradiance. These two groups of experimental evidence are split into two subsections.

3.1. Direct Radiometric Evidence

If the hypothesis of secular solar irradiance variations is true, it might be detectable in the satellite observations made by Willson and Hudson [1988, 1991] using the active cavity radiometer (ACRIM) on the Solar Maximum Mission (SMM). Willson and Hudson [1991] point out the SMM/ACRIM measurements in early 1980 diverge from the solar irradiance models based upon variations caused by facular emission, active network emission, and sunspot blocking. The difference noted by Willson and Hudson may be an indication of another source of solar irradiance variation. The Nimbus 7 measurements support the SMM/ACRIM measurements in indicating a modeling, as opposed to a measurement problem, exists in 1980.

When activity is low or zero, a long-term trend in the irradiance might reveal itself. Since the Nimbus 7 measurements are noisy during the solar minimum, we examined all

---

**Table 1. Phase Relationship of the Solar Indices**

<table>
<thead>
<tr>
<th>Index</th>
<th>Year(s) of 20th Century Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunspot structure (U/W)</td>
<td>1934</td>
</tr>
<tr>
<td>Fraction of sunspots without umbrae</td>
<td>1933</td>
</tr>
<tr>
<td>Rates of sunspot decay</td>
<td>not available</td>
</tr>
<tr>
<td>Solar cycle lengths (1-2-1 filter)</td>
<td>1937.5 ± 5.5</td>
</tr>
<tr>
<td>Solar cycle lengths (this study)</td>
<td>1940</td>
</tr>
<tr>
<td>Normalized rate of solar cycle decay</td>
<td>1920–1931</td>
</tr>
<tr>
<td>Equatorial solar rotation</td>
<td>1924–1934</td>
</tr>
</tbody>
</table>

---

**Figure 9.** A plot showing the combined solar irradiance model using the models in Figure 8 and adding a solar activity component. The error bars show the relative disagreement among the different techniques used to derive the irradiance variations. For 1700–1874, models based upon cycle length, cycle decay rate, and mean level of solar activity are used. For 1875–1992, up to five solar indices are used.
the SMM/ACRIM data from 1985 to 1987. Days with sunspot blocking, based upon the photometric sunspot index (PSI) of Willson and Hudson [1991], are discarded yielding 271 quiet days. The quiet days are not all alike, but vary by several tenths of a watt per square meter. The scatter is caused by variations in the number of faculae present on quiet days and to a lesser extent by the continuing presence of small unresolved sunspots. Quiet days occurring just before a sunspot appears or just after a sunspot disappears are brighter than other quiet days. Starting early in 1987 some of quiet days are influenced by very small sunspots since the daily standard deviations of the ACRIM measurements become larger. Sorting the quiet days into 6-month groups and averaging reveals a small upward trend from mid-1985 through 1986. This trend is not statistically significant and because of residual faculae does not allow the presence of a background trend to be resolved. The composite model predicts that a zero slope for irradiance must occur at each solar minimum which can only be shortened or lengthened by the any underlying trend. Self-consistent measurements of two or more solar minima, sufficient to detect differences at the level of a few hundredths of a percent, are required if radiometric observations are to detect the postulated changes.

Abbot at the Smithsonian Astrophysical Observatory (APO) made extensive ground-based measurements of the extraterrestrial solar irradiance from 1923 to 1954 [e.g., Hoyt, 1979c]. These measurements suffer from errors primarily arising from the inability to remove all atmospheric influences. For example, the Chilean volcanic eruptions in 1932 depressed the APO solar constant values to their lowest values. Therefore the APO observations are not of sufficient precision to resolve the validity of the sunspot structure hypothesis. Pettit [1932] made many observations of the solar ultraviolet flux in the late 1920s. He finds high values of solar ultraviolet measurements at 0.32 μm occur when solar activity is high. However, his active Sun ultraviolet irradiance exceeds his quiet Sun values by ~50%. Modern observations and theory [e.g., Lean, 1991] suggest variations at this wavelength are <1%. Pettit’s observations must have some flaw arising from instrumental problems or from data reduction errors.

In summary, the radiometric measurements do not as yet provide any evidence for a long-term secular variation in solar irradiance except for the discrepancy between the models and satellite measurements in 1980. This lack of evidence is perhaps no surprise since the predicted effect is at or beyond past, and perhaps future, capabilities.

3.2. Indirect Measures of Solar Irradiance and Diagnostic Measurements

If there is some change in the convective energy transport with time, there will be a change in the temperature gradient in the photosphere. A temperature gradient change will cause \( b_p \) and \( c_p \) (in equation (2)) to vary. The coefficient \( a_p \) will change if there is a global increase in the effective temperature of the Sun. Changes in limb darkening are probably subtle and difficult to detect and may appear as solar diameter variations. Kuhn et al. [1998] have examined the changes in solar limb brightness from 1983 to 1987 and claim that a portion of the variations are explained by faculae, but another part of the variations are explained by changes in the effective temperature of the photosphere. Kroll et al. [1990] have measured changes in solar limb darkening which they postulate may be caused by changes in convective energy transport. Kroll et al.’s limb darkening variations appear to have a component that is not well correlated with solar activity changes.

Additional experimental evidence for changes in convective energy transport comes from the examination of the width of the carbon line at 5380 Å [Livingston, 1990; Livingston and Holweger, 1982]. This line is formed relatively deep in the photosphere and shows a monotonic increase in width from 1978 to 1990. Coupled with measured variations of equivalent widths of other lines formed at different depths in the photosphere, the temperature gradient apparently is changing secularly, implying a secular change in the convective energy transport, with changes in \( b_p \) and \( c_p \) causing a solar irradiance change. Theoretical models cannot, as yet, provide a quantitative estimate of the solar irradiance variations arising from this effect. It does suggest that the solar limb darkening is changing secularly and that a component of solar irradiance variations exists which is independent of the level of solar activity.

Yet another indication of changes in the convective energy transport in the photosphere comes from measurements of the bisectors of Fraunhofer lines [Livingston, 1982]. These measurements show that the asymmetry of the iron line at 5250 Å changed between 1976–1977 and 1980–1981. The change can be explained by a change in the velocity of solar convection, but a theory relating these changes to an irradiance change is not yet developed.

Finally, Schatten [1979] shows there are secular changes in the brightness of the great red spot on Jupiter. Internally self-consistent Jovian observations cover the period from 1892 to 1948 and show the red spot was brightest around 1928. The spot brightness has a 0.63 correlation with the composite irradiance model compared to a 0.37 correlation with Wolf sunspot number. Schatten attributes the spot brightness variations to changes in the solar ultraviolet radiation. These observations are consistent with the existence of a long-term secular solar irradiance variations.

The indirect measures seem to provide more support for a secular change in solar irradiance than do the direct measurements. The theories for solar convection, however, are as yet not sufficiently developed to convert these indirect measures into a quantitative value for the change in solar irradiance. They do indicate changes in solar convection may be occurring which are independent of solar activity.

4. The Climate Connection

If the solar irradiance is varying as postulated in the above discussion, it can be expected to have some effect on the temperature of the Earth. In this section we empirically examine the two questions: (1) is the Earth responding in a manner consistent with an external forcing and (2) do the Earth’s temperature variations and the model solar irradiance variations correlate with each other?

A solar forcing will cause the two hemispheres of the Earth to vary in parallel. The amplitudes of the responses will differ because the two hemispheres have different amounts and distributions of land and ocean. To check for evidence of an external forcing, we first separately detrend the northern and southern hemisphere temperature records
of Hansen and Lebedeff [1987] using the slope of a linear regression fit through the hemispheric mean temperatures. This upward trend in temperature over the last century can be attributed to a greenhouse warming, an urban heat island effect [e.g., Karl and Jones, 1989], changes in the distribution and number of sampling stations [Willmott et al., 1991], changes in the diurnal temperature sampling [Edwards, 1987], changes in shelter construction, thermometer type, or local exposure [Mitchell, 1953], and other effects. Removing the trends in temperature reduces the influence of these extraneous effects. For 1880–1990 the detrended temperature variations of the two hemispheres have a correlation coefficient of 0.55. The nondetrended temperature variations have a 0.80 correlation. Both correlations are significant at the 99.9% confidence level or better indicating that it is highly probable an external forcing is being imposed. A random walk type of climatic change would not be expected to produce a positive correlation between the two hemispheres.

Further evidence for the external forcing of climate changes comes from the study of Ardaut and others [1992]. Using the Nimbus 7 Earth Radiation Budget (ERB) and THIRS/TOMS data sets, they show that the Earth’s temperature is stable in the sense that any variation is forced back toward an equilibrium set point. The Earth therefore behaves like a thermostat with a fixed set point. Any long-term drift away from the set point is therefore unlikely. The set point can be altered by changing its value through a change in the composition of the atmosphere (e.g., the greenhouse effect) or by an external forcing (e.g., changes in surface albedo or changes in solar irradiance).

The combined model for solar irradiance seen at the Earth (Figure 9) extends over ~290 years so it would be useful to compare it a hemispheric or global model for climate variations over this time. Groves and Landsberge [1979a, b] and Landsberg [1981] provide the only published reconstruction of the temperature of the northern hemisphere from 1579 to 1880. Their temperature anomalies can be extended to the present by using Hansen and Lebedeff’s temperature anomalies. All other published reconstructions of climate over this timescale are local reconstructions. With fewer and fewer thermometer observations available as one goes back in time, the uncertainties in Groves’s estimations of temperature become larger.

The combined solar irradiance model is correlated with the temperatures of the northern hemisphere of the Earth (Figure 10). On a decadal timescale the solar irradiance model can explain ~71% of the variance during the past 100 years and ~50% of the variance since 1700. The model irradiance and measured temperatures share many similarities, but several differences are worth commenting on. The irradiance model predicts the mid to late 1700s are warmer than Groves and Landsberg deduce. Biffia et al. [1990], however, indicate that 1748–1767 was the warmest 20-year group of summers in the last two centuries and 1761 was the warmest summer in Scandinavia. Groves and Landsberg’s temperature reconstruction uses thermometer measurements predominantly from Europe, so there may be a problem in the temperature reconstruction. Another difference occurs for the 1830s, when Groves and Landsberg’s temperatures show a much sharper peak than the irradiance model. Finally, around 1850 the model irradiance and temperatures go in opposite directions. Smyth [1856] claimed the Sun was getting brighter at this time, which is the opposite of our model results. Clearly more work in reconstructing both the solar irradiance and the temperature variations is required. Statistically, the temperature variations are consistent with a solar forcing, although physically the amplitude of the solar variations imply the Earth would be more sensitive to solar forcing than is generally believed, as mentioned in section 2.4.

5. Summary and Discussion

There is plausible evidence for long-term changes in solar irradiance. Over the last two decades, diagnostic measurements of the equivalent width of lines, the limb darkening of the Sun, and line bisectors all indicate secular changes in solar convection, the photospheric temperature gradient, and solar irradiance are possible. Additional evidence for long-term irradiance changes come from such proxy measures as sunspot structure, sunspot decay rates, the length of solar cycles, the normalized solar cycle decay rate, the equatorial solar rotation rate, and the time rate of change of the solar diameter. The variations in these indices can plausibly be explained as arising from a common source, namely secular changes in solar convective energy transport or convective velocities. We recognize that such changes fall outside the domain of usual theories of stellar structure, but then all the observed solar variations do so too. Without any consideration of the arguments put forth in this paper, it seems more plausible for all these solar proxies to play some role in the varying solar irradiance than it would be for all these variations to exist with an invariant solar brightness.

For all the proxy models considered the solar output varies by less than about ±0.2% in the last century. The solar convection zone stores approximately 10^{45} ergs. A perturbation in radiative flux of 0.2% lasting for one century amounts to 1 part in 40,000 of the total thermal energy stored in the convection zone. For comparison, the thermal energy storage in the Earth’s atmosphere has varied by ~1 part in 500 over the last century. Energetically, there seems little reason to rule out these irradiance variations.
timescale of the variations, the deeper the likely source for the perturbation will be. Relatively short variations from sunspots and faculae, lasting days, are the result of perturbations in the top few thousand kilometers below the photosphere. The root source of the longer variations may arise from deep within the convection zone, perhaps at its base or just below its base, because of the observed solar rotation changes. Endal et al. [1985] and Nesmes-Ribes and Mangeney [1992] explore possible mechanisms for irradiance variations on the timescale of decades to centuries. Candidate mechanisms include (1) β perturbations in which stochastic variations in the energy transport arise from the finite number of convective cells involved, and (2) β perturbations in which changes in pressure, perhaps arising from changes in the strength of the magnetic field, alter the rate of energy transport. Plausible arguments can be made in behalf of both mechanisms.

The postulated irradiance variations are only a few tenths of a percent over periods of several decades and therefore will be difficult to detect. Present measurement plans do not appear to provide sufficient redundancy and overlap to detect low-level secular irradiance variations.

The correlation of the solar indices and modeled solar irradiance with the Earth’s temperature are significant at better than the 99% confidence level. However, if the amplitude of the solar irradiance variations is only 0.14% from 1880 to 1940 and if the Earth’s climate responded with a 0.5 °C warming, then the climate would be much more sensitive to solar forcing than is commonly assumed. The direct effect of a 0.14% increase in solar irradiance could only account for a 0.23 K increase if the sensitivity to solar influences is 1.67 °C per 1% increase in solar output (based upon the last decade of satellite observations). The amplitude of the solar variations remains highly uncertain with most estimates ranging from 0.14% to 0.38% over the past century. With the higher-amplitude estimates there are less problems connecting the solar variations to climatic changes. On decadal timescales the climate sensitivity can be expected to be larger due to several positive feedbacks. Potential positive feedbacks include a decrease in the ice and snow cover, an increase in plant absorptivity as it becomes greener in a warmer world, an increase in absorption by water surfaces as wind velocities decrease (based upon changes in the length of the day), and changes in plant orientation and albedo as wind velocities change. The last three potential feedback loops are not included in present day climate models. The postulated solar irradiance variations may explain part of the Earth’s interannual temperature variations although their effects would be most evident in long-term trends in temperature on hemispheric and global spatial scales.

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