

1.) A 10.6 kg object oscillates at the end of a vertical spring that has a spring constant of $2.05 \times 10^4 \frac{N}{m}$. The effect of air resistance is represented by the damping coefficient $b = 3.0 \text{ N}\cdot\text{s}/\text{m}$.

- (a) Calculate the frequency of the damped oscillation.
- (b) By what percentage does the amplitude of the oscillation decrease in each cycle?
- (c) Find the time interval that elapses while the energy of the system drops to 5.00% of its initial value.

▷ (a) If an oscillator experiences a damping force, its oscillation frequency is given by :

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} , \text{ with } \omega_0^2 = \frac{k}{m} .$$

$$\omega = \sqrt{\frac{2.05 \times 10^4}{10.6}} - \left(\frac{3}{2 \times 10.6}\right)^2 = 43.98 \text{ r.s}^{-1} .$$

$$\text{or } f = \frac{\omega}{2\pi} = 7 \text{ Hz} .$$

b) The oscillator's position is described by :

$$x(t) = A e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$$

The amplitude of the system depends now on the damping coefficient (b) and time t . It can be represented as :

$$x(t) = A(t) \cos(\omega t + \phi), \text{ with } A(t) = A e^{-\frac{b}{2m}t}$$

After one cycle, the system's amplitude evolves from $A(t)$ to $A(t+T)$, where T is the period of the motion. Then

$$A(t+T) = A e^{-\frac{b}{2m}(t+T)}, \text{ and the ratio between amplitudes is}$$

$$\frac{A(t+T)}{A(t)} = \frac{A e^{-\frac{b}{2m}(t+T)}}{A e^{-\frac{b}{2m}t}} = e^{-\frac{b}{2m}T}; \quad T = \frac{1}{f}$$

Therefore, in one cycle the amplitude decreases in approximately 2%. Check :

$$\begin{aligned} \frac{A(t+T)}{A(t)} &= e^{-b/2mf} = e^{-(3/(2 \times 10.6 \times 7))} \\ &= \underline{\underline{0.98}} \\ &= 0.98 \quad (2\%) \end{aligned}$$

c) We might assume that the energy of a damped oscillator is proportional to the square of its amplitude:

$$E \propto A^2 \quad \text{or}$$

$$E \propto \left(e^{-\frac{b}{2m}t}\right)^2 \quad \text{since} \quad A \mapsto A(t) \propto e^{-\frac{b}{2m}t}$$

Then, the energy's oscillator can be written as:

$$E(t) = E_0 e^{-\frac{b}{m}t}, \quad \text{where } E_0 \text{ corresponds to}$$

its initial value. After some particular time t' , the energy drops to 5% of E_0 . Mathematically:

$$E(t') = 0.05 E_0 = E_0 e^{-\frac{b}{m}t'} \quad \text{or}$$

$$0.05 = e^{-\frac{b}{m}t'}; \quad \text{For } b = 3 \text{ N.s/m and } m = 10.6 \text{ kg}$$

$$\boxed{t' = 10.6 \text{ s}}$$

After 10.6 s the system has lost a 95% of its initial energy.

2.) A car rounds a banked curve as in Fig. 1. The radius of curvature of the road is R , the banking angle is θ , and the coefficient of static friction is μ_s . (a) Determine the range of speeds the car can have without slipping up or down the road. (b) Find the minimum value for μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100\text{m}$, $\theta = 10.0^\circ$ and $\mu_s = 0.1$ (slippery conditions?)

SOL.

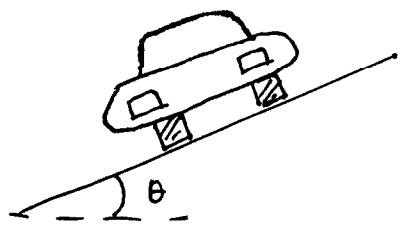


Fig. 1

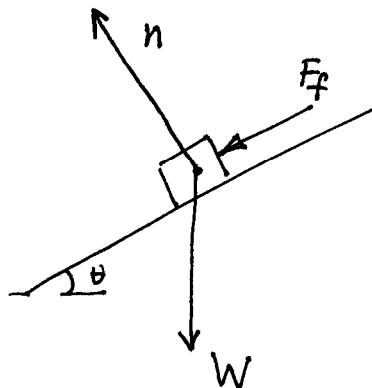


Fig. 2

"Slipping up" situation: Diagram of forces.

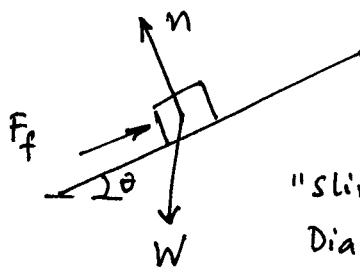


Fig. 3

"Slipping down" situation:
Diagram of forces.



(a) Let's consider the case illustrated in Fig. 2.

The car is equilibrium in the vertical direction. Thus, from $\sum F_y = 0$ we have

$$n \cos \theta - F_f \sin \theta - W = 0 \quad (1)$$

The magnitude of the static friction is proportional to the normal n ; $F_f = \mu_s n$

From (1), the normal force becomes into:

$$n = \frac{w}{\cos \theta - \mu_s \sin \theta} . \quad (2)$$

The Newton's second law for x-direction (radial) gives

$$n \sin \theta + F_f \cos \theta = m \frac{v^2}{R} \quad (3).$$

Combining (2) & (3), we have

$$\frac{w (\sin \theta + \mu_s \cos \theta)}{(\cos \theta - \mu_s \sin \theta)} = m \frac{v^2}{R}, \quad \text{or}$$

$$v = \sqrt{g R \frac{(\tan \theta + \mu_s)}{(1 - \mu_s \tan \theta)}} \quad (4)$$

case in Fig 2.

For the case depicted in Fig. 3, similar analysis leads to

$$v' = \sqrt{g R \frac{(\tan \theta - \mu_s)}{(1 + \mu_s \tan \theta)}} \quad (5)$$

Relation (4) corresponds to the maximum speed of the car without slipping. Likewise, v' in relationship (5) gives the minimum speed possible for non-slipping condition.

b) The minimum speed is zero when
 $\tan\theta = \mu_s$.

c) For these numerical values, v is in the range of
 $\{8.6 \text{ m/s}, 16.6 \text{ m/s}\}$.

3) A single bead can slide with negligible friction on a wire that is bent into a circular loop of radius 15.0 cm, as in Fig. 3.1. The circle is always in a vertical plane and rotates steadily about its vertical diameter with a period of 0.450 s. The position of the bead is described by the angle θ that the radial line, from the center of the loop to the bead, makes with the vertical. At what angle up from the bottom of the circle can the bead stay motionless relative to the turning circle?

Diagram of Forces

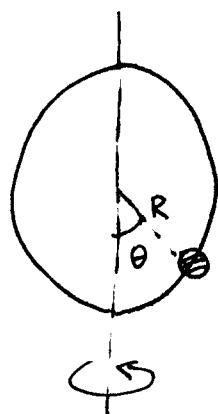
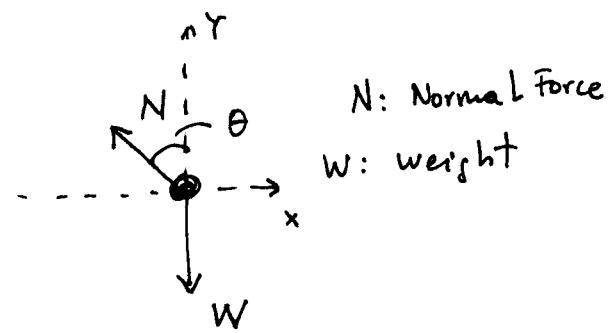


Fig 3.1



- The bead stays in equilibrium along y -axis, thus:
 $\sum F_y = 0 ; \quad N \cos \theta = W . \quad (1)$
- Newton's second law for the radial direction (along x , in this case) gives

$$(2) \quad \sum F_x = \frac{m v^2}{R'} , \quad \text{where } R' \text{ is the radius of the circle associated to the trajectory of the bead. In terms of the unknown angle } \theta$$

$$R' = R \sin \theta . \quad (3)$$

Eq. (2) can be written as:

$$N \sin \theta = m \frac{v^2}{R'} . \text{ Recalling that } v = \omega R'$$

and $\omega = \frac{2\pi}{T}$, where T is the period of motion, we obtain

$$N \sin \theta = m \omega^2 R' = m \left(\frac{2\pi}{T} \right)^2 R \sin \theta . \quad (4)$$

or

$$N = \frac{4\pi^2 R m}{T^2} \quad (5)$$

Inserting (5) into Eq (1), we get:

$$\frac{4\pi^2 R m}{T^2} \cos \theta = mg , \text{ or}$$

$$\boxed{\theta = \cos^{-1} \left(\frac{g T^2}{4\pi^2 R} \right)} \quad (6)$$

The bead will stay motionless in an angle of:

$$\theta \approx 70.42^\circ .$$

Notice that real solutions on (6) are restricted by

$$\frac{g T^2}{4\pi^2 R} \leq 1 .$$

The bead leaves its bottom position for values of T smaller than 0.78s.