Amplitudes of Rossby waves based on the Bjerknes circulation

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Background: quantifying wave amplitude

Traditionally, wave (eddy) is defined as departures from the longitudinal average (zonal mean). There are two difficulties with this. First, the wave-mean dichotomy becomes ambiguous when the wave amplitude is large, because the zonal mean is already altered by the wave. Second, since the zonal mean state varies in time and space, the local wave amplitude does not necessarily give a sense of 'waviness.' For example, eddy kinetic energy may be large along a jet, but the flow can be least wavy there because the kinetic energy of the zonal mean wind is much greater. To avoid these difficulties, a wave amplitude diagnostic that does not depend on the dynamical state of the flow is necessary.

A new diagnostic: planetary circulation

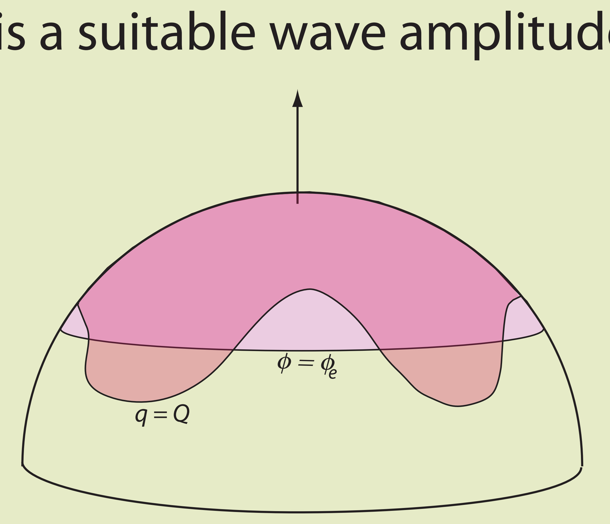
We define wave amplitude geometrically, using the meridional displacement of a quasi-material contour on the isentropic surface. This is achieved in terms of surface integral of the Coriolis parameter, or the planetary circulation as per Stokes' theorem. For example, the waviness of a potential vorticity (PV) contour ($q = Q$, where q is PV and Q is its value) may be defined as

$$\Delta C = C_P^* - C_P, \quad C_P^* \equiv \int_{\phi \ge \phi_e} f\, dS, \quad C_P \equiv \int_{q \ge Q} f\, dS \qquad (1)$$

where $f = 2\Omega \sin\phi$ is the Coriolis parameter, and $\phi = \phi_e$ is the latitude of the zonal circle that encloses the same area as the PV contour (equivalent latitude). Note $C_P^* = 2\pi\Omega a^2 \cos^2\phi_e$ (a is the planetary radius). (1) may be rewritten as

$$\Delta C = \int_{\substack{\phi \ge \phi_e \\ q < Q}} f\, dS - \int_{\substack{\phi < \phi_e \\ q \ge Q}} f\, dS \ge 0\,, \qquad (2)$$

where the last inequality follows from the fact that f is an increasing function of latitude and that the area of integral is the same for the two terms in the second expression. The magnitude of ΔC increases as this area increases (i.e., as the contour becomes wavier). Thus, ΔC is a suitable wave amplitude norm. Although the shape of the contour is arbitrary, in the small amplitude limit, (2) becomes



$$\Delta C \approx a^2 \cos\phi_e \int_0^{2\pi} d\lambda \int_{\phi_e}^{\phi_e + \Delta\phi} f\, d\phi$$
$$\approx 2\pi\Omega a^2 \cos^2\phi_e \left\langle (\Delta\phi)^2 \right\rangle = C_P^* \left\langle (\Delta\phi)^2 \right\rangle \qquad (3)$$

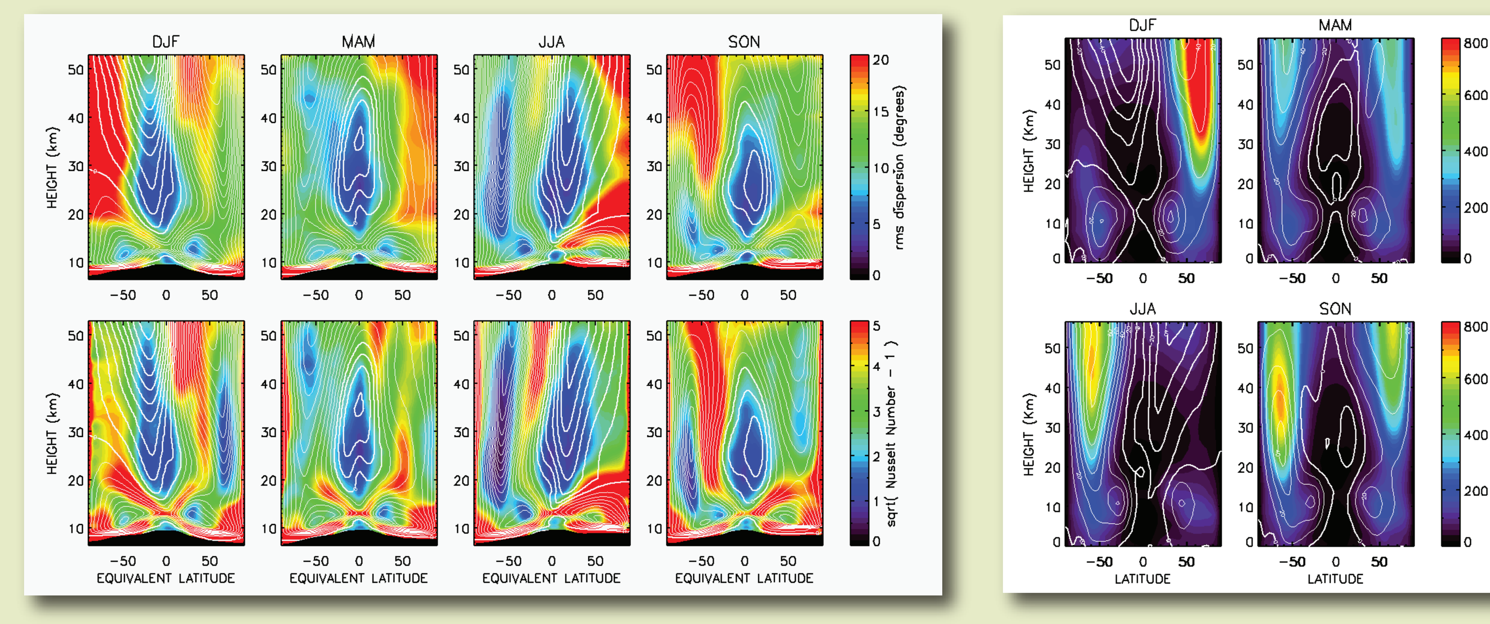
where $\phi_e + \Delta\phi$ is the latitude at which $q = Q$, λ is longitude, and the angle bracket denotes zonal average. Therefore,

$$\eta \equiv (180/\pi)\sqrt{\Delta C / C_P^*}$$

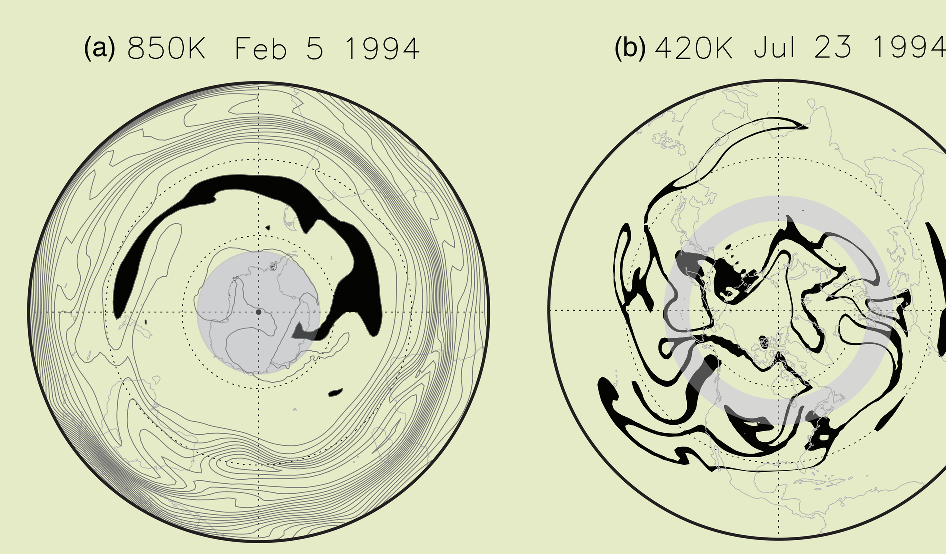
is the rms meridional displacement of the contour in degrees. Note that ΔC and η depend only on the instantaneous location of the contour and do not involve fluid motion. Furthermore, since C_P^* is invariant with time at a fixed ϕ_e, $\eta(\phi_e, t)$ is a wave amplitude relative to a fixed reference. Therefore, it is an absolute measure of wave amplitude. Although the formalism does not require a particular variable to define a contour, PV (and its variants) is the natural choice because it corresponds to ϕ_e one-to-one, and an equation for ΔC can be readily derived from the Bjerknes circulation theorem.

Diagnostic: η–climatology 1992-2005

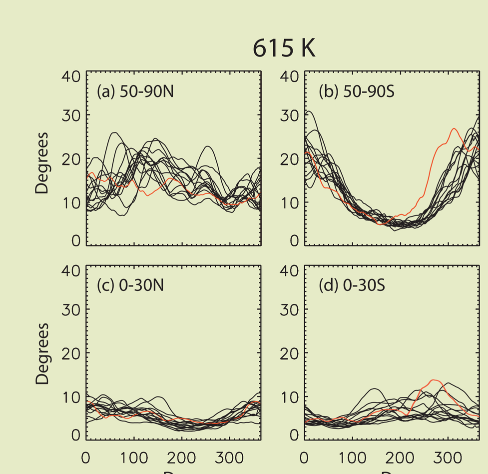
We diagnose η using a PV-like numerical tracer derived from the Met Office Stratospheric Analysis winds by solving isentropic advection-diffusion problem (top row of the left panel below.)



In the top row of the left panel, η is shown in color and relative circulation divided by the length of the zonal circle at $\phi = \phi_e$ is shown in contours (westerly regions are dashed). η is small throughout the year in the easterly region of the (sub)tropics. It is also small at the edge of the winter polar vortex (SH in particular) and around the upper tropospheric jets. This demonstrates that the **PV contours are least wavy in the jets**, lending support to the Rossby elasticity argument (Baldwin et al. 2007). Large η is found in the summer Antarctic stratosphere, spring Arctic stratosphere, summer lower stratosphere in NH, and spring upper Antarctic stratosphere. **There is remarkable correlation with effective diffusivity** (bottom row): this is because effective diffusivity is a measure of contour length (Nakamura 1996), and to lengthen a material contour, one must first displace it from the zonal circle. The right panel shows the rms eddy geopotential height using the same data. This traditional diagnostic is generally maximal along (or near) the jet axes, the opposite pattern of η.

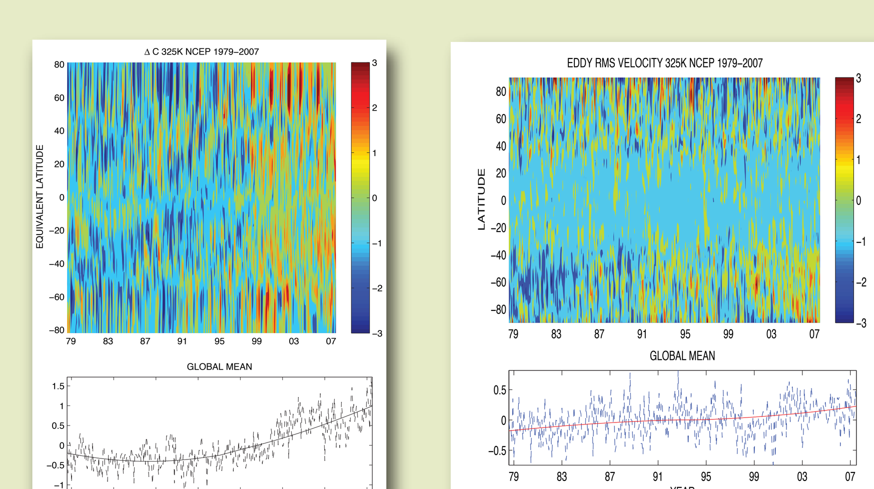


The large η in the Antarctic summer reflects remnants of the vortex air, broken up earlier in the season and trapped at low latitudes (see panel a on the left). Note that the motion of the patch is close to solid body rotation. In the summer lower stratosphere in the NH (panel b), on the other hand, the flow is highly deformational due to monsoon, and the ring of air is characterized by a contorted geometry.



To appreciate the interannual and seasonal variability, 14 annual cycles (1992-2005) of average η at 615 K are shown on the left for four bands of ϕ_e (50-90N, 0-30N, 0-30S, and 50-90S). In the high latitudes, the NH is characterized by a much greater interannual variability than seasonal variability, whereas the SH is dominated by the seasonal cycle. A notable exception is 2002, shown in red, in which a major warming was observed for the first time in the Antarctic. That year shows an unusually large η in the spring. The seasonal variability is much weaker at low latitudes.

Diagnostic: long-term trend?



ΔC anomaly at 325 K based on 1979-2007 NCEP reanalysis PV (left). It seems to show a globally increasing trend. Trend is much weaker in the eddy rms velocity (right, the traditional zonal mean method) and hard-pressed by interannual variability. Is the ΔC trend real? It may or may not. For example, the switchover from TOVS to ATOVS around 1998 may have contributed to a better resolution of eddy, and hence an apparent increase of wave amplitude.

Verdict: potentially useful

Being independent of the dynamical state of the atmosphere, the new diagnostic is an objective measure of wave amplitude and it may be used as a fingerprint of long-term changes in the dynamics. The trend analysis applied to the reanalysis products is marred by the changes in observational platforms etc., but it would be worth testing the diagnostic in long-term climate change simulations (e.g. IPCC AR4 runs).

Sponsor: NSF

For more info: http://geosci.uchicago.edu/~nnn/Solomon_Nakamura_GRL.pdf