Sudden Stratospheric Warmings as Noise–Induced Transitions

Thomas Birner[(1)] & Paul D. Williams[(2)]

(1) Department of Physics, University of Toronto, Toronto, ON, Canada

(2) Department of Meteorology, University of Reading, Reading, U.K.

contact: thomas@atmosp.physics.utoronto.ca

preprint: http://www.atmosp.physics.utoronto.ca/people/thomas/papers/ssw_accepted.pdf

1. Background, Concepts, Summary

- **Sudden Stratospheric Warmings (SSWs):** conventionally considered to be associated with planetary wave activity
- **Can small-scale variability, e.g. due to breaking gravity waves, lead to SSWs when planetary wave activity is not strong enough to cause a SSW by itself?**
- study SSWs using recently proposed highly truncated version of **Holten–Mass (1976) stratospheric wave–mean flow model** (Ruzmaikin et al., 2003)
- this low–order model exhibits **multiple stable equilibria** corresponding to the undisturbed vortex and an SSW-state, respectively
- momentum forcing due to quasi-random **gravity wave actitvity** is introduced **as additive noise** in evolution equation for the mean flow
- study stochastic system using concept of **First Passage Times (FPTs):** initialized at undisturbed state, numerically integrate system many times in order to derive statistics of transitions to the SSW-state
- solve **Fokker-Planck equation** corresponding to the stochastic system numerically in order to derive stationary probability density function
- **Even small to moderate strengths of stochastic gravity wave forcing can lead to SSWs for cases where the deterministic system does not predict a SSW.**

2. Holton & Mass (1976)

QG equations for linearized PV and zonal mean flow, β–channel, 60°N±30°, 10km ≤ z ≤ 80km:

$$
\begin{aligned}
(\partial_t + \overline{u}\partial_x)\, q' + \beta'\partial_x\psi' &+ \frac{f_0^2}{\rho}\partial_z\left(\frac{\alpha\rho}{N^2}\partial_z\psi'\right) = 0\\
\partial_t\left[\partial_{yy}\overline{u} + \frac{f_0^2}{N^2\rho}\partial_z(\rho\partial_z\overline{u})\right] &= -\frac{f_0^2}{N^2\rho}\partial_z\left[\alpha\rho\partial_z(\overline{u}-U_R)\right]\\
&+ \frac{f_0^2}{N^2}\partial_{yy}\left[\rho^{-1}\partial_z\left(\rho\overline{\partial_x\psi'\partial_z\psi'}\right)\right]
\end{aligned}
$$

$$\left[\, q' = \nabla^2\psi' + \frac{f_0^2}{\rho}\partial_z\left(\frac{\rho}{N^2}\partial_z\psi'\right) \quad \text{and} \quad \beta' = \beta - \partial_{yy}\overline{u} - \frac{f_0^2}{\rho}\partial_z\left(\frac{\rho}{N^2}\partial_z\overline{u}\right)\right]$$

Newtonian Cooling: $\alpha = \left(1.5 + \tanh\frac{z - 25\,\mathrm{km}}{H}\right)\cdot 10^{-6}\mathrm{s}^{-1},\ U_R(z,t) = U_R(0,t) + \Lambda(t)z$

Ansatz: **l=1, k=2**

$$
\begin{aligned}
\overline{u}(y,z,t) &= U(z,t)\sin ly\\
\psi'(x,y,z,t) &= \mathrm{Re}\{\Psi(z,t)e^{ikx}\}e^{z/2H}\sin ly
\end{aligned}
$$

Wave forcing at lower boundary:

$$\Psi(0,t) = \frac{g}{f_0}h(t)$$

3. Ruzmaikin et al. (2003) + Noise

- discretize Holton & Mass (1976) very coarsly in z (only three levels)
- obtain low order system consisting only of three coupled ODEs for the real (X) and imaginary (Y) parts of the wave streamfunction and the mean flow (U), respectively, at the mid–level z = 25 km
- represent small scale effects, e.g. due to quasi-random gravity wave activity as additive noise term in the evolution equation of the mean flow

$$\partial_z U|_{z=k\Delta z} = \frac{U_{k+1}-U_{k-1}}{2\Delta z} \quad \text{and} \quad \partial_{zz}U|_{z=k\Delta z} = \frac{U_{k+1}-2U_k+U_{k-1}}{(\Delta z)^2} \qquad k = 0,1,2$$

$$\Psi = X + iY$$

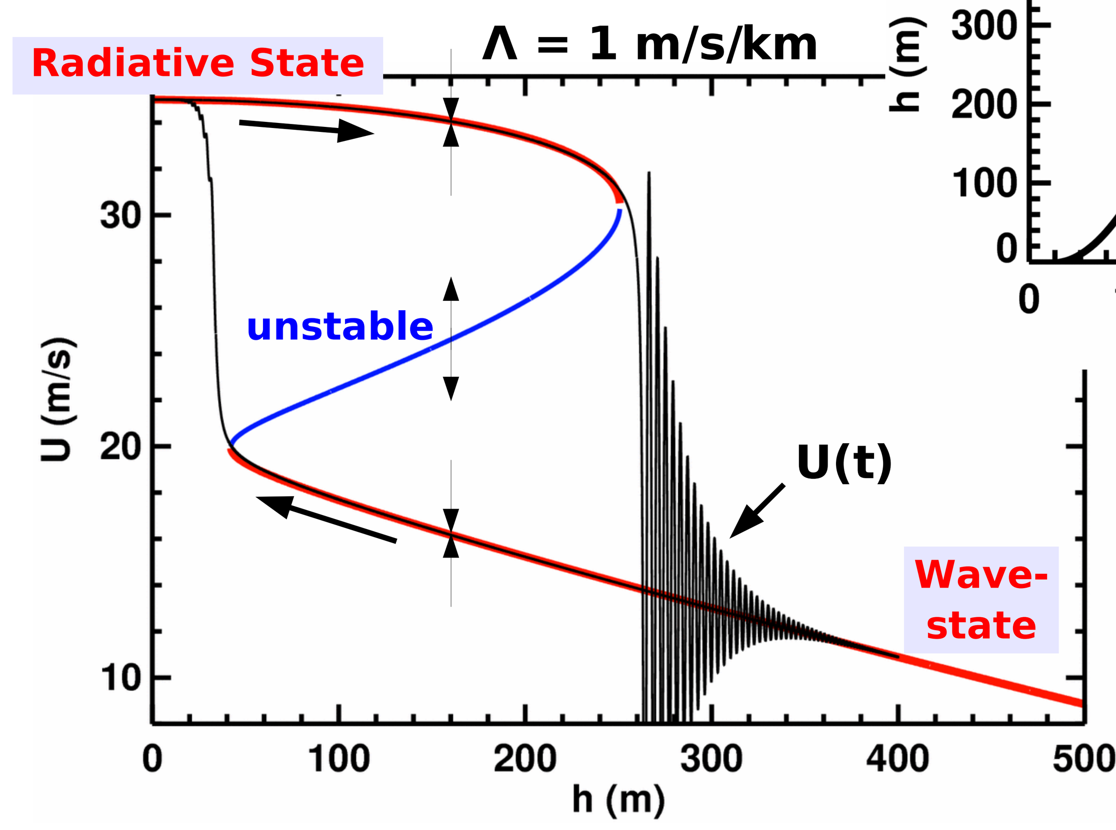
$$
\boxed{
\begin{aligned}
\dot{X} &= -\alpha_1 X - rY + sUY - \xi h\\
\dot{Y} &= -\alpha_1 Y + rX - sUX + \zeta hU\\
\dot{U} &= -\alpha_2(U - U_R) - \eta hY + \chi_a
\end{aligned}
}
$$

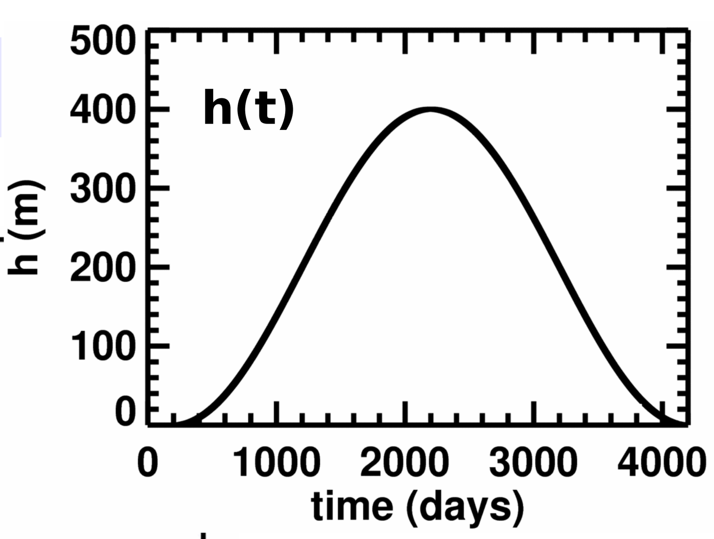
Gaußian white noise: $\overline{\chi_a(t)\chi_a(t')} = \sigma_a^2\delta(t-t')$

- control parameter: h = wave forcing at lower boundary
- Here: $(\alpha_1)^{-1}$ ~ 30 Days, $(\alpha_2)^{-1}$ ~ 20 Days, U_R fixed
- all coefficients determined through external parameters (see appendix in Ruzmaikin et al., 2003)

4. Stationary Solutions of the deterministic System

- U_{stat} as a function of wave forcing h



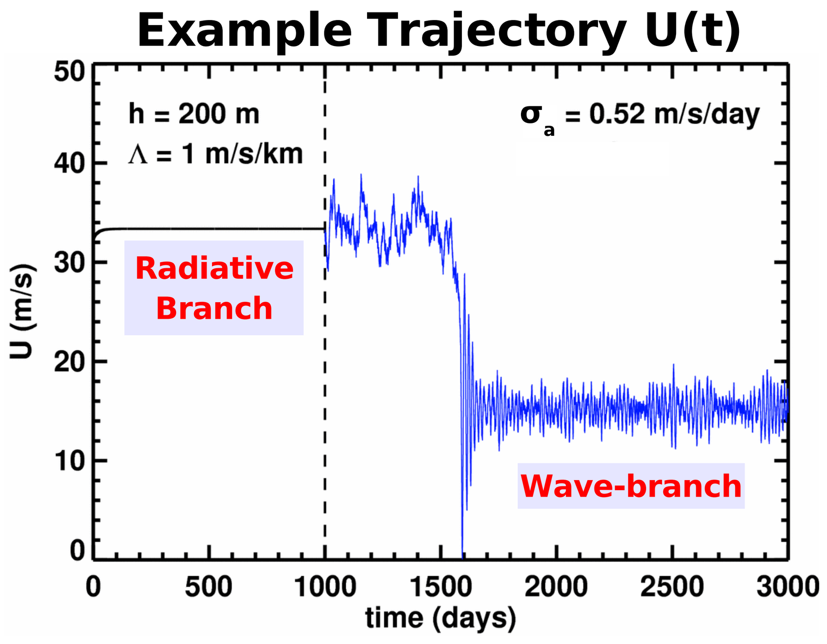


- system is bi–stable for: 40m < h < 250m
- **wave-state corresponds to SSW**

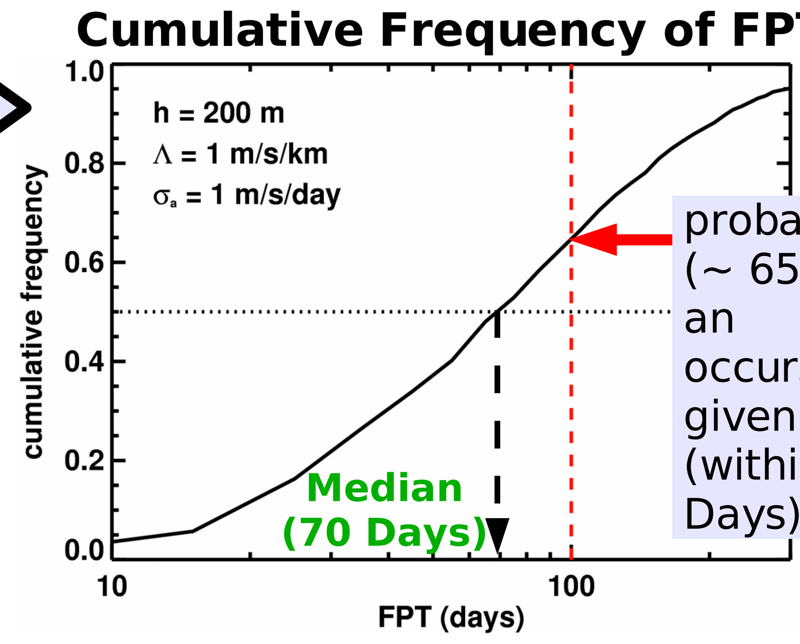
5. First Passage Times (FPTs)

= time taken to first undergo transition from radiative state to wave state after noise is switched on (marked by cross–over of stationary solution

Example Trajectory U(t)



Cumulative Frequency of FPT

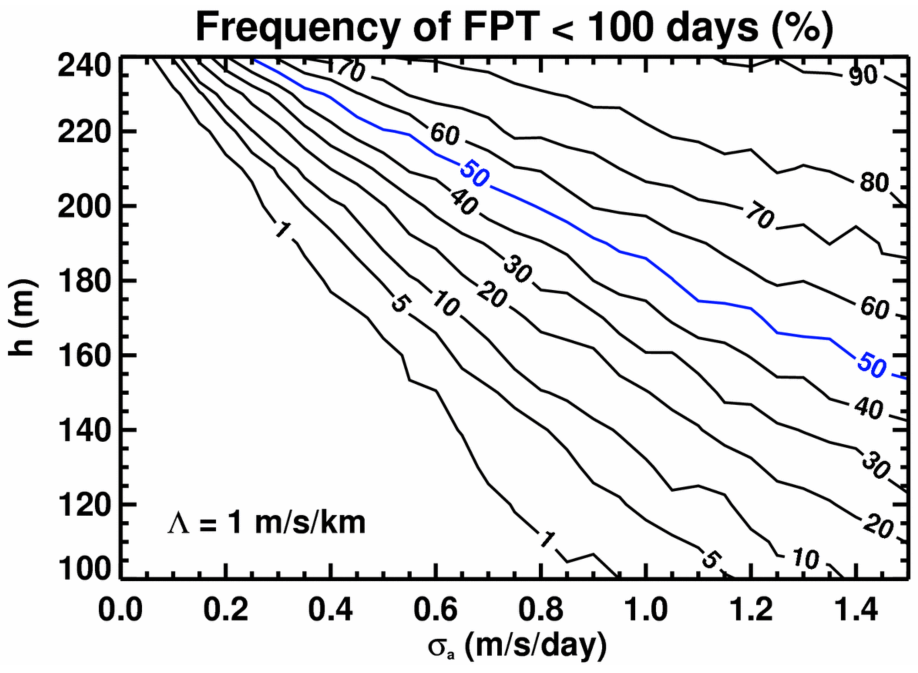


probability (~ 65%) that an SSW occurs in a given winter (within 100 Days)

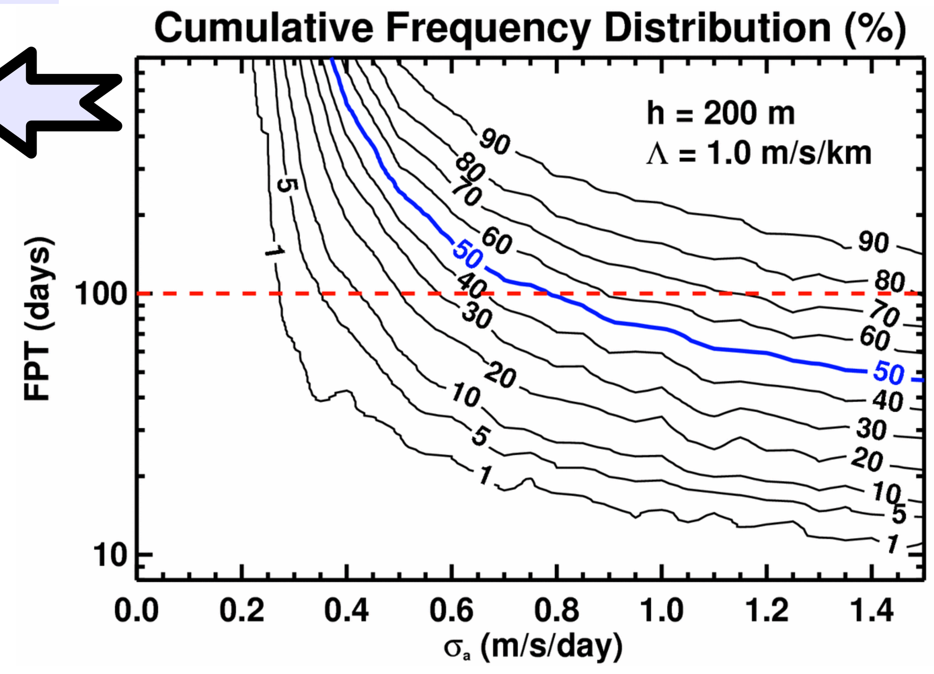
varying σ_a

Probability that an SSW occurs within 100 Days as a function of h and σ_a

Frequency of FPT < 100 days (%)



Cumulative Frequency Distribution (%)



6. Fokker–Planck Equation

For a system governed by: $\frac{d\mathbf{x}}{dt} = \mathbf{A}(\mathbf{x}) + \cancel{\mathbf{B}(\mathbf{x})\eta^M} + \eta^A$ ← **additive noise**

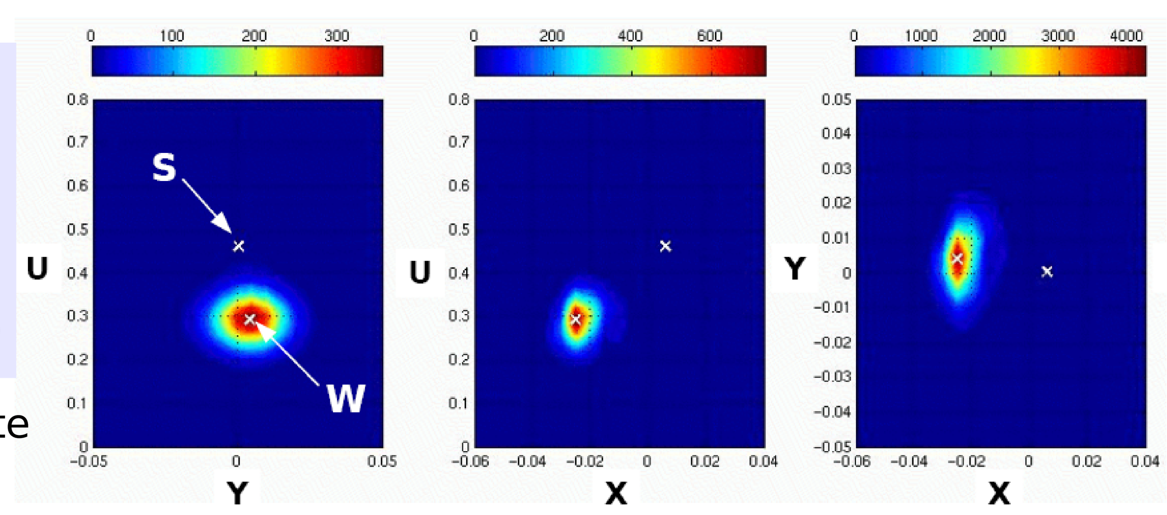
the Fokker–Planck Equation is (e.g. Sura, 2002): $\frac{\partial p(\mathbf{x},t)}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}A_i\,p(\mathbf{x},t) + \frac{1}{2}\sum_i(\sigma_i^A)^2\frac{\partial^2}{\partial x_i^2}p(\mathbf{x},t)$

Stationary solutions:

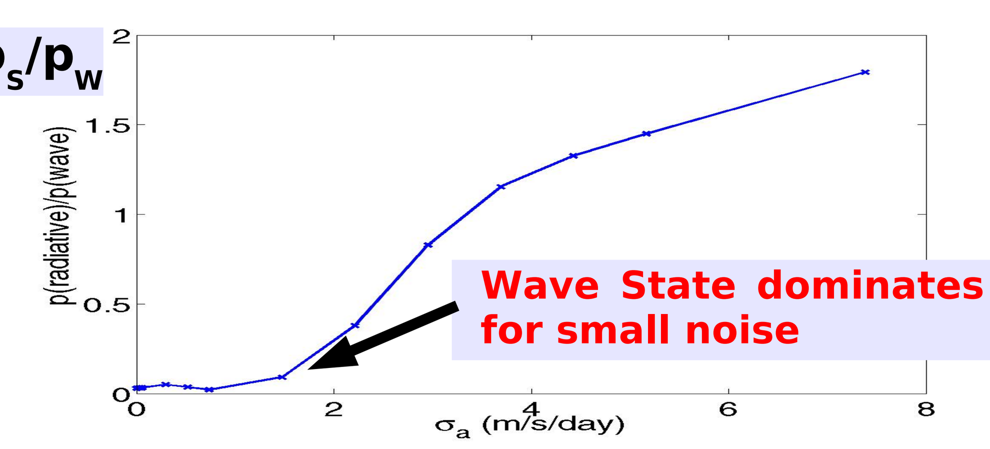
(h = 100 m, Λ = 1 m/s/km, σ_a = 0.8 m/s/day)

S – Radiative State

W – Wave State



Occupation statistics from stationary Fokker–Planck solutions:



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- Ruzmaikin, A., J. Lawrence, and C. Cadavid, 2003: A simple model of stratospheric dynamics including solar variability. *J. Climate, 16*, 1593–1600.
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Acknowledgements:

- useful discussions: Ted Shepherd, Adam Monahan, Paul Kushner, Jahan Davoudi