The Response of Tropospheric Climate to Weak Perturbations

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Motivation

Many recent studies have considered the response of the zonally averaged tropospheric circulation, for example to changes in the extra tropical stratosphere (Kushner & Polvani 2004), to changes in the lower stratosphere that might be associated with the solar cycle, (Haigh et al. 2005) and to surface friction (Chen et al. 2007). A vital part of the response is the change in tropospheric eddy fluxes. The strong coupling between the eddy fluxes and the mean flow makes it very difficult to provide a clear physical mechanism for the response, for example by arguing that changes in the mean flow imply changes in eddy structure.

An alternative approach is to focus on a weak perturbation, where there is a linear operator which predicts the response. The properties of this operator control, for example, the typical spatial structure of the response and its dependence on the location and type of perturbation. The operator encodes the coupled changes in eddy flux and mean flow. One way to calculate the operator is to deduce it from a large number of perturbation experiments. Another is to exploit the fluctuation-dissipation theorem (FDT) which provides a method of calculating the response to forcing of a system by simply observing its variability.

The FDT has recently been used, with some success, by Gritsun & Branstator (2007) as a predictor of atmospheric response to localised forcing. Ring & Plumb (2007,2008) have applied the FDT to the problem described above of the response of the zonally averaged circulation. Their conclusion is that whilst the FDT captures qualitative aspects of the response it makes significant quantitative errors.

Here we consider further the predictive skill of the FDT. In order to draw clear conclusions we must take account of

- Weather the forcing problem being considered is in the linear regime.
- The statistical uncertainty of the forced response.
- The statistical uncertainty in the predictions of the FDT.

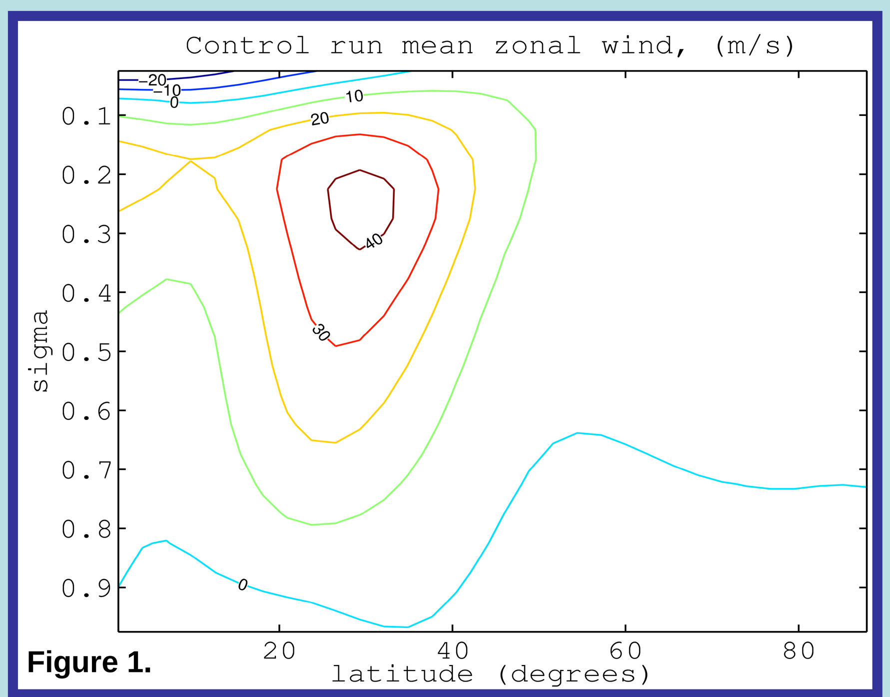


Figure 1.

Experimental Setup

- Dry, primitive equation, simplified global circulation model, Hoskins & Simmons (1975), Simmons & Burridge (1981).
- The model's climate was maintained using the forcing and drag scheme described by Held & Suarez (1994), with no orography, the only modification being the surface friction parameter k_f.
- The statistical certainty of our calculation depends upon small autocorrelation timescales which were found, by experiment, to be given by high surface friction, (k_f=2 day^{-1}), and short sixth order hyperdiffusion timescales of 0.1 days for the smallest wavelengths.
- T21L5 and T42L20 resolution. Only a single hemisphere is required to test the FDT. Data was output in the form of zonal mean fields.
- At each resolution, 10 control runs, 10 forced runs and one run with negative forcing, each over 10^6 days with a 10^4 day spin up. Fields were recorded once per day.
- For forced runs, one tenth of the forcing used by Ring & Plumb (2007) trial three, located at 48° latitude was used. The peak forcing was 0.1 m/s/day.
- An example of the climatological zonal mean wind produced by the T42 experiment is given in figure 1.

Statistical Uncertainty

The response of an atmospheric system is often located in the regions of the highest variance and is therefore not easily distinguished from statistical error. The mean (figure 2) and standard deviation (figure 3), calculated over 10 runs, of the mean zonal mean wind response of the 10^6 day T42 model is plotted. In this particular case the standard deviation of, or uncertainty in, the response is about 25% of the mean. A 10^4 day run produces a standard deviation of about 250% of the mean.

Linear Regime

The responses to a positive and negative forcing of a linear system are equal and opposite. Therefore a measure of the non-linearity of the response is

$$u_{\text{non-lin}} = \frac{1}{2}\sqrt{\frac{\sum_i\left(\delta\mathbf{u}_-(i)+\delta\mathbf{u}_+(i)\right)^2}{\sum_i\left(\delta\mathbf{u}_+(i)\right)^2}}$$

where the subscripts denote a positive or negative forcing vector and i denotes the grid point. This is the combined geometrical length of the non-linearity in units of the response vector. We measure $u_{\text{non-lin}}$ to be 0.26±0.06 and 0.17±0.09 for the respective T21 and T42 models.

Truncation

For convenience we restrict our attention to the zonal wind and the system is truncated to only the first principal component, leaving a one dimensional time series. The first principal component contains 57±13% and 95±1% of the variability for the respective T21 and T42 experiments.

Calculation of a Forced Response

The linear response vector $\delta\mathbf{u}$ and an applied forcing vector $\delta\mathbf{f}$ are related by a response matrix **L**

$$\delta\mathbf{u} = \mathbf{L}\delta\mathbf{f} \quad (1)$$

To calculate the linear response matrix **L** of the GCM to a forcing we use both the integral method given by Gritsun & Branstator (2007)

$$\mathbf{L} = \int_0^\infty \mathbf{C}(\tau)\mathbf{C}(0)^{-1}d\tau \quad (2)$$

and the log method given by Penland (1989)

$$\mathbf{L} = \left[\frac{-1}{\tau}\log\left(\mathbf{C}(\tau)\mathbf{C}(0)^{-1}\right)\right]^{-1} \quad (3)$$

where $\mathbf{C}(\tau)$ is the lagged covariance matrix at a lag of τ. For practical use, the upper limit of integration in the integral calculation and the lag τ used in the log method, are free parameters that must be carefully chosen.

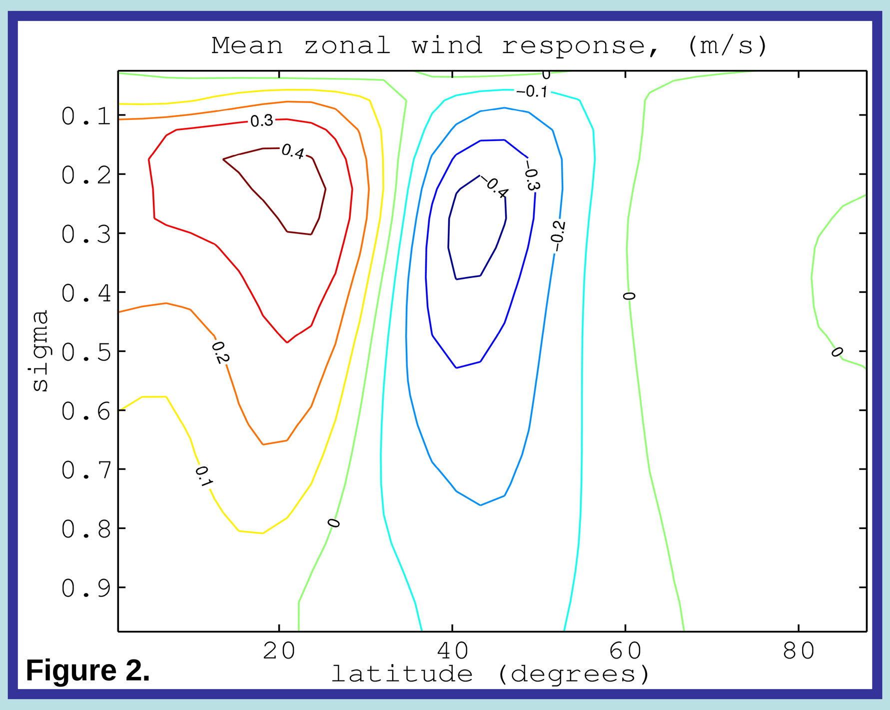


Figure 2.

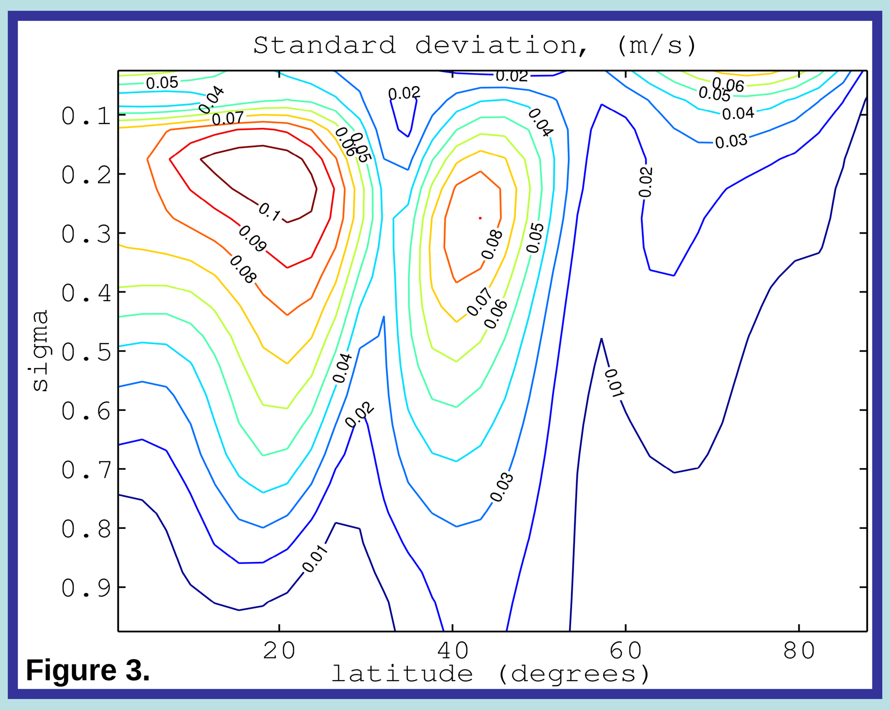


Figure 3.

Autocorrelation

The autocorrelation function is closely related to the lag τ covariance matrix C(τ) used in equations (2) and (3) and a calculated response must take account of statistical error and systematic bias.

Statistical Error

The standard deviation of the time mean of a GCM variable is proportional to $1/\sqrt{n}$ where n is the length of the run. The same relation is true for the autocorrelation of long runs.

Systematic Error

The principal component autocorrelation function is systematically biased and has a large standard deviation for short runs. For a T21L20 model (figure 4), samples of a length specified by the x axis were taken from a 10^6 day run. The mean (circle) and standard deviation (cross) of these samples is plotted. The dashed line is a $1/\sqrt{n}$ fit. Similar plots may be produced from an ensemble. This behaviour is a property of the autocorrelation function, so a linear Markov model produces the same result. The length of run required for a small enough sample autocorrelation bias is dependent upon the true autocorrelation. It varies from model to model.

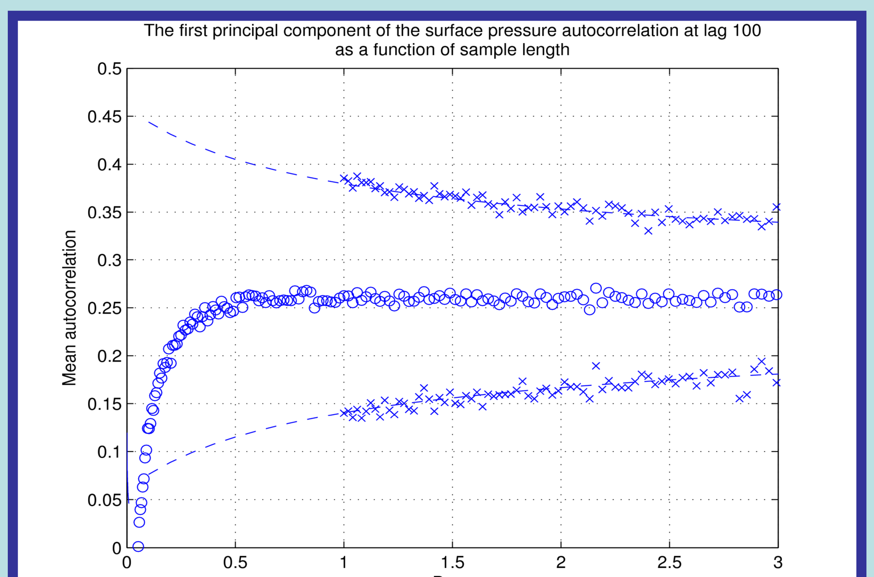


Figure 4.

Results and Conclusions

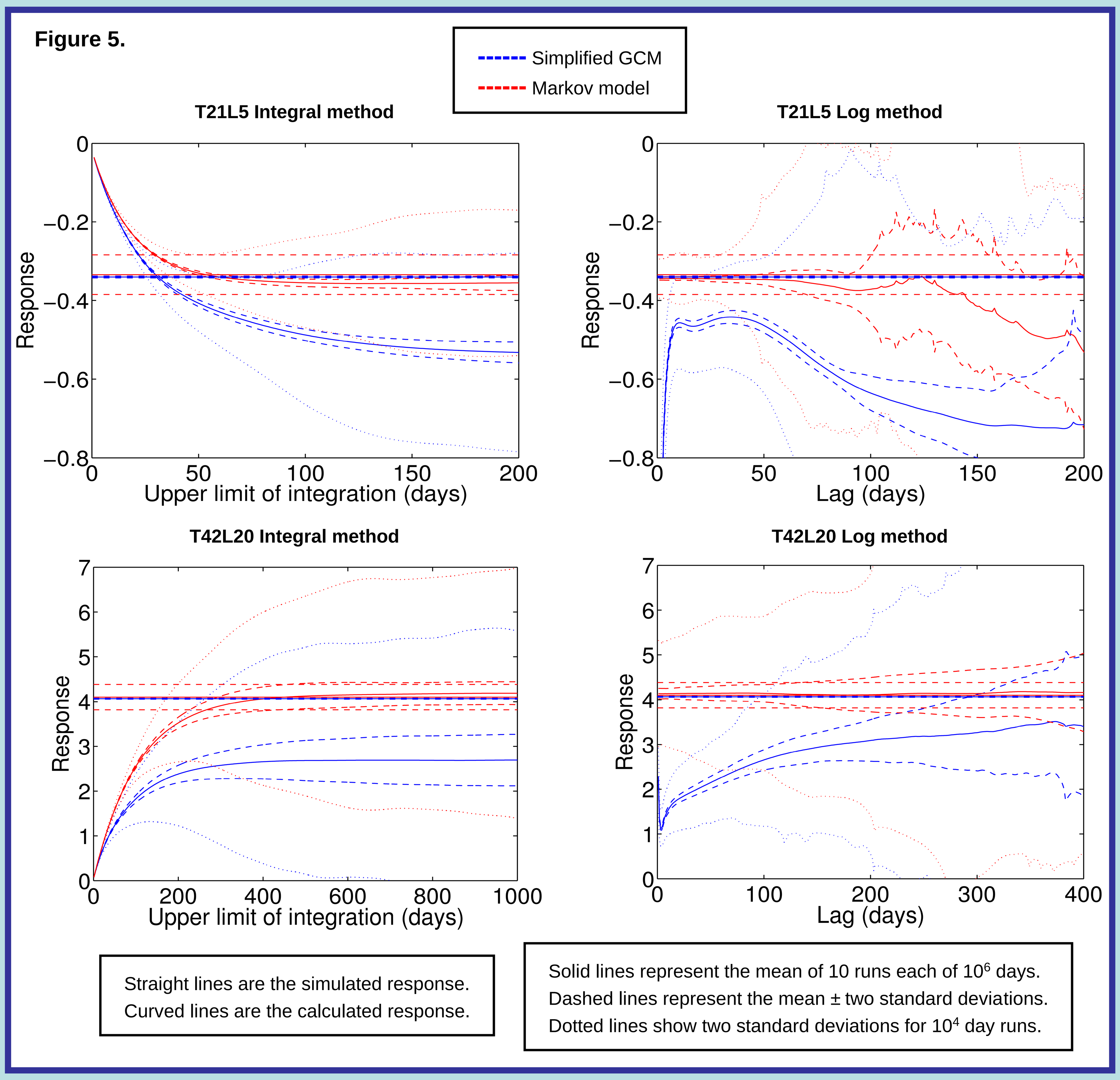
The one dimensional response of the first principal component was calculated directly from a control and a forced run. Using equation (1), **L** was then calculated and used to drive a one dimensional linear Markov model, producing identical length runs.

The response calculated using equation (2) demonstrates that both the solution converges and the standard deviation increases with a larger upper limit of the integral. Applying equation (3) to the Markov model, both the standard deviation and the bias increase with lag τ. The fact that the response predicted by the log method changes significantly as a function of lag τ suggests that a linear model is inadequate.

The length of the runs used here are sufficient to quantitatively separate the predictions of the FDT and the actual GCM response. The dotted lines illustrate that these results are difficult to interpret if we restrict ourselves to an ensemble of 10 shorter (10^4 days) runs. Systematic biases over timescales less than 10^6 days are also accounted for.

Conclusions - The four plots below (figure 5) demonstrate that the linear FDT implemented by equations (2) and (3) works well for a linear Markov model and that it does not work as well for the GCM runs described here. What is the possible reason for this result? Equation (3) requires that the system is linear, equation (2) makes the weaker assumption that the pdf. of the system is Gaussian. The non-Gaussianity of the statistics of the GCM could therefore be the reason for the quantitative disagreement between the actual response and that predicted by the FDT.

Figure 5.



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