Using data assimilation to improve climate models: The stratosphere

Manuel Pulido

Department of Physics, University of Toronto and UNNE, Argentina

S. Polavarapu, T. Shepherd, J. Thuburn.

1. **Estimation of the missing forcing.**
   - **Twin experiments**
   - **Realistic experiments**
2. **Parameter estimation.**
   - **Variational data assimilation**
   - **Genetic algorithm**

Source of errors in climate models: subgrid effects

GCMs do not resolve all the motion scales, because of this, they are not able to capture the systematic momentum forcing that is produced by small-scale waves.

There is no simple way to infer this systematic momentum deficit (missing drag) in a GCM.

If one computes the difference between the true state and the model state, the result is a combination of different source of errors, recent and past, which once they are generated are advected and interact with other parts of the system.

Is there an objetive one to find the source of the momentum deficit, i.e., the exact time and position where the momentum error was produced?

Then if we force the momentum equation with this 4D vector field (RHS forcing term), the model will evolve exactly as the true state.

Using data assimilation to diagnose ’missing drag’

4DVar can be used to estimate unknown parameters of a model.

There is no background information (perfect ignorance), so the cost function is defined as

$$J = \frac{1}{2}\sum_{i=1}^{n}(H[\mathbf{y}_i] - \mathbf{x}_i)^T \mathbf{R}^{-1}(H[\mathbf{y}_i] - \mathbf{x}_i)$$

where x_i is the model state, y_i are the observations. The state is given by the model evolution from t_0 to t_i

$$\mathbf{x}_i = M(\mathbf{x}_0, \boldsymbol{X}, t_i)$$

The model state is a function of the initial condition and also of the ’missing drag’. Then $J = J(\mathbf{x}_0, \boldsymbol{X})$

Therefore, if we know x_0 the control space of the cost function is only the field X. The minimum of the cost function gives the ’missing drag’ (Pulido and Thuburn, 2005).

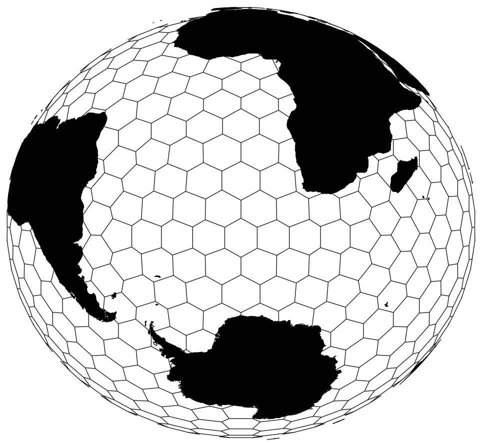
Middle atmosphere dynamical model, $M(\mathbf{x}_0, \boldsymbol{X}, t_i)$

The dynamical model is based on the fully nonlinear, hydrostatic primitive equations, with an isentropic vertical coordinate and a hexagonal-icosahedral horizontal grid (Thuburn 1994).

$$\partial_t \sigma + \nabla \cdot (\sigma \mathbf{u}) + \partial_\theta (\sigma \dot{\theta}) = 0$$

$$\partial_t (\sigma Q) + \nabla \cdot (\sigma Q \mathbf{u} - \hat{\mathbf{k}} \times \dot{\theta} \partial_\theta \mathbf{u}) = X_\zeta$$

$$\partial_t \delta + \nabla \cdot [\sigma Q \hat{\mathbf{k}} \times \mathbf{u} + \nabla(\Psi + \frac{\mathbf{u}^2}{2}) + \dot{\theta} \partial_\theta \mathbf{u}] = X_\delta$$



Horizontal icosahedral grid.

The bottom boundary condition is set at $p \approx 100mb$, where a time dependent observational Montgomery potential is imposed.

A realistic parametrisation of radiative transfer is used (Shine 1987; Shine and Rickaby 1989).

The Adjoint Model

The gradient of the cost function is calculated with the adjoint model.

The missing forcing is assumed constant within an assimilation window length.

1. Develop the tangent linear of the dynamical model:

$$\begin{bmatrix} \delta x^{n+1} \\ \delta X^{n+1} \end{bmatrix} = \begin{bmatrix} M'(x^n) & I \\ 0 & I \end{bmatrix} \begin{bmatrix} \delta x^{n} \\ \delta X^{n} \end{bmatrix}$$

2. Develop the adjoint from the tangent linear model

$$\begin{bmatrix} \delta \hat{x}^{n-1} \\ \delta \hat{X}^{n-1} \end{bmatrix} = \begin{bmatrix} M'^T(x^n) & 0 \\ I & I \end{bmatrix} \begin{bmatrix} \delta \hat{x}^{n} \\ \delta \hat{X}^{n} \end{bmatrix}$$

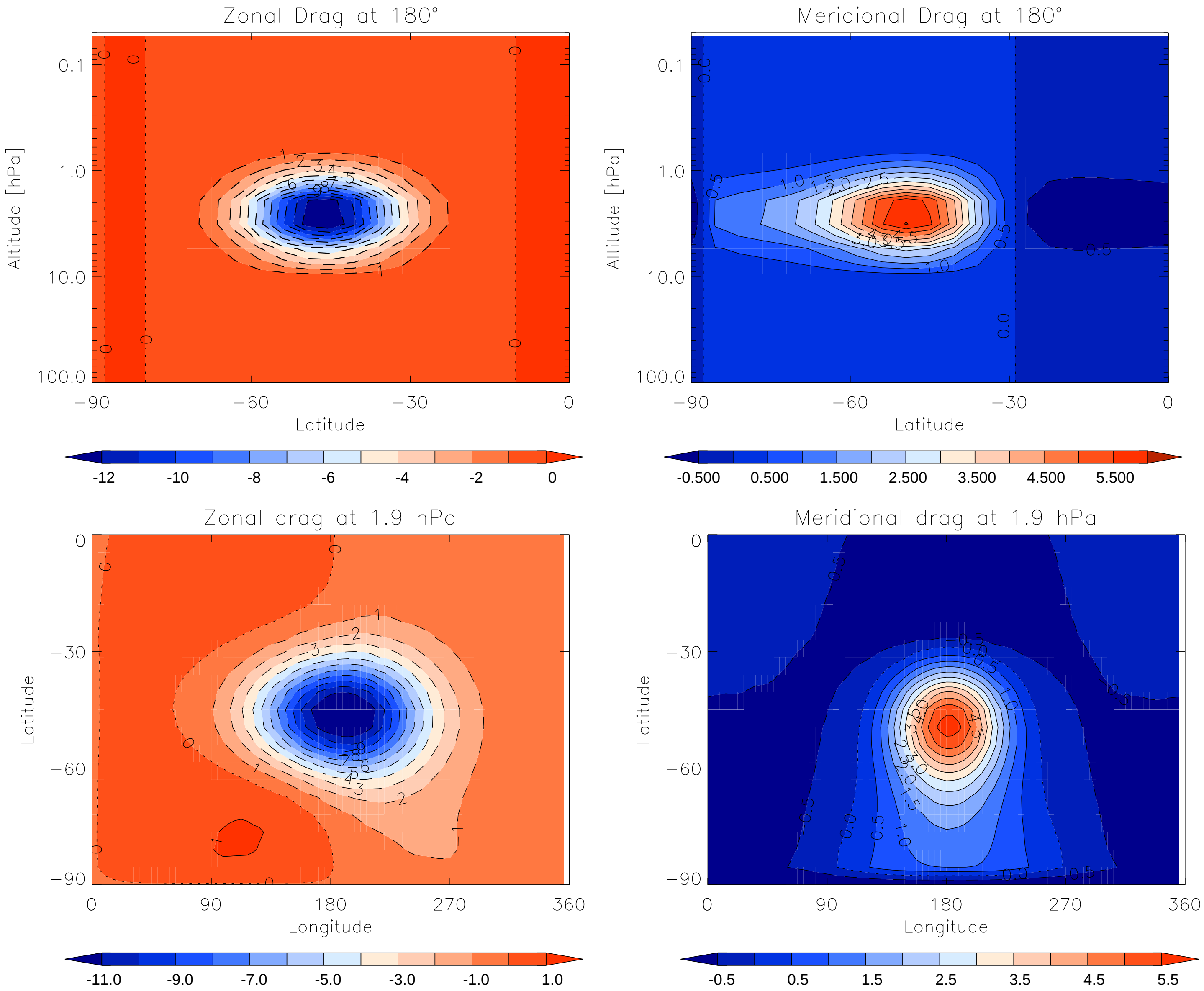
3. Finally, the gradient of the cost function is given by

$$\begin{bmatrix} \frac{\partial J}{\partial x^0} \\ \frac{\partial J}{\partial X^0} \end{bmatrix} = \sum_{n=0}^{N-1} \begin{bmatrix} M'^T(x^n) & 0 \\ I & I \end{bmatrix} \cdots \begin{bmatrix} M'^T(x^{N-1}) & 0 \\ I & I \end{bmatrix} \begin{bmatrix} \frac{\partial J}{\partial x_N} \\ \frac{\partial J}{\partial X^N} \end{bmatrix}$$

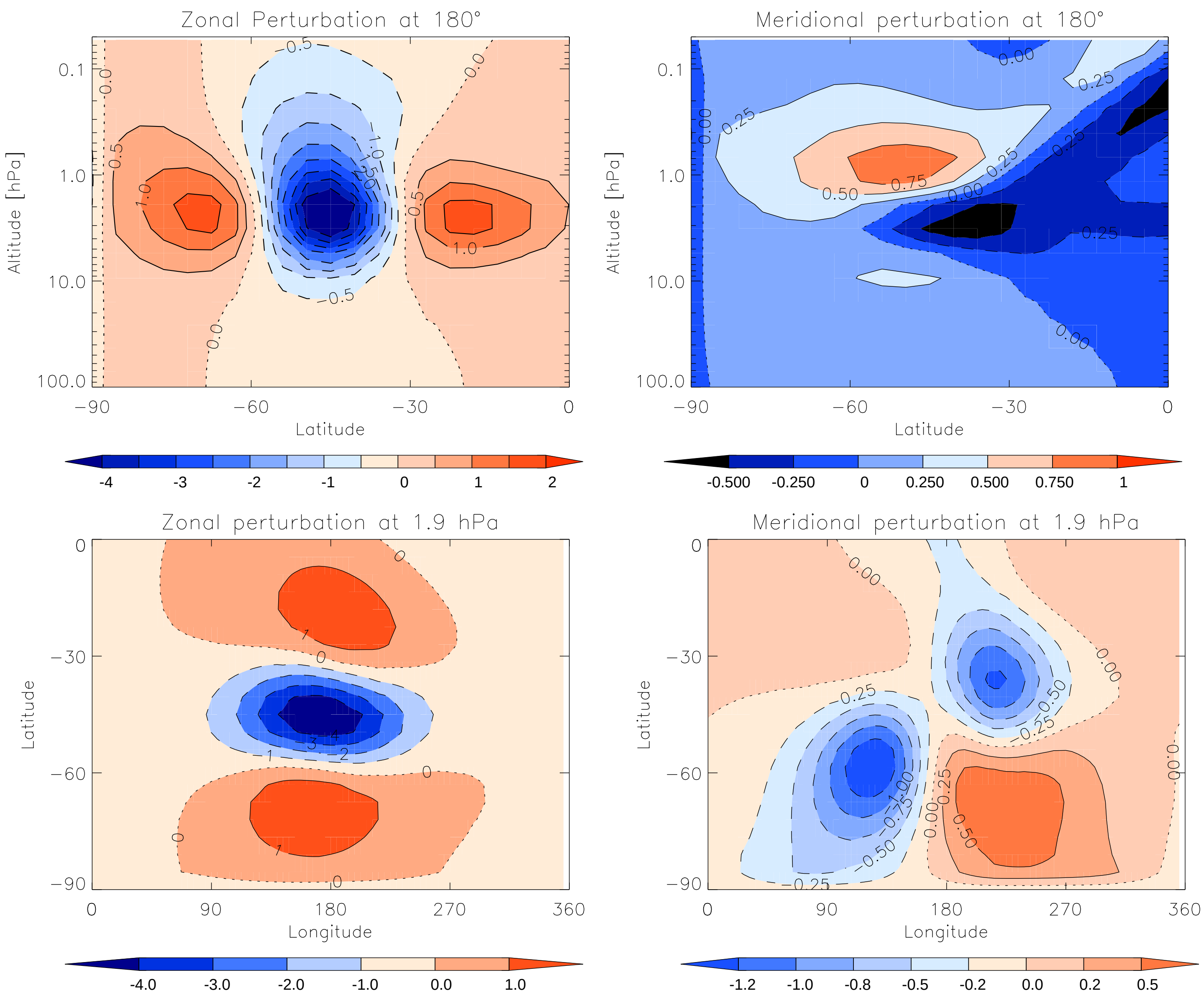
Twin experiments

Experiment:

- Gaussian forcing used as the prescribed drag for the twin experiments.
- The adiabatic evolution is started from resting condition with an isothermal atmosphere.
- The model evolution with the prescribed drag is taken as the observation.



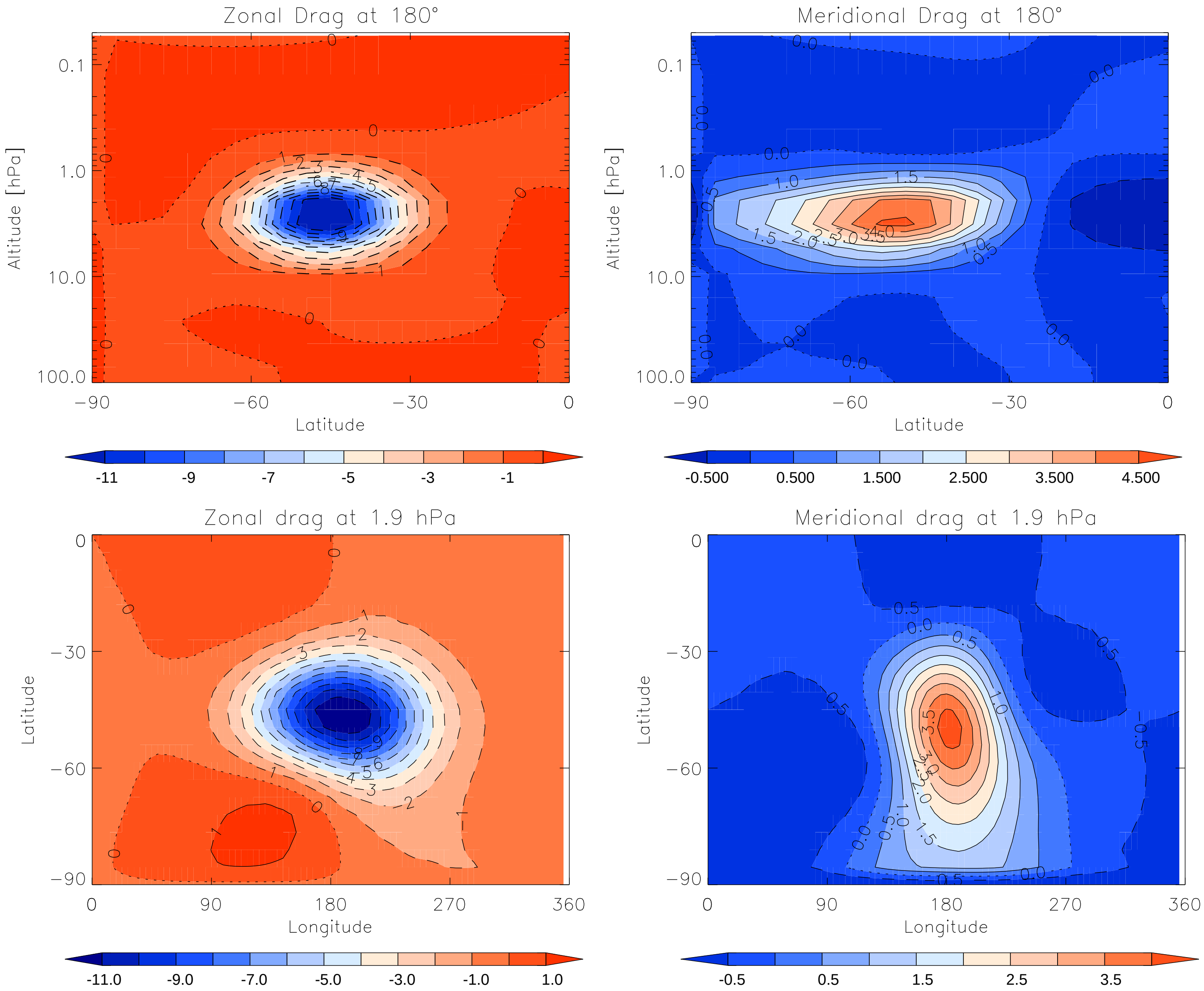
Flow response. 'The observations'



Flow response to the applied drag at $t = 1$ day. This could be interpreted as a crude budget calculation:

$$\mathbf{X} = [\mathbf{u}(1\mathrm{d}) - \mathbf{u}(0)]/1\mathrm{d}$$

Estimated drag



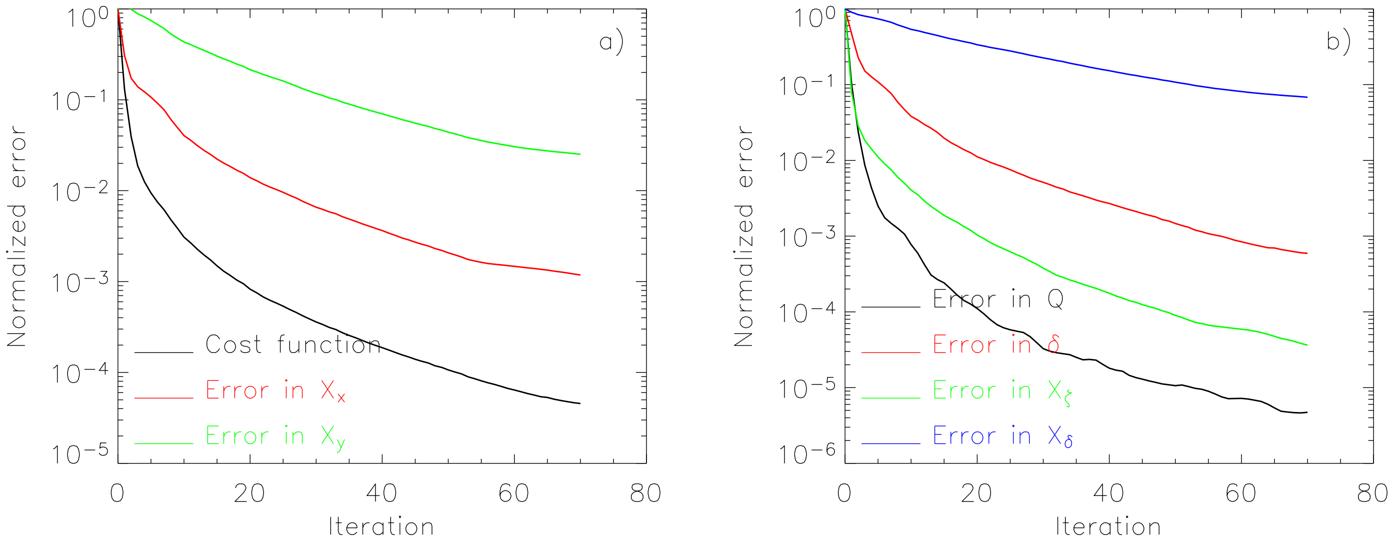
Estimated drag after 25 minimisation iterations.

Observations are: $\sigma^*(1\mathrm{d})$, $Q^*(1\mathrm{d})$ and $\delta^*(1\mathrm{d})$. So that

$$J = \sum(\delta - \delta^*)^2 + \overline{\sigma}^2(Q - Q^*)^2 + (\tau\overline{\sigma})^{-2}(\sigma - \sigma^*)^2$$

The error in the drag estimation is smaller than 1 m/s/day (Pulido and Thuburn 2005).

Convergence

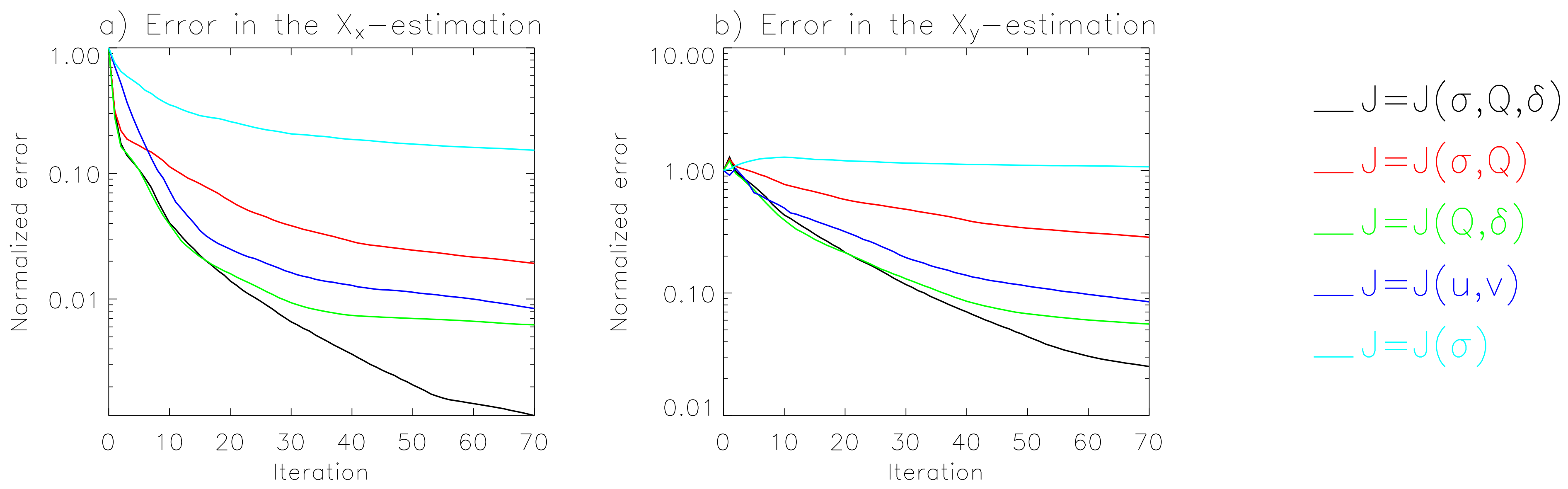


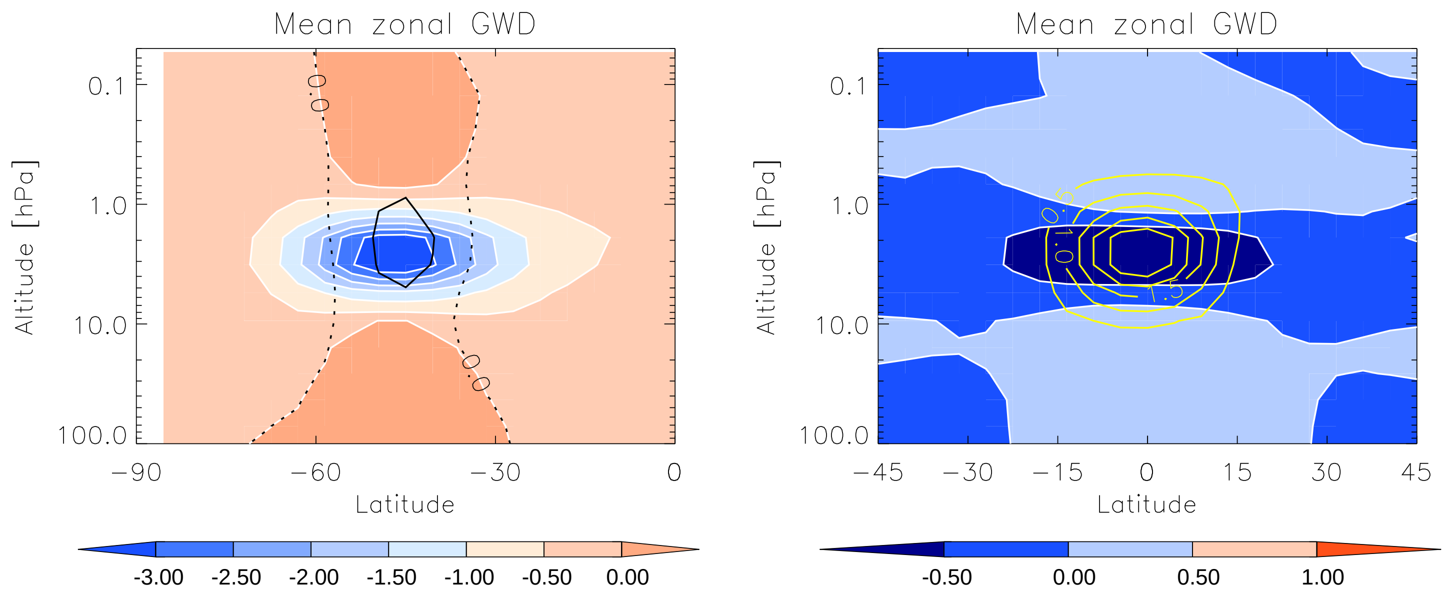
Error as a function of minimisation iteration.

25 minimisation iterations are enough to find a good drag estimate.

The rotational component of drag is better estimated than divergence component.

Limited observational information





Experiment $J(\sigma)$ with prescribed drag at midlatitudes (left panel) and at the equator (right panel).

Shading contours are the estimations and contours are the errors.

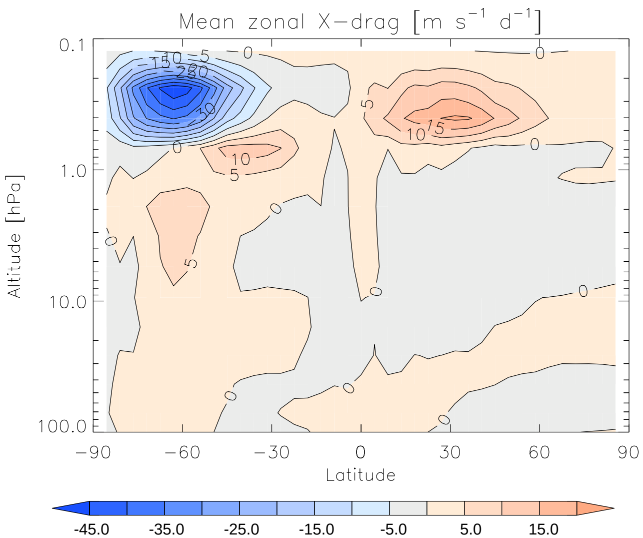
Real estimations: Met Office analysis

Observations: Met Office middle atmosphere analyses.

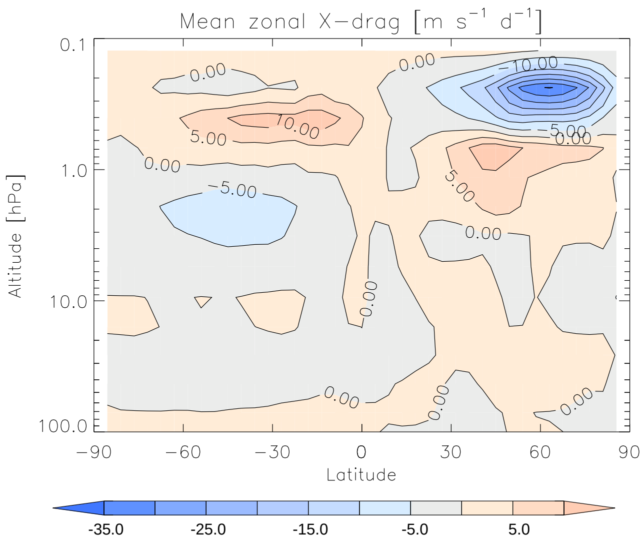
Initial condition: for the first assimilation window of each month is taken from MO analyses, for subsequent windows we use our analyses.

Cost function: potential vorticity and pseudo-density (function of temperature only) are used as observed variables which are taken from MO analyses.

Control space: Curl of drag only.



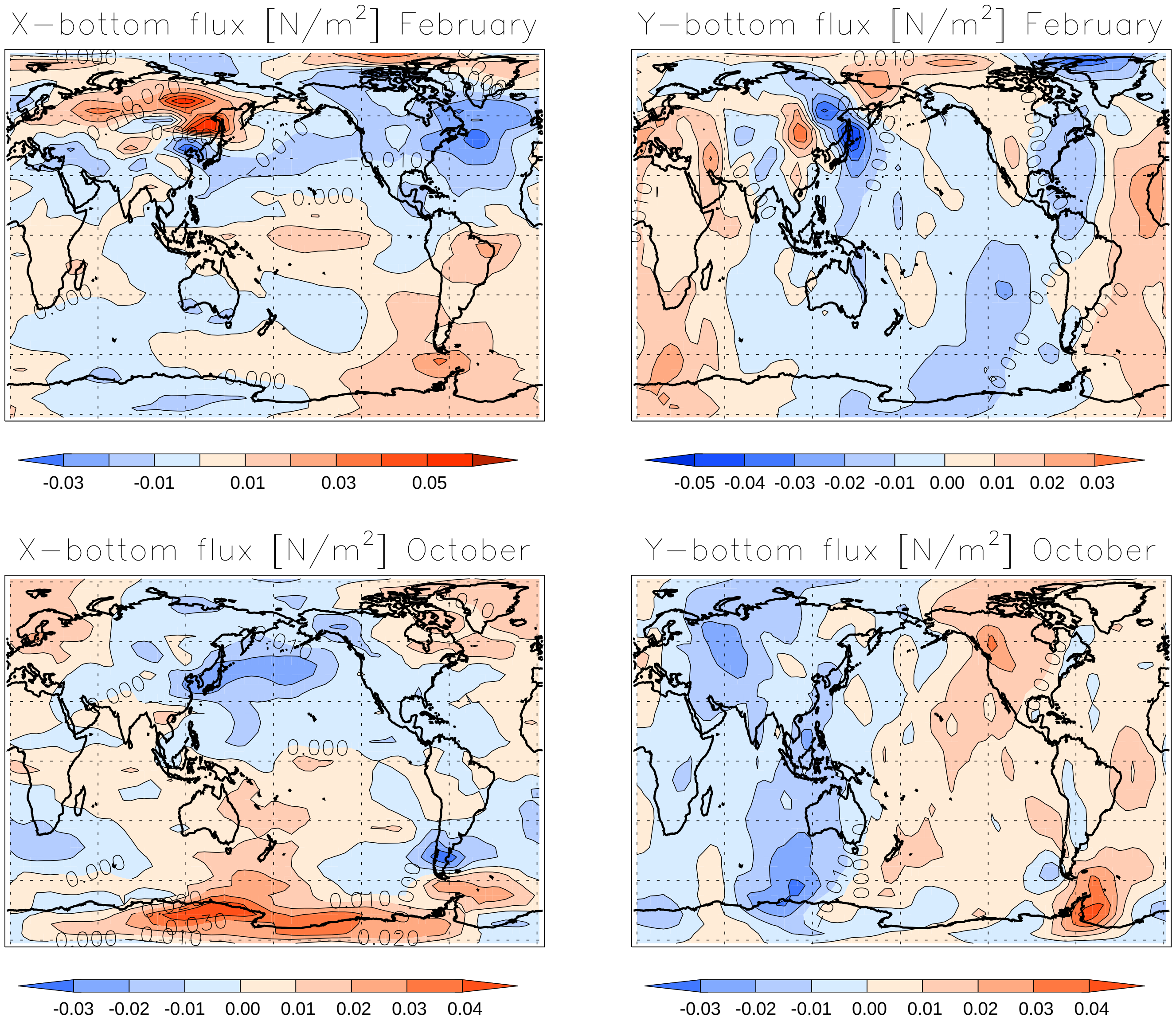
July



December

Estimated zonal mean monthly averaged zonal drag

Bottom momentum flux: Sources?



Integrating drag and neglecting the top momentum flux:

$$F_b = \int_{\theta_b}^{\theta_t} \sigma X_x \mathrm{d}\theta.$$

For February (upper panels) and October (Lower panels)

Pulido and Thuburn (2008).

A further step: Parameter estimation

Can GW schemes with optimum parameters reproduce the estimated missing drag?

Scinocca (2002) scheme assumes a launch EP momentum flux spectrum is given by

$$E(c, z_l) = \frac{4E*}{\pi c_*^2} c \left[1 + \left(\frac{c}{c_*} \right)^4 \right]^{-1}$$

$c_* \equiv \frac{N_l \lambda_*}{2\pi}$ is the characteristic phase speed and E_* the total momentum flux.

The dissipation of the waves is activated when a component of the spectrum exceeds a saturation threshold given by

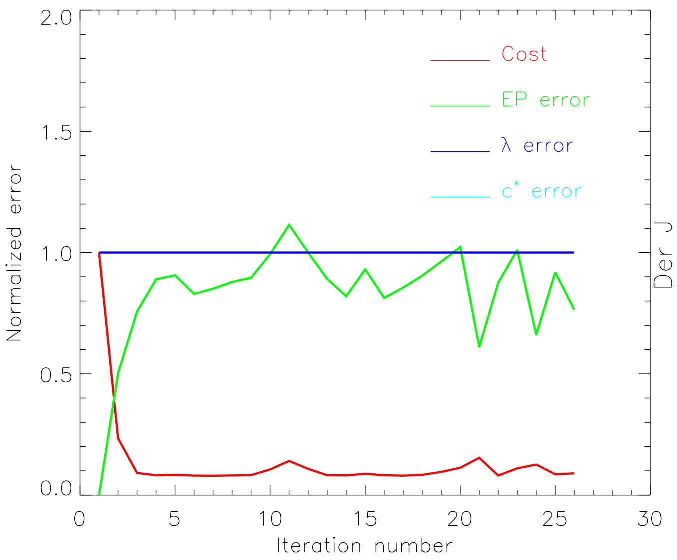
$$E_s(c, z) = \frac{S_* E_*}{c_*^2} \frac{\rho(z) N_l}{\rho_l N(z)} \frac{[c - u(z)]^2}{c}$$

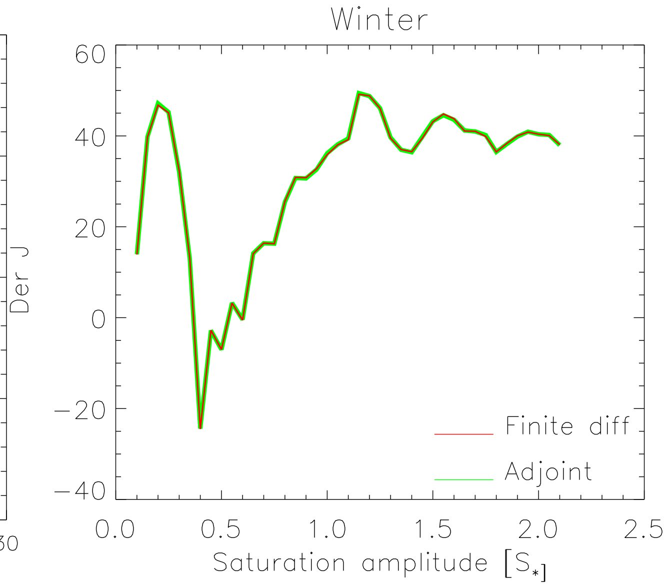
The momentum flux that is eliminated and the drag are given by

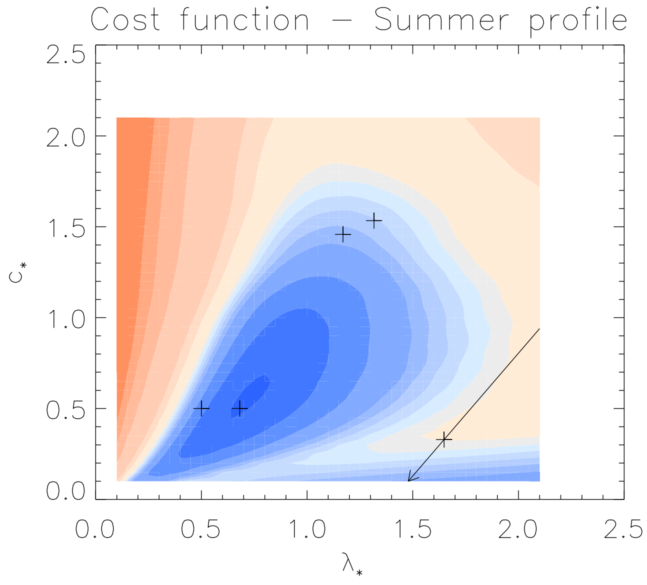
$$E_T(z) = E_* - \int_0^{c_c} [E(c, z_l) - E_s(c, z)] \, \mathrm{d}c \qquad X = \rho^{-1} \partial_z E_T.$$

Optimum parameters: Variational data assimilation

The cost function is defined as: $J = (\mathbf{x} - \mathbf{y})^T \mathrm{R}^{-1} (\mathbf{x} - \mathbf{y})$ where $\mathbf{y}$ is the observed GWD profile and $\mathbf{x} = X(E_*, \lambda_*, S_*)$ is the forcing resulting from the GW scheme.





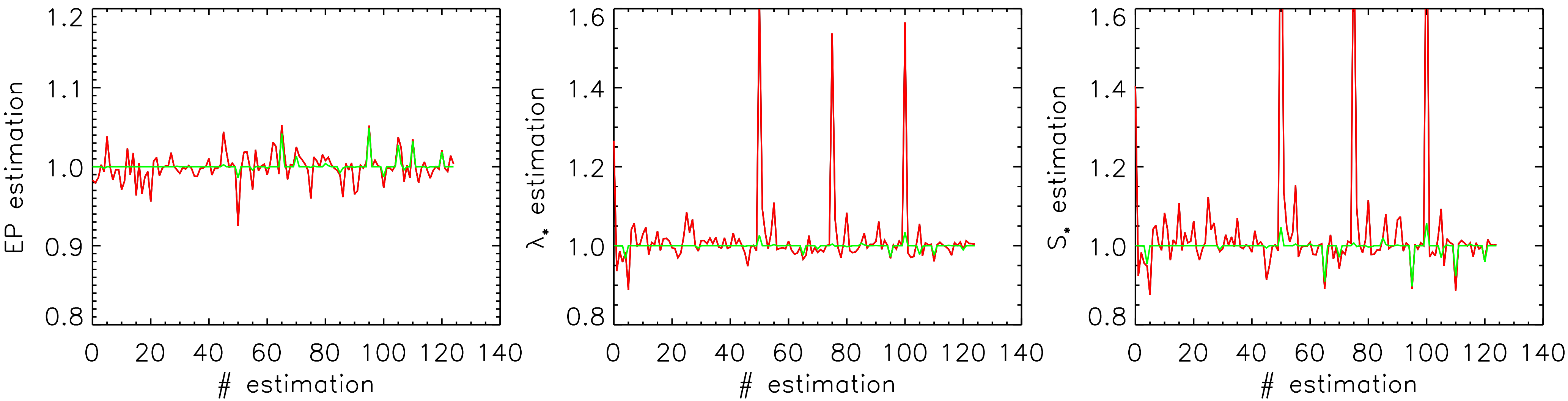


Variational data assimilation similar to Drag estimation, the minimization is performed by a conjugate gradient method.

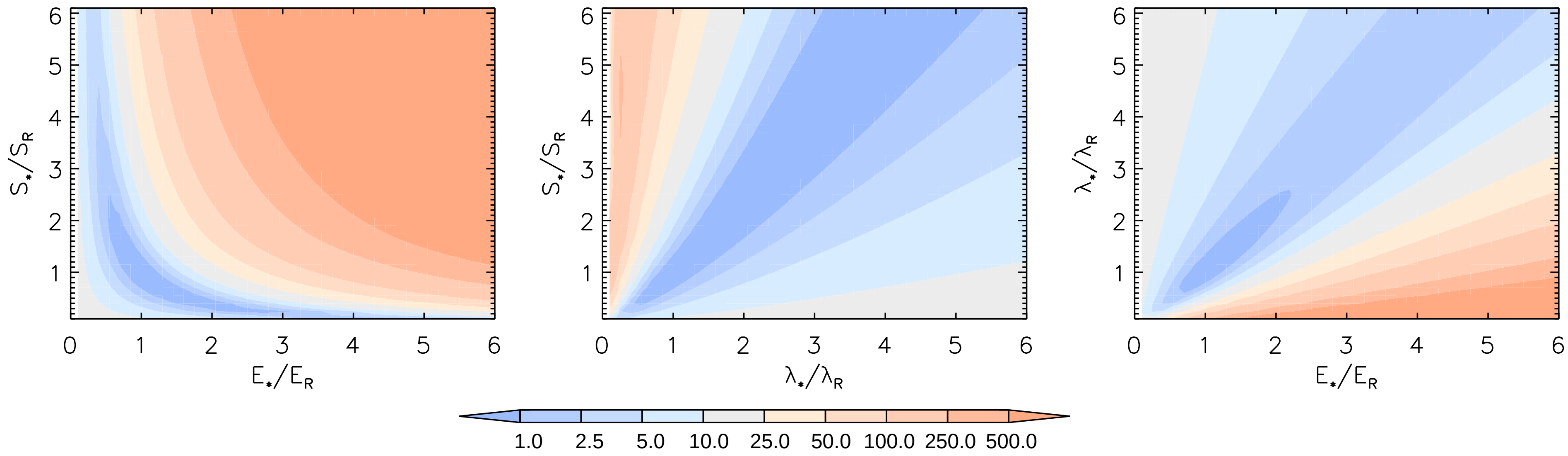
Optimum parameters: Genetic algorithm

A genetic algorithm developed in NCAR by Charbonneau and Knapp (1995) is used to minimize the cost function.

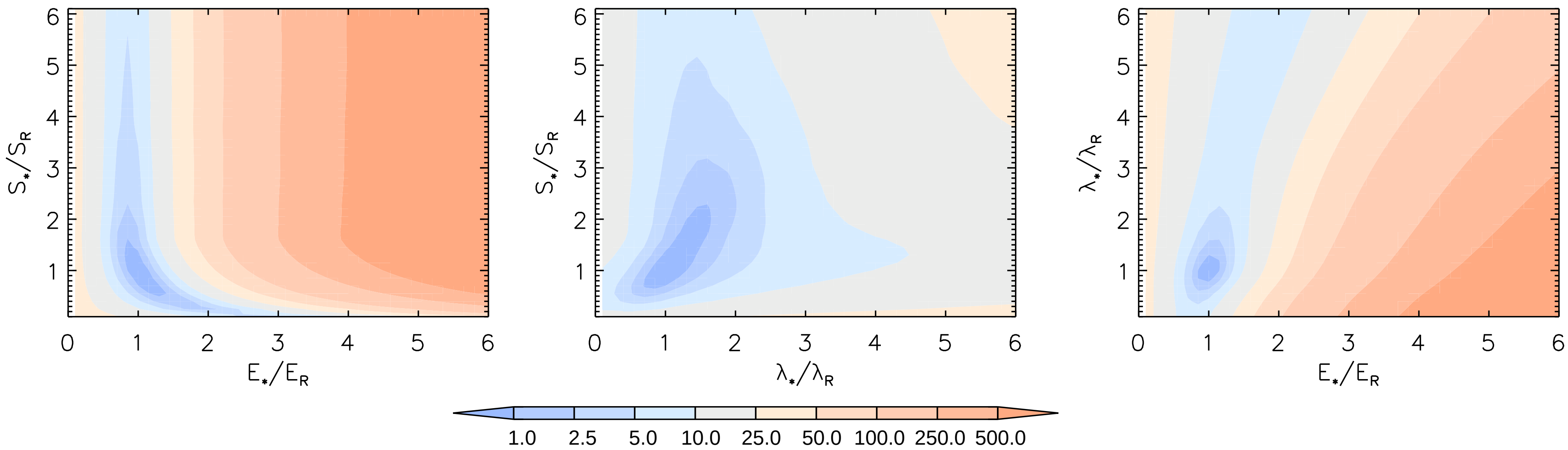
- The minimization is perfomed in a constrained domain.
- We set the number of individuals in a population to 100 and the number of generations to 200.



Cost function geometry. R matrix?



$\mathrm{R}^{-1} = \mathrm{I}$ i.e. the observed variable is GWD, $\mathbf{y} = X$.

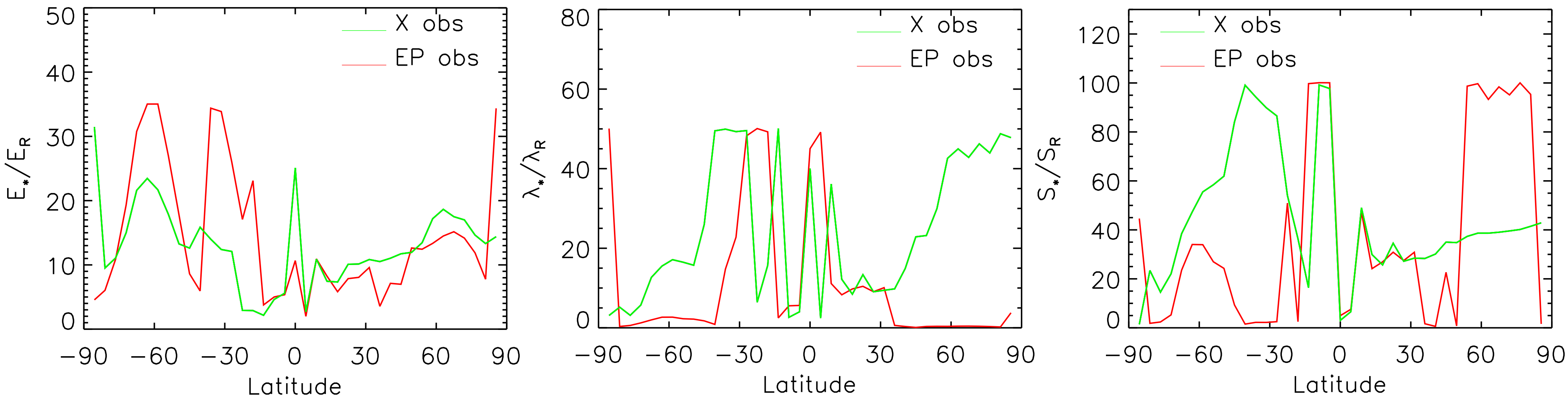


$\mathrm{R}^{-1} = \rho^T \rho$ i.e. the observed variable is EP flux divergence, $\mathbf{y} = \rho X$.

Estimated parameters

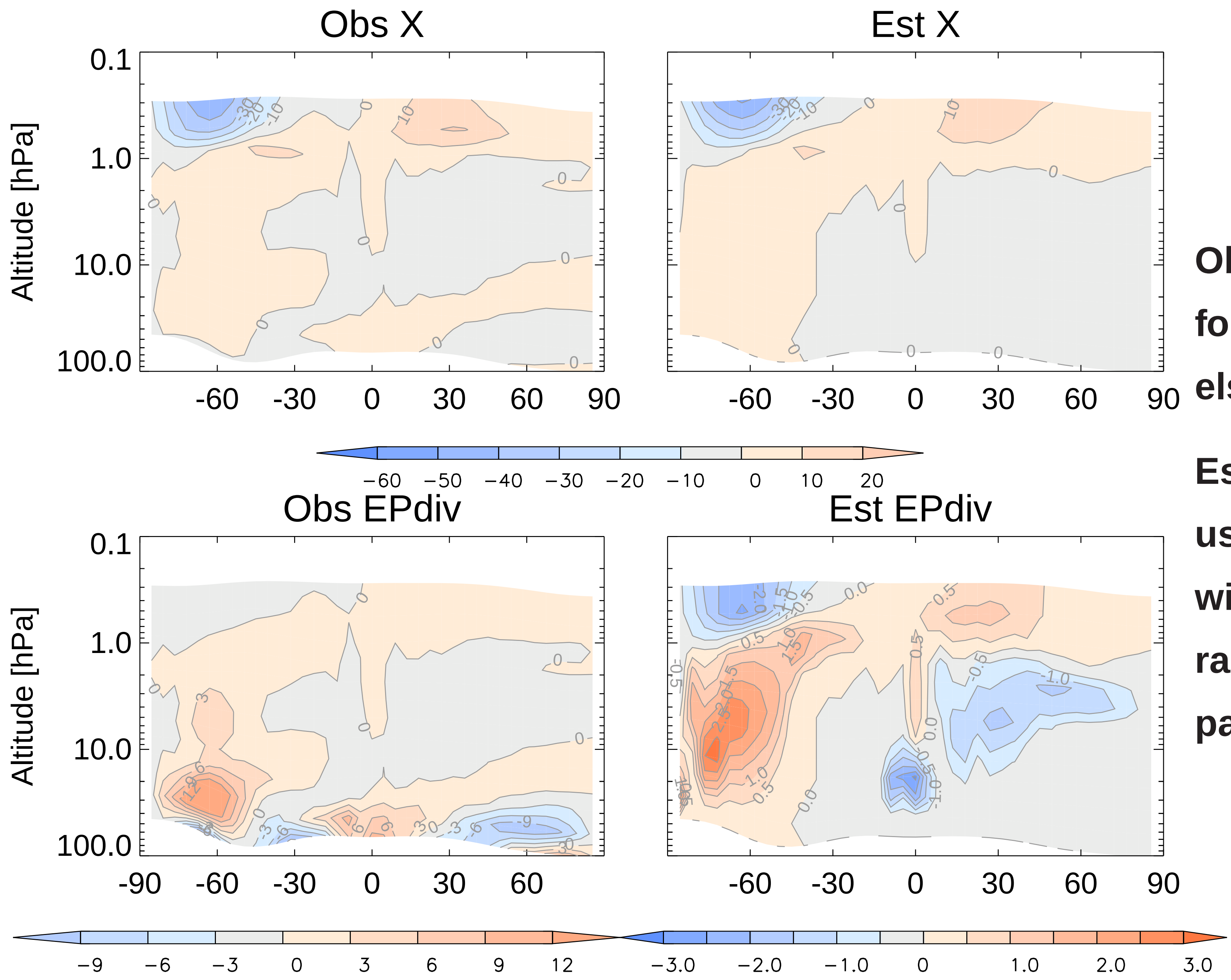
Zonal wind and temperature is taken from Met Office analysis.

The GWD field estimated with the ASDE-4DVar technique (Pulido and Thuburn, JC 2008) for July 2002 is used as observational forcing profile $\mathbf{y}$.



Parameters E_* (left) λ_* (middle) and S_* (right) estimated for Met Office analysis in July 2002

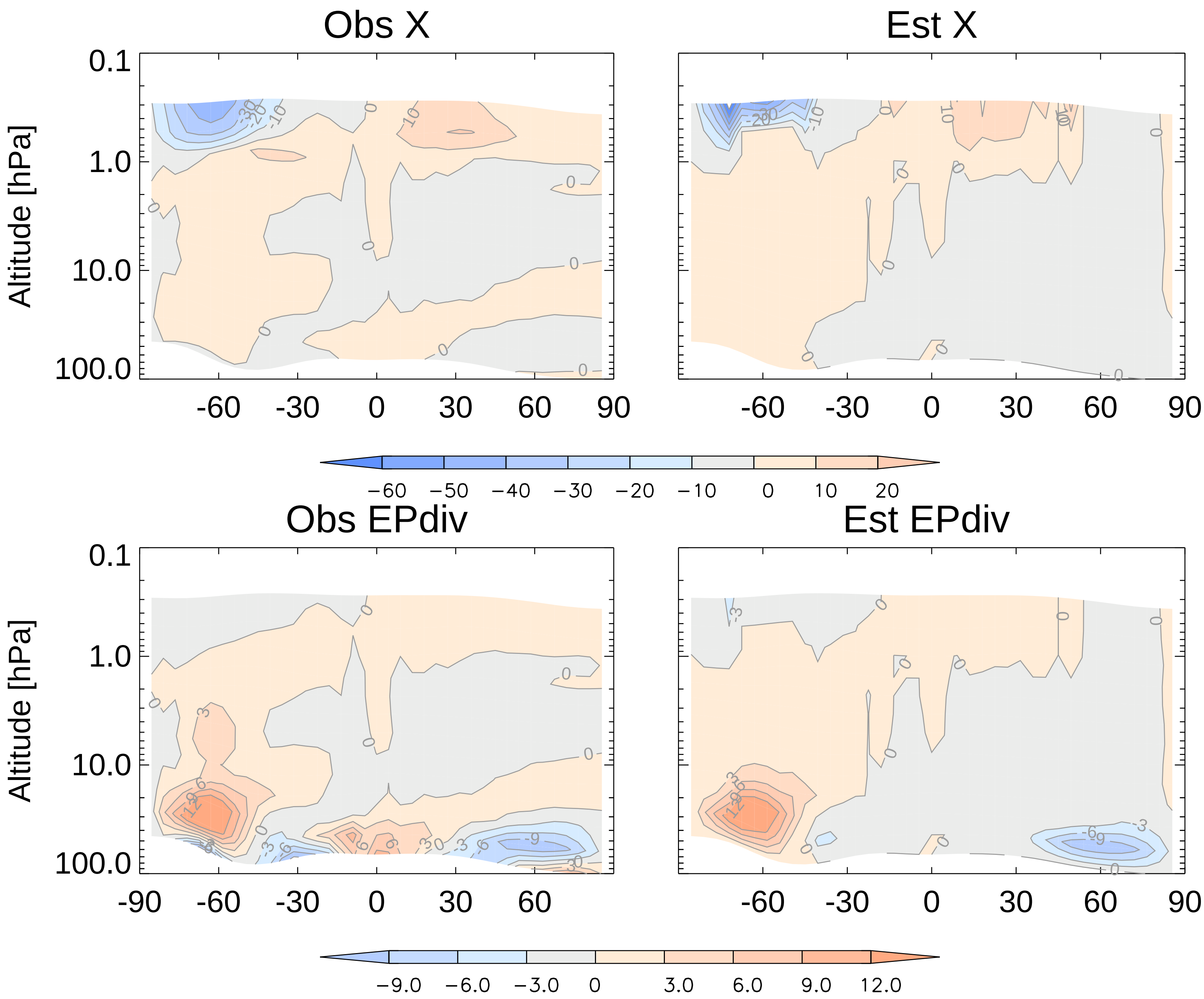
Case $\mathbf{y} = X$: Estimated GWD and EP div



Observed ASDE forcing (left panels).

Estimated forcing using GW scheme with optimum parameters (right panel).

Case $\mathbf{y} = \rho X$: Estimated GWD and EP div



Observed ASDE forcing (left panels).

Estimated forcing using GW scheme with optimum parameters (right panel).

Conclusions

Estimating the source of missing momentum:

- **Variational data assimilation may be used to estimate the missing force for a given climate model.**
- **The 4DVar technique appears to give robust results with very good convergence.**
- **It is able to estimate the ’launch’ momentum flux**

Estimating parameters of GW schemes:

- **Variational data assimilation may be not useful for estimating parameters of physical parameterizations, since the sensitivity is usually nonlinear.**
- **A genetic algorithm works well for this low dimension problem.**
- **The technique is able to reproduce the ’observed missing forcing’.**
- **A physical interpretation of the estimations is not direct.**