Relevant papers to SPARC-DA from M07 and Data Assimilation Using Modulated Ensembles

Craig H. Bishop, Daniel Hodyss
Naval Research Laboratory
Monterey, CA, USA
July 21, 2009

Overview

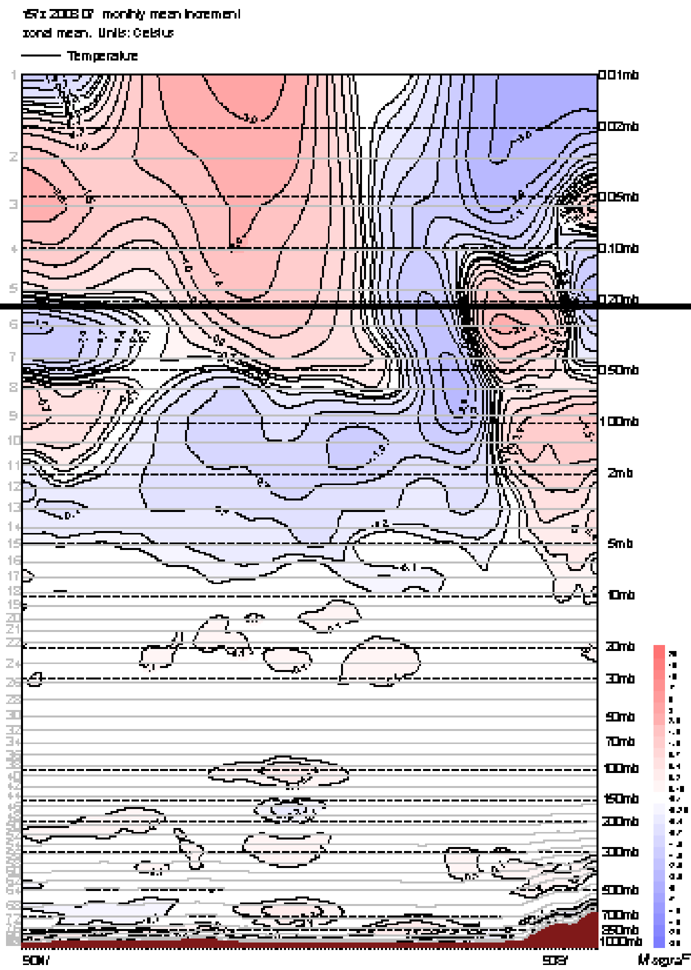
- Tremolet and M07
- Motivation for adaptive ensemble covariance localization
- Method
- Results from preliminary comparison of DA performance with operational covariance model using pseudo-obs
- Show that adaptive localization enables ensemble based TLM
- Conclusions

Balance in the Stratosphere

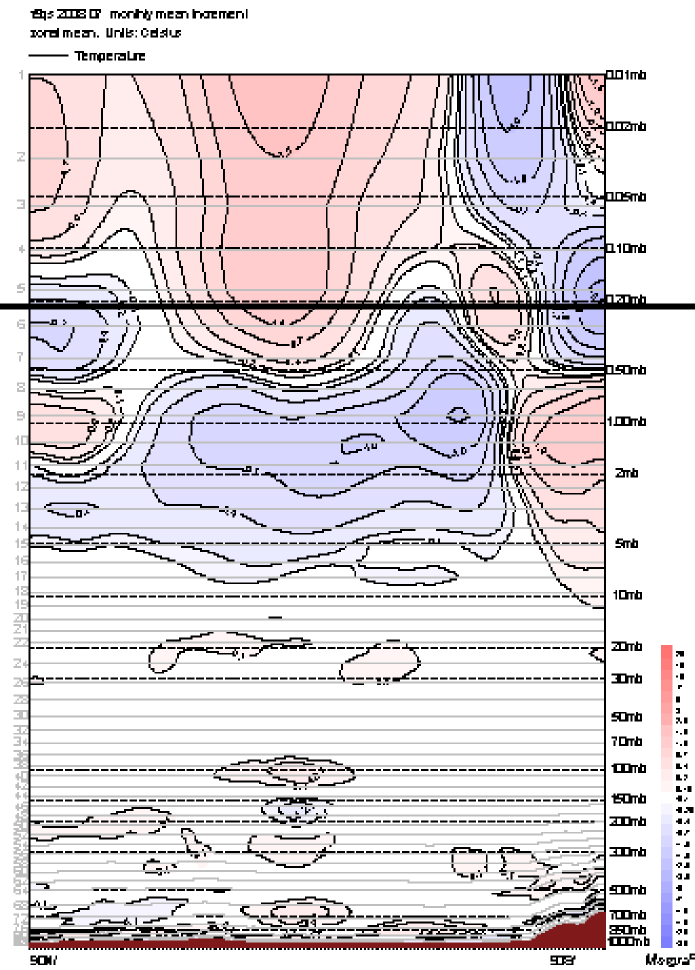
- Weak constraint 4D-Var experiments produced model errors with very large wind components in the stratosphere.
- These *model errors* could not be attributed to the model.
- The large wind errors were traced back to incorrect specification of the balance terms in J_b for the stratosphere.
- Not using the balance constraints above 20hPa improved the analysis in weak and strong constraint 4D-Var.
- The development of weak constraint 4D-Var helped identify errors in the data assimilation system, not only model error.

Balance in the Stratosphere

Control



No Balance in Stratosphere



20 mb

The mean temperature increment is smaller and smoother without applying the balance operators in the stratosphere (July 2008).

High Initial-time Sensibility Observed for the Tropospheric NAM Predictability in the Stratospheric Sudden Warming

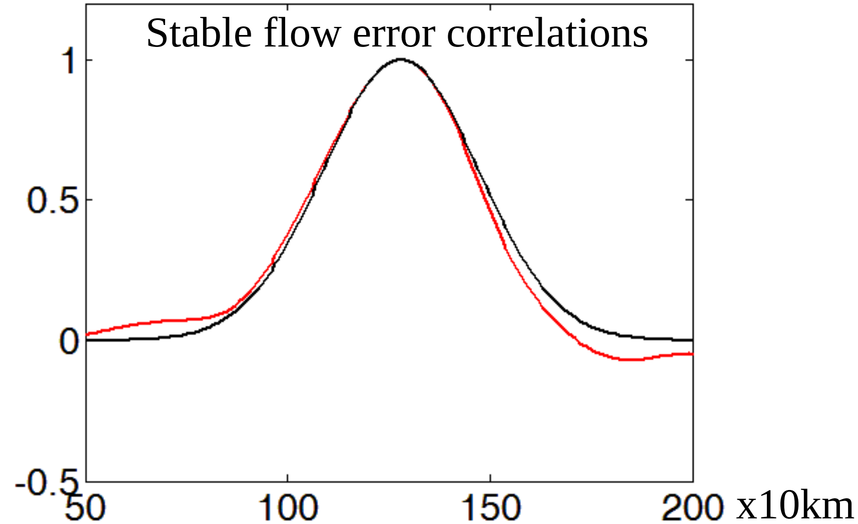
Y. Kuroda

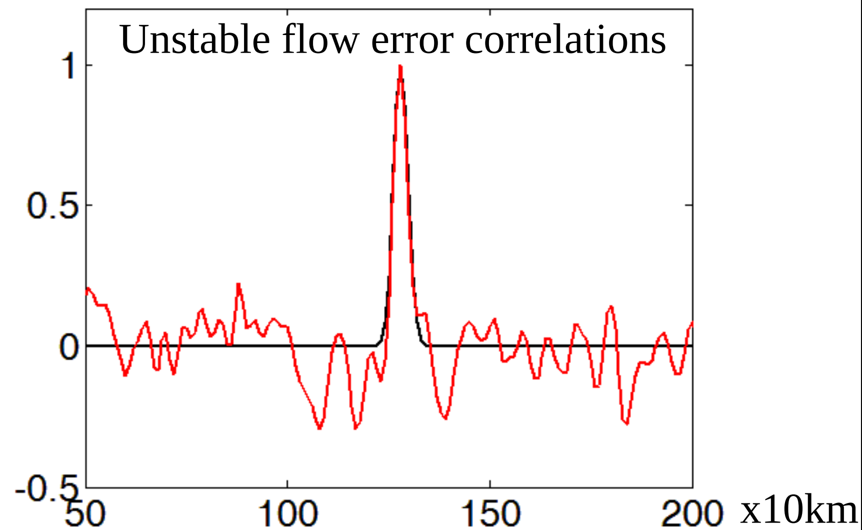
Meteorological Research Institute, Tsukuba, Japan

Very high predictability in the tropospheric NAM to a few months was simulated by ensemble runs of the general circulation model when the initial time was set just before the key day of the major stratospheric sudden warming (SSW) in January 2004 (Kuroda, GRL2008). In this study, we examined how the predictability of the tropospheric NAM varies with the change of the initial time. It is found that higher statistical significance of the NAM-predictability is obtained with advancement of initial time to the key day of the SSW. Such higher predictability is obtained until initial time is set to the key day. However, abrupt large decrease in the predictability is observed after the key day, and higher predictability cannot be obtained afterward. The reason of such abrupt decrease in the predictability after the key day is also discussed.

M07 and Data Assimilation Using Modulated Ensembles

Motivation

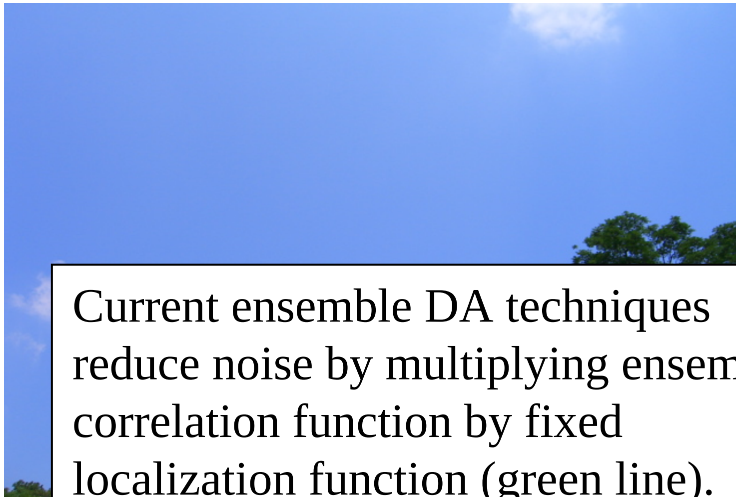


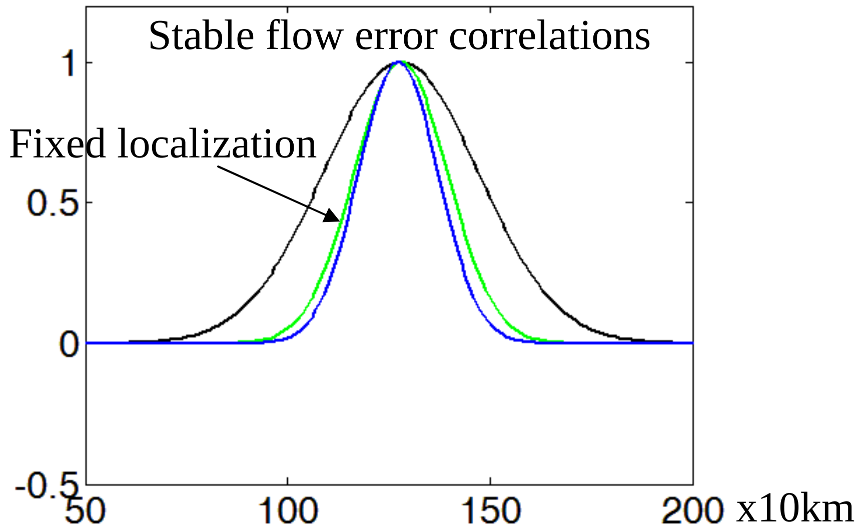


Ensembles give flow dependent, but ***noisy*** correlations

M07 and Data Assimilation Using Modulated Ensembles

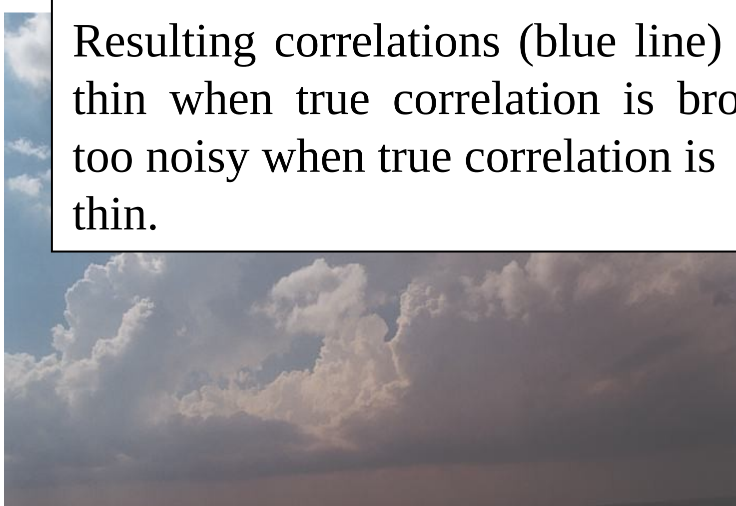
Motivation

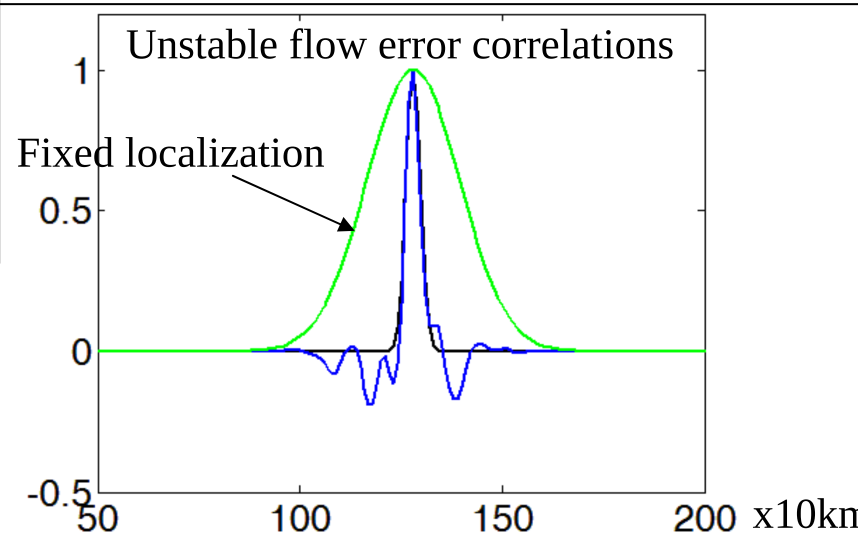




Current ensemble DA techniques reduce noise by multiplying ensemble correlation function by fixed localization function (green line).

Resulting correlations (blue line) are too thin when true correlation is broad and too noisy when true correlation is thin.



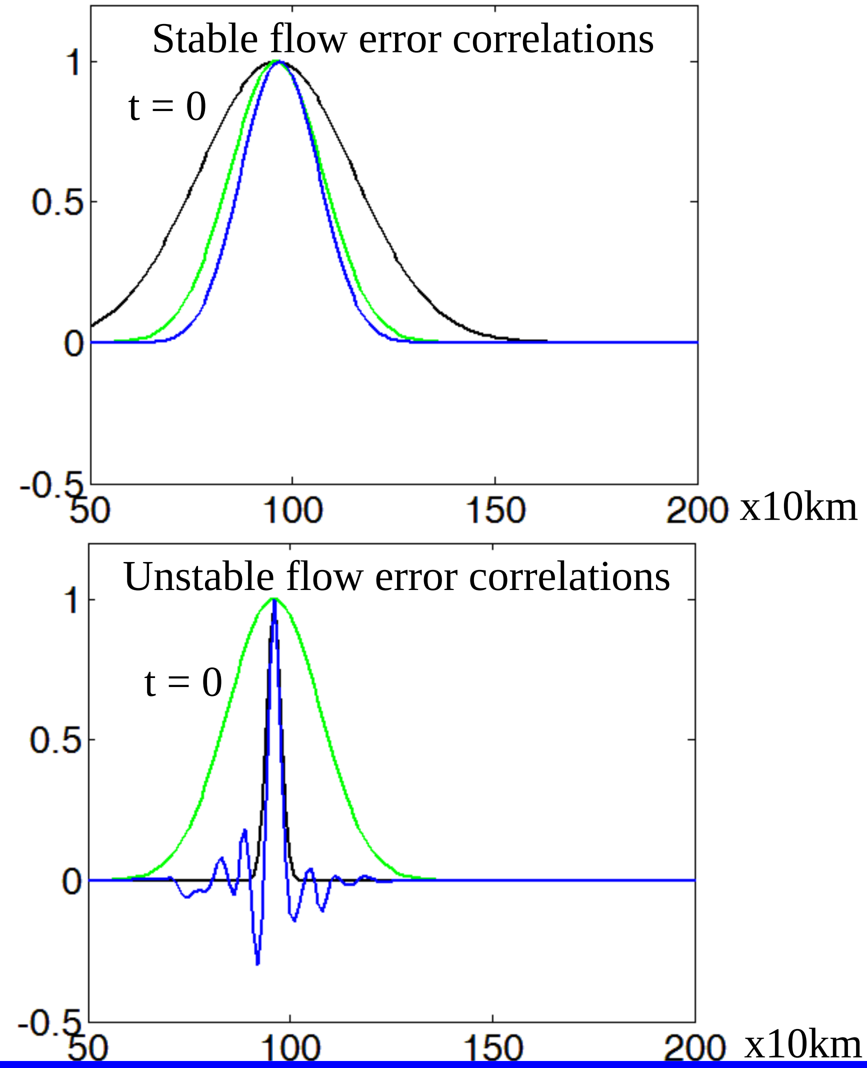


Today's fixed localization functions limit adaptivity

M07 and Data Assimilation Using Modulated Ensembles

Motivation

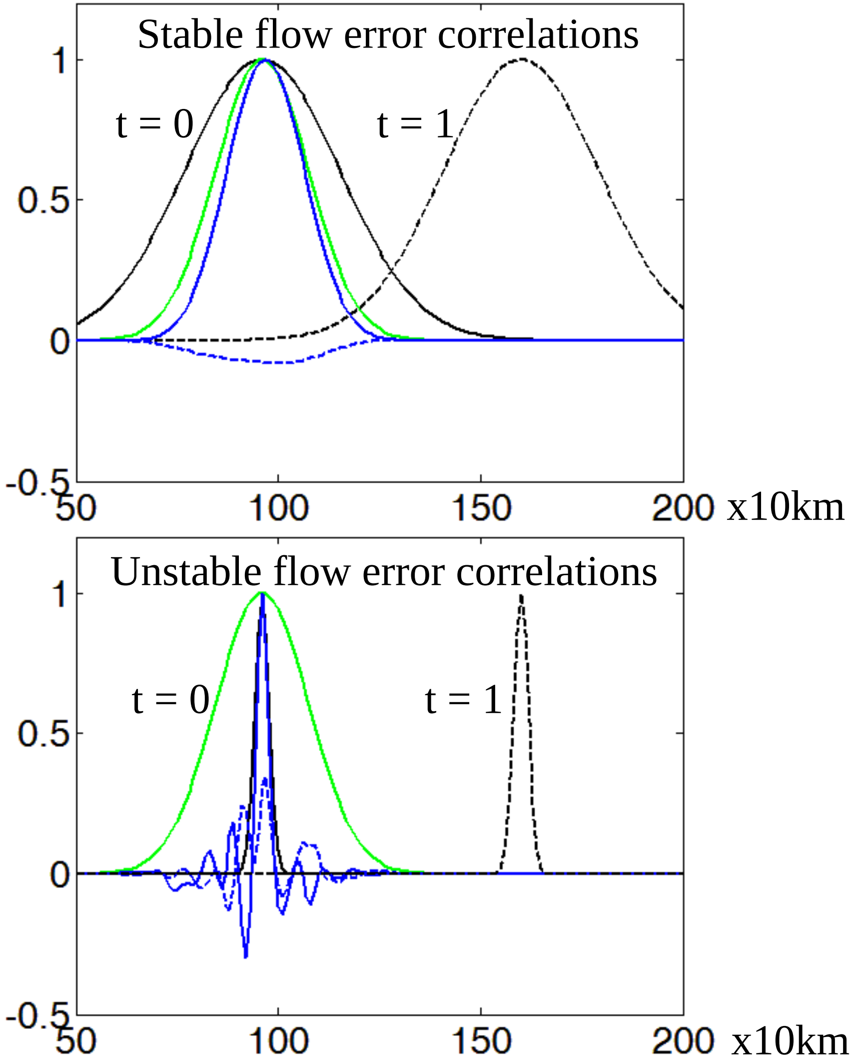
- Current ensemble localization functions poorly represent propagating error correlations.



Today's fixed localization functions limit ensemble-based 4D DA

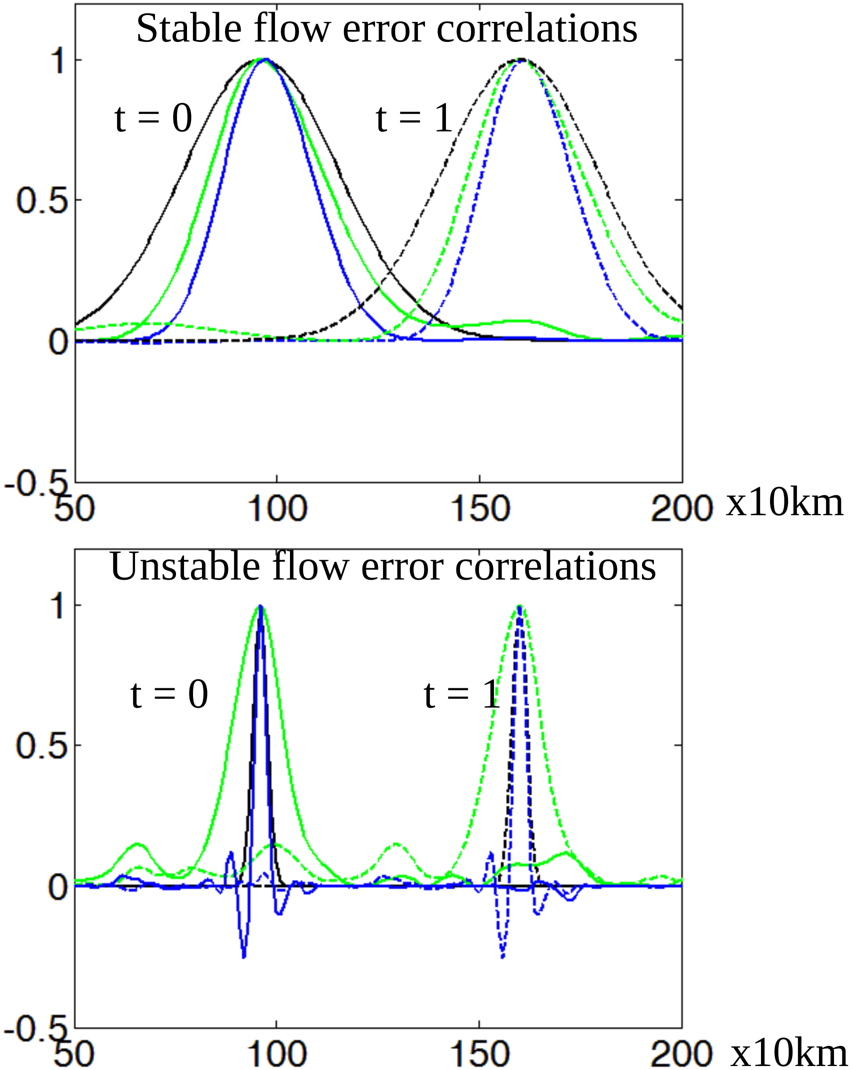
Motivation

- Current ensemble localization functions poorly represent propagating error correlations.



Today's fixed localization functions limit ensemble-based 4D DA

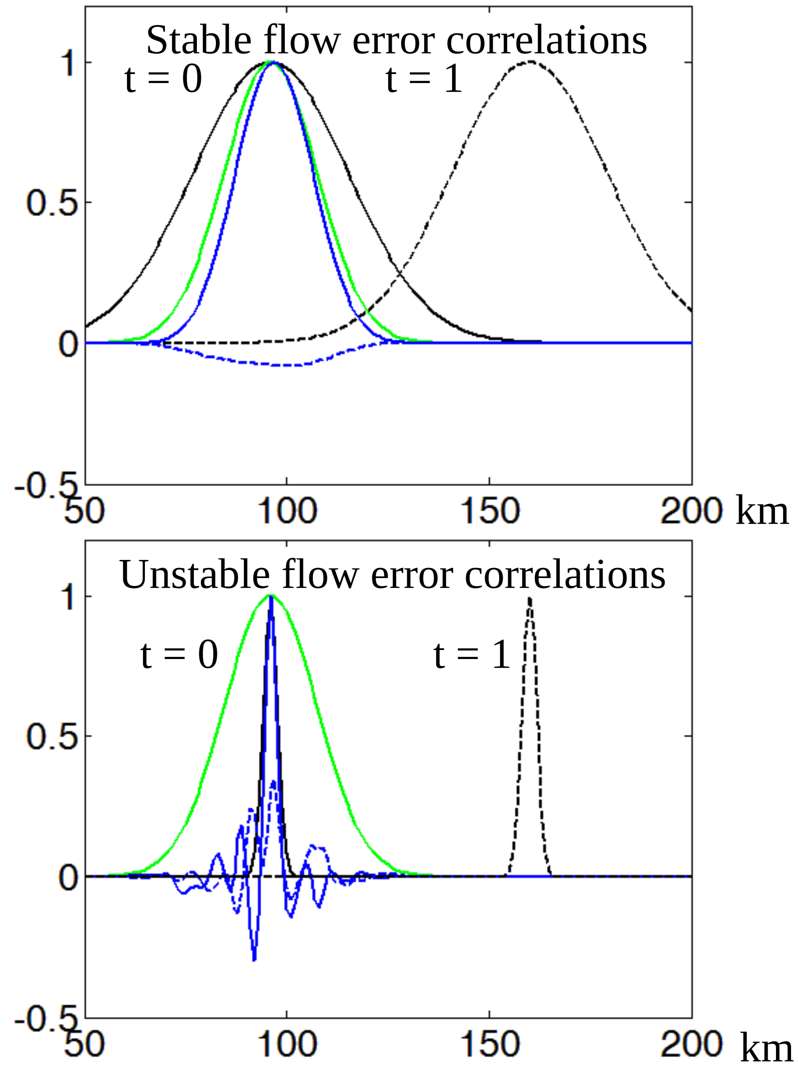
- Green line now gives an example of one of the adaptive localization functions that are the subject of this talk.



Want localization to adapt to width and propagation of true correlation

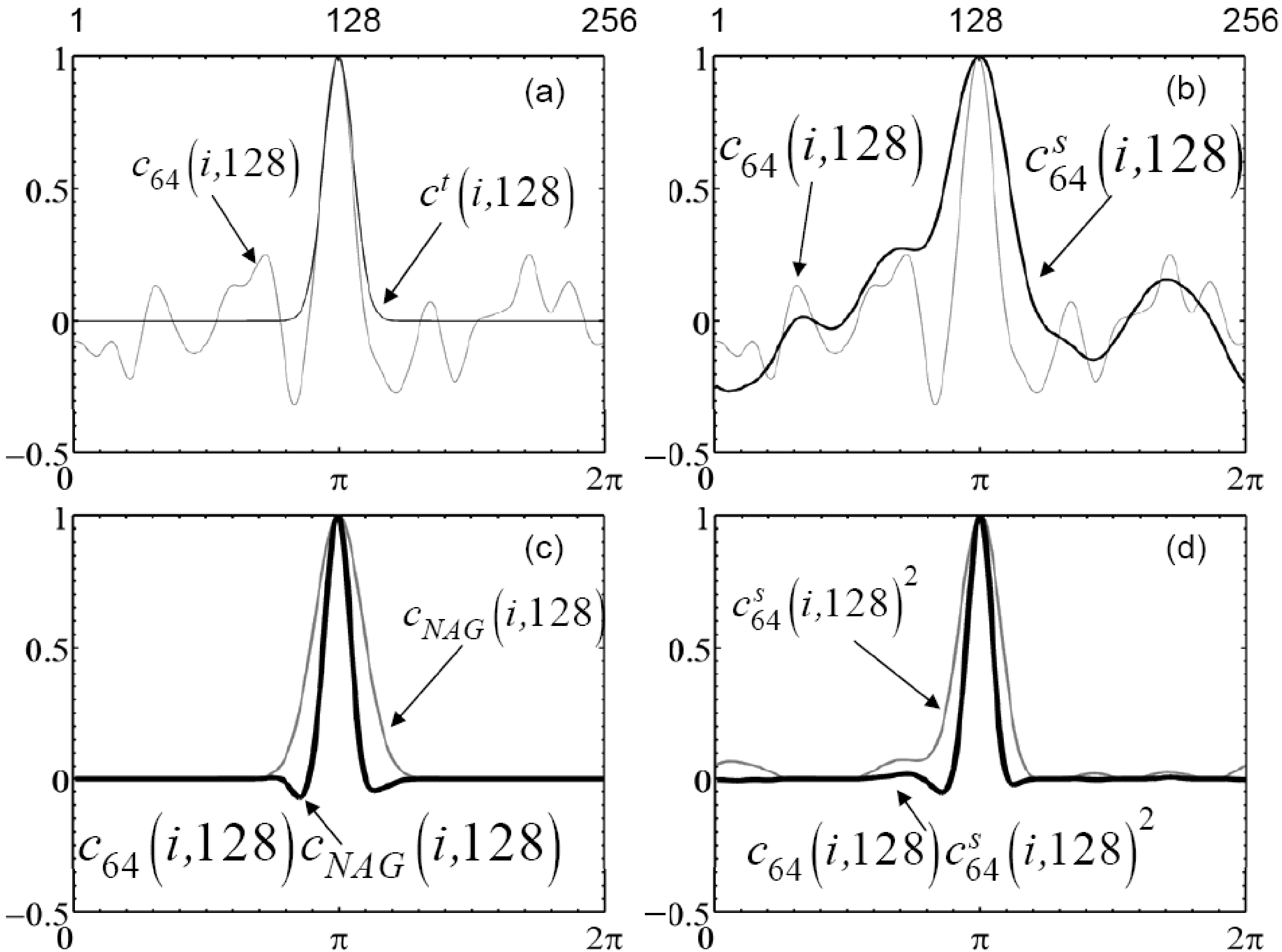
Motivation

Current ensemble localization functions do not adapt to the spatial scale of raw ensemble correlations and they poorly preserve propagating error correlations.



Method

An adaptive ensemble covariance localization technique (Bishop and Hodyss, 2007, QJRMS)



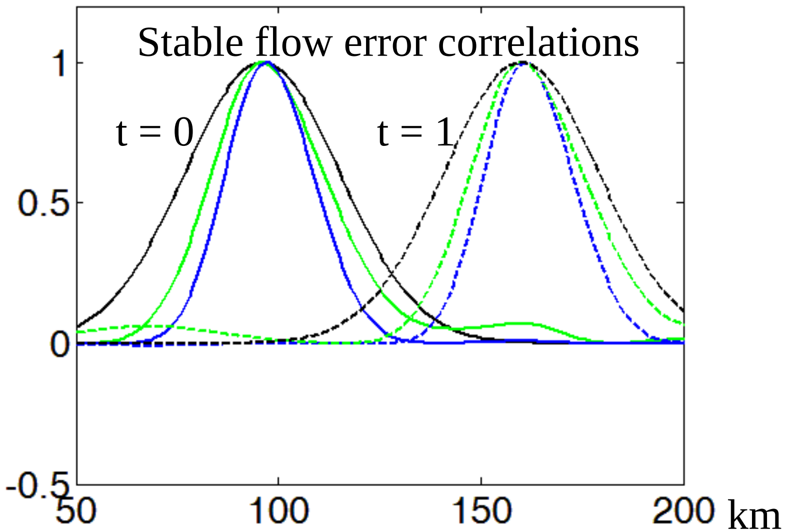
In matrix form,

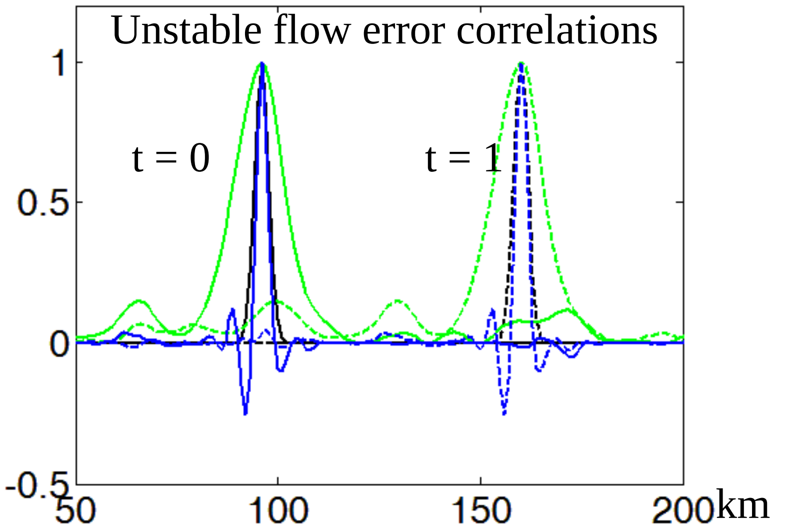
$$\mathbf{P}^f = \mathbf{P}^f_K \odot \mathbf{C}_s \odot \mathbf{C}_s$$

M07 and Data Assimilation Using Modulated Ensembles

Method

- Green line now gives an example of one of the adaptive localization functions that are the subject of this talk.





Key Finding: Moderation functions based on smoothed ensemble correlations provide scale adaptive and propagating localization functions.

Modulated ensembles and localization

(Bishop and Hodyss, 2009 a and b, Tellus)

Consider covariance of mth and nth elements of $\left(\underline{\mathbf{z}}_k \odot \underline{\mathbf{z}}_j^s \odot \underline{\mathbf{z}}_i^s\right)$ given by

$$\sum_{k=1}^{K}\sum_{j=1}^{K}\sum_{i=1}^{K}\left(z_{mk}z_{mj}^{s}z_{mi}^{s}\right)\left(z_{nk}z_{nj}^{s}z_{ni}^{s}\right)=\left(\sum_{k=1}^{K}z_{mk}z_{nk}\right)\left(\sum_{j=1}^{K}z_{mj}^{s}z_{nj}^{s}\right)\left(\sum_{i=1}^{K}z_{mi}^{s}z_{ni}^{s}\right)=\left(\underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}^s \odot \underline{\mathbf{C}}^s\right)_{mn}$$

where $\underline{\mathbf{P}}_K^f=\sum_{k=1}^{K}\underline{\mathbf{z}}_k\underline{\mathbf{z}}_k^T$ and $\underline{\mathbf{C}}^s=\sum_{j=1}^{K}\underline{\mathbf{z}}_j^s\underline{\mathbf{z}}_j^{sT}$. Hence,

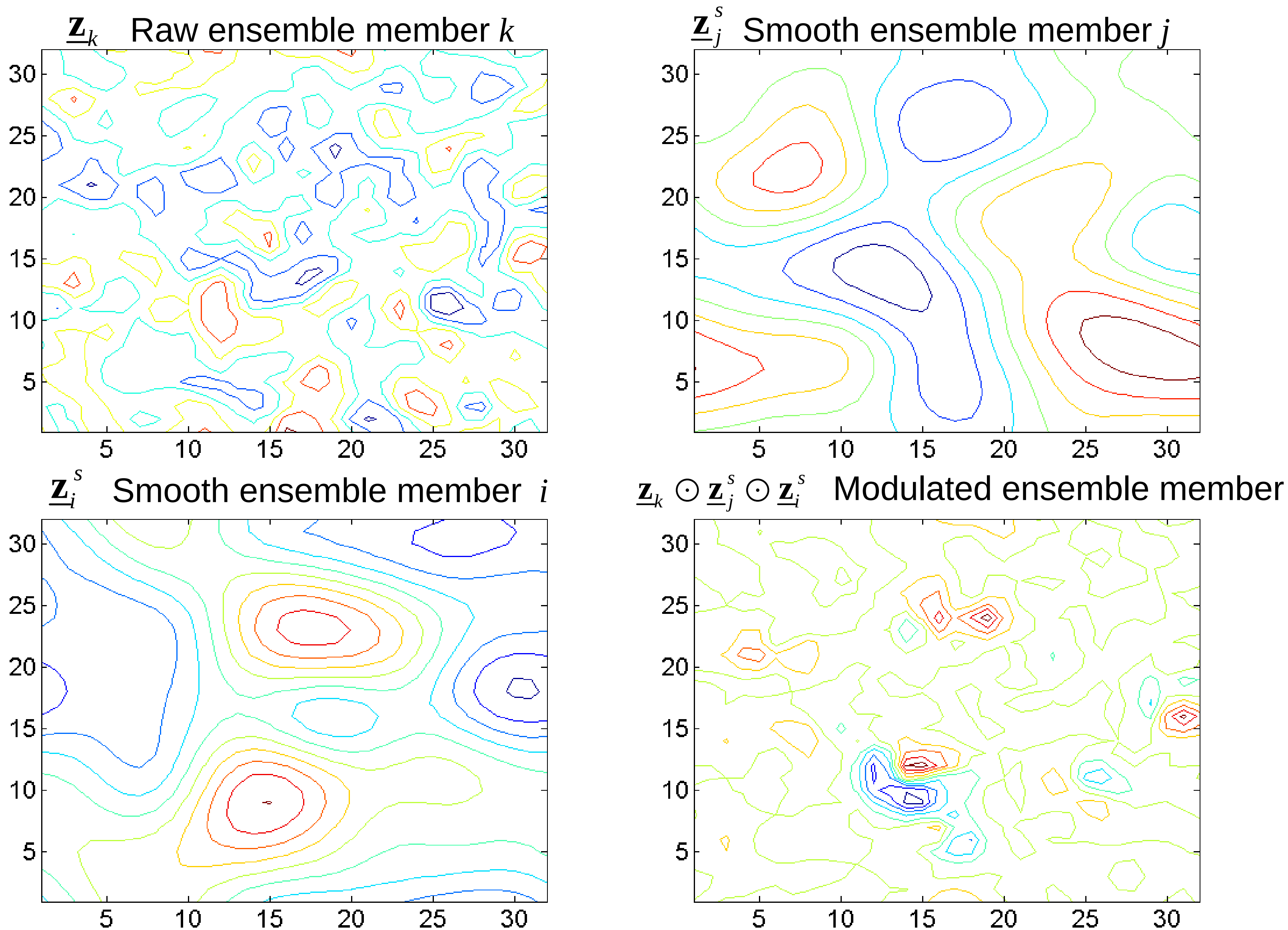
$$\sum_{k=1}^{K}\sum_{j=1}^{K}\sum_{i=1}^{K}\underbrace{\left(\underline{\mathbf{z}}_k \odot \underline{\mathbf{z}}_j^s \odot \underline{\mathbf{z}}_i^s\right)}_{\text{Modulated ensemble member}}\left(\underline{\mathbf{z}}_k \odot \underline{\mathbf{z}}_j^s \odot \underline{\mathbf{z}}_i^s\right)^T=\underline{\mathbf{Z}}_D\underline{\mathbf{Z}}_D^T=\underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}^s \odot \underline{\mathbf{C}}^s$$

Thus, the covariance of the modulated ensemble is the localized ensemble covariance.

For $K=128$, the modulated ensemble contains over 2 million members. Up to $K^2\left(K+1\right)/2=1{,}056{,}768$ of these are likely to be linearly independent.

M07 and Data Assimilation Using Modulated Ensembles

Modulated ensembles and localization



Modulated ensembles enable global 4DVAR

Both incremental and non-incremental AR are possible.

Non-incremental weak constraint AR is as follows.

Since $\underline{\mathbf{P}}^f = \underline{\mathbf{Z}}_D \underline{\mathbf{Z}}_D^T$ where $\underline{\mathbf{Z}}_D$ is the very large modulated ensemble, Step 1 is to solve

$$\left\{ \left[\underline{\mathbf{R}}^{-1/2} \underline{\mathbf{H}} \underline{\mathbf{Z}}_D \right] \left[\underline{\mathbf{R}}^{-1/2} \underline{\mathbf{H}} \underline{\mathbf{Z}}_D \right]^T + \mathbf{I} \right\} \mathbf{v} = \underline{\mathbf{R}}^{-1/2} \left[\underline{\mathbf{y}} - \underline{H} \left(\underline{\mathbf{x}}^f \right) \right]$$

for $\mathbf{v}$ using (for example) conjugate gradient. Step 2 is the postmultiply

$$\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f = \underline{\mathbf{Z}}_D \left[\underline{\mathbf{R}}^{-1/2} \underline{\mathbf{H}} \underline{\mathbf{Z}}_D \right]^T \mathbf{v}$$

Hybrid mixes of ensemble based TLMs and initial covariances with those from NAVDAS are straightforward.

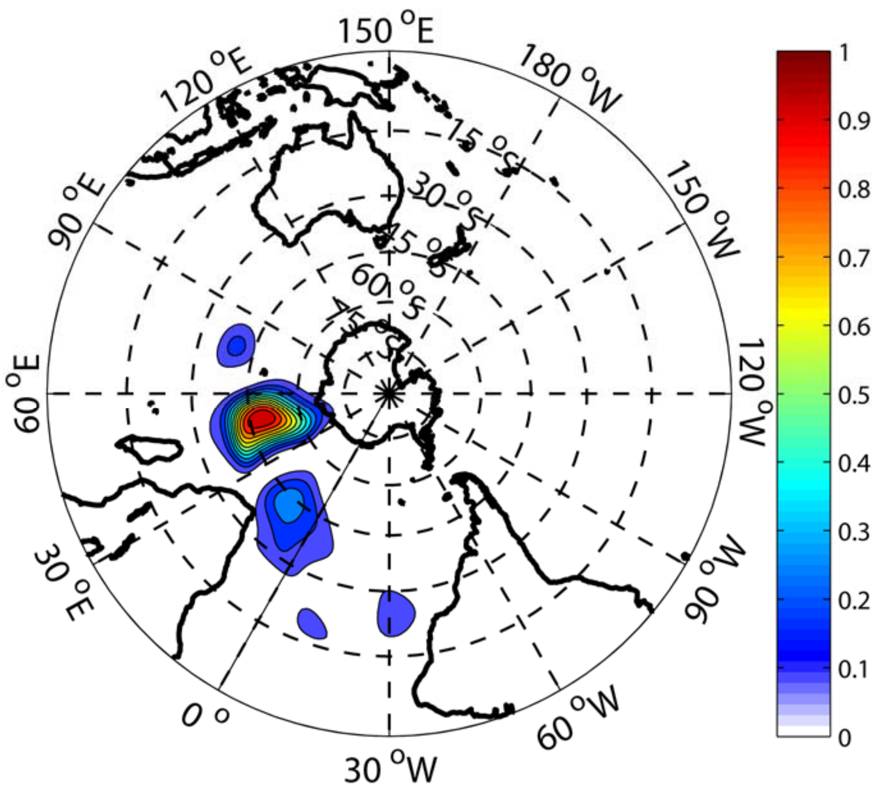
Model space "primal" 4D-VAR form is also straightforward (El Akkraoui et al., 2008, QJRMS) .

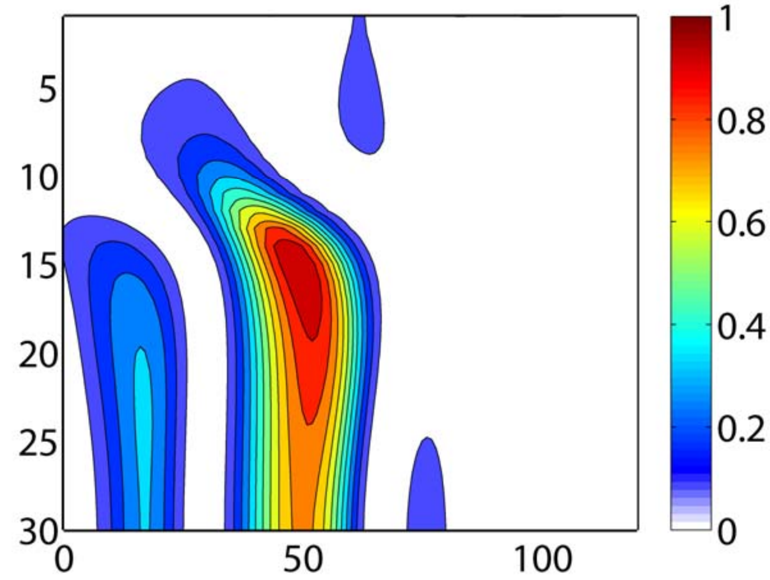
M07 and Data Assimilation Using Modulated Ensembles

Application to global NWP model

Example of a column of the localization $\underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$ with $K = 128$

06Z





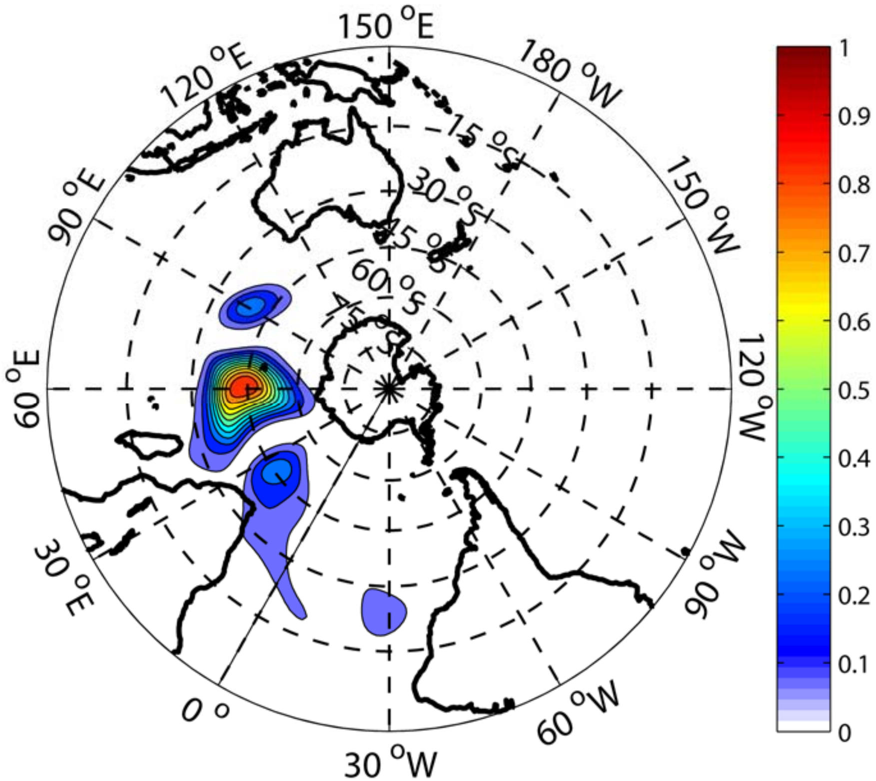
Ensemble based localization moves about 1000 km in 12 hrs. This is >=half-width of a typical LETKF observation volume (~900km).

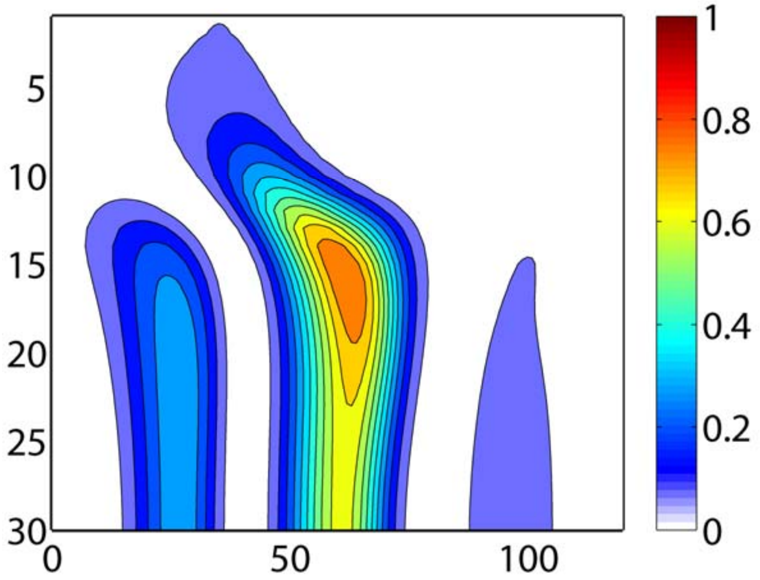
M07 and Data Assimilation Using Modulated Ensembles

Application to global NWP model

Example of a column of the localization $\underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$ with $K = 128$

18Z



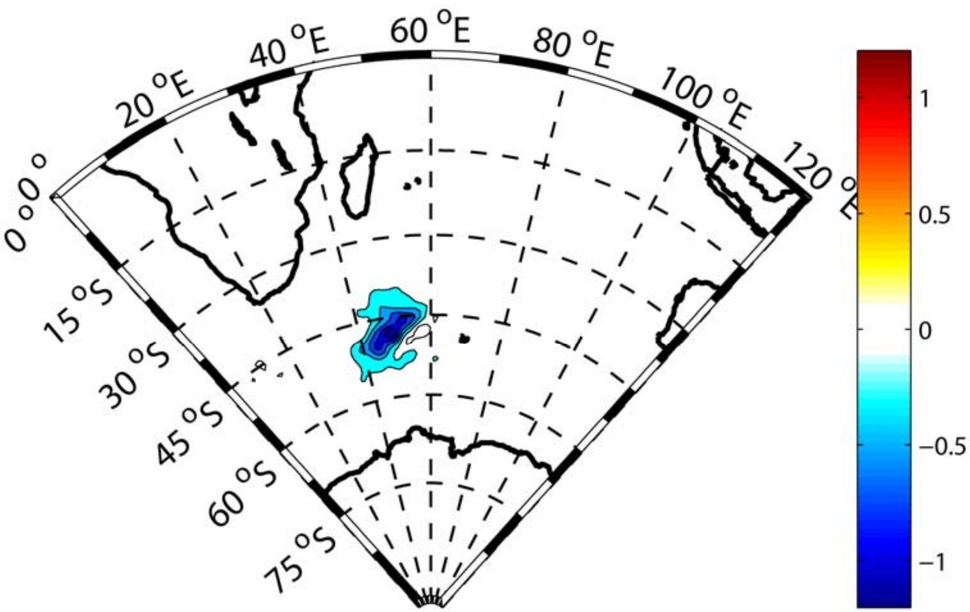


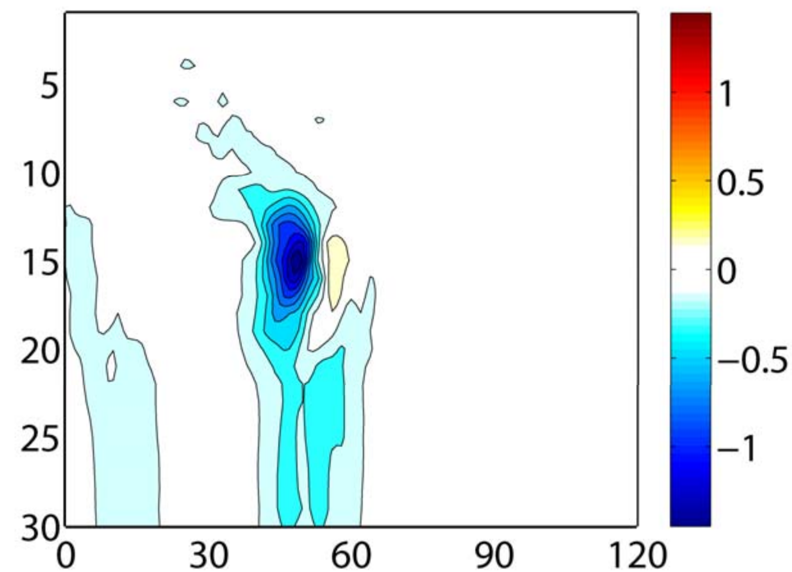
Ensemble based localization moves about 1000 km in 12 hrs. This is >=half-width of a typical LETKF observation volume (~900km).

Application to global NWP model

Example of a column of $\underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$ with $K = 128$

<*vv*> Increment 06Z





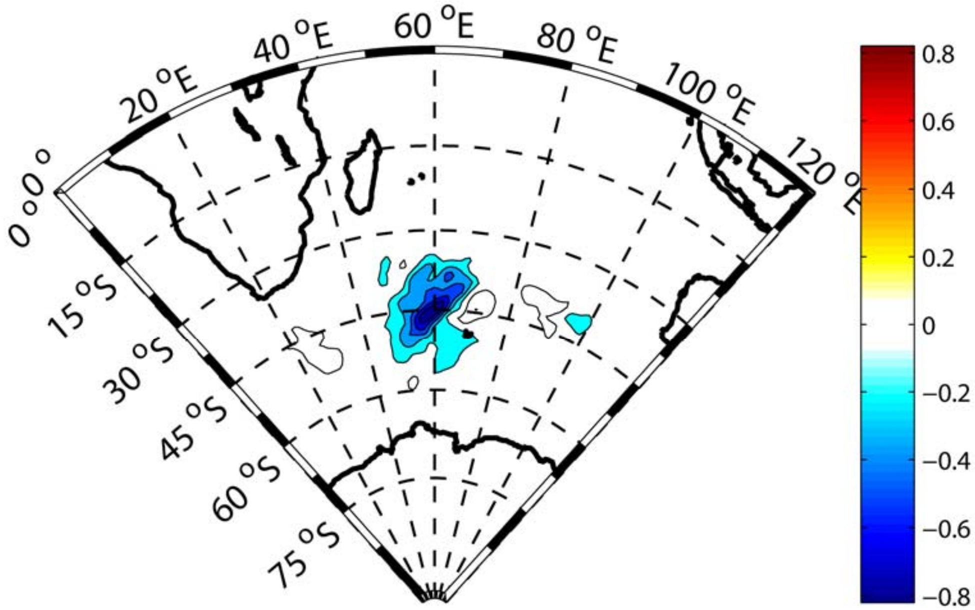
Statistical TLM implied by mobile adaptively localized covariance propagates single observation increment 1000 km in 12 hrs.

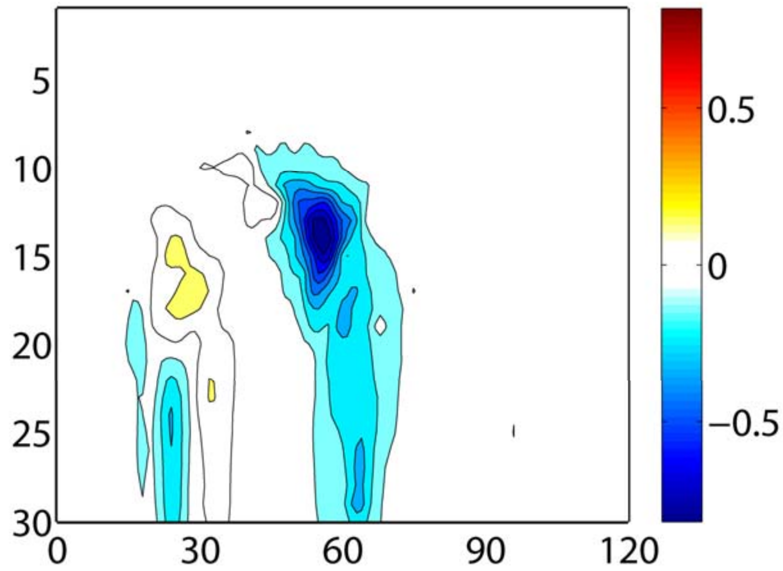
M07 and Data Assimilation Using Modulated Ensembles

Application to global NWP model

Example of a column of $\underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$ with $K = 128$

<vv> Increment 18Z





Statistical TLM implied by mobile adaptively localized covariance propagates single observation increment 1000 km in 12 hrs.

Mobile adaptive localization provides a statistical TLM/PFM

$$\mathbf{x}^a(18)-\mathbf{x}^f(18)=M\left[\mathbf{x}^a(6)\right]-M\left[\mathbf{x}^f(6)\right]\approx\mathbf{M}\left[\mathbf{x}^a(6)-\mathbf{x}^f(6)\right]$$

Statistically, the Best Linear Unbiased Estimate of $\mathbf{M}$ is

$$\mathbf{M}=\left\langle\left[\mathbf{x}^a(18)-\mathbf{x}^f(18)\right]\left[\mathbf{x}^a(6)-\mathbf{x}^f(6)\right]^T\right\rangle\left\langle\left[\mathbf{x}^a(6)-\mathbf{x}^f(6)\right]\left[\mathbf{x}^a(6)-\mathbf{x}^f(6)\right]^T\right\rangle^{-1}$$

$$\approx\left\langle\varepsilon_{18}^f\varepsilon_6^{fT}\right\rangle\left\langle\varepsilon_6^f\varepsilon_6^{fT}\right\rangle^{-1}$$

and hence $\mathbf{M}$ is readily derived from any four-dimensional $\underline{\mathbf{P}}^f$.

(Compare with "statistical 4D-VAR" TLM/PFM discussed

in Lorenc and Payne, 2007, QJRMS)

Mobile adaptive localization provides an initial time covariance $\mathbf{P}^f_0$ and a statistical **M**. Can the quality of such models of $\mathbf{P}^f_0$ and **M** match the quality of their non-ensemble counterparts?

Experiment 1: Test of Initial adaptively localized covariance $\mathbf{P^f_0}$

- Forecast error is difference between a T119L30 6Z-18Z forecast (the first guess) and a T119L30 30-42 hr forecast valid at the same time (the simulated truth).
- In this case, 440,000 evenly spaced obs of u,v, and T are simulated. (Every 3rd level in z, every 2nd grid point in x and y, no obs poleward of 80). (8 million variables).
- All obs are taken at 6Z (the beginning of DA window)
- Rms ob error for u and v is 2 ms^-1
- Rms ob error for T is 2 K
- 128 Ensemble Transform (ET) ensemble members=>localized ensemble has rank <1,056,768
- Compare $\underline{\mathbf{P}}^f = \underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$ with NAVDAS. Arguably, NAVDAS has the advantage because forecast error is the result of 4 NAVDAS analysis corrections (using real obs) made over the preceding 24 hrs.

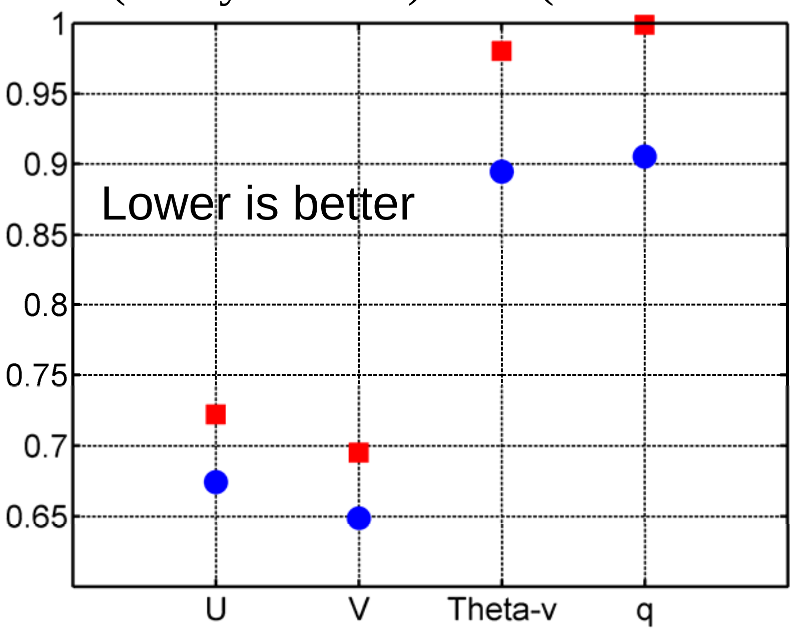
M07 and Data Assimilation Using Modulated Ensembles

Comparison of background covariances

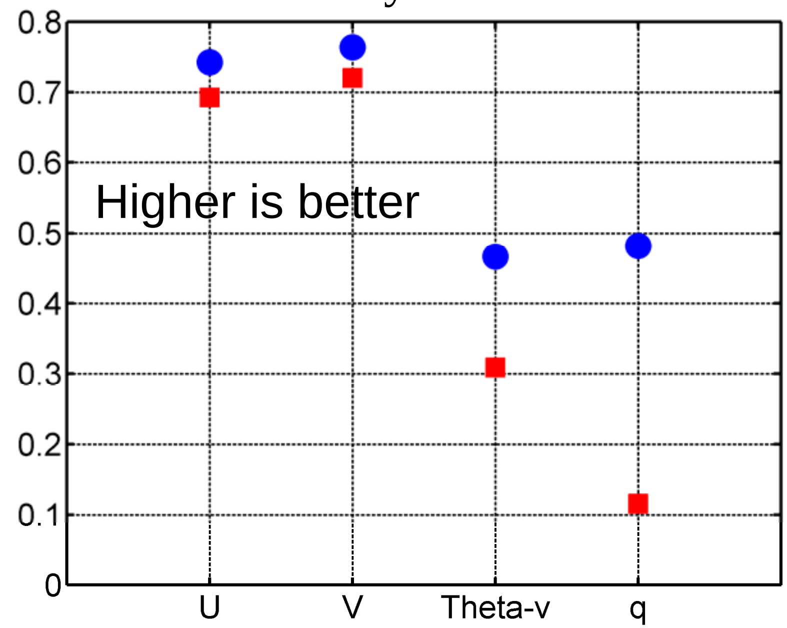
Red square → NAVDAS

Blue Circle → $\underline{\mathbf{P}}^f = \underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$

RMS(Analysis Error)/RMS(Forecast Error)

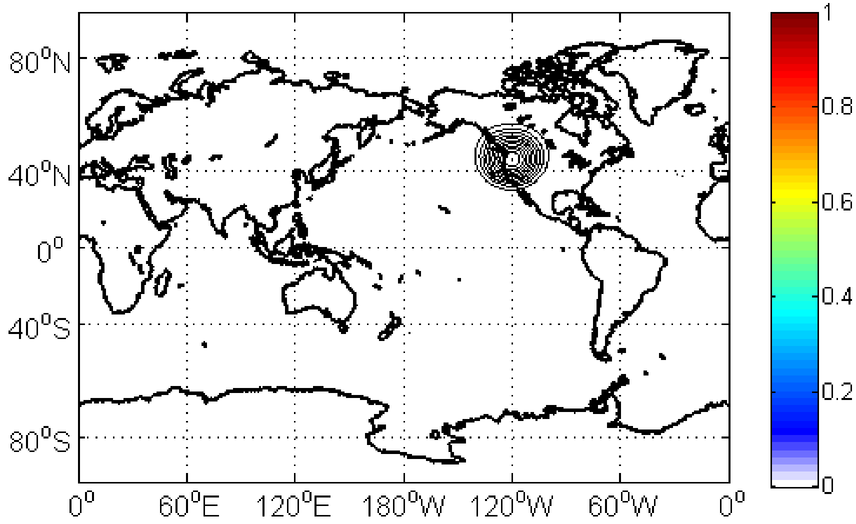


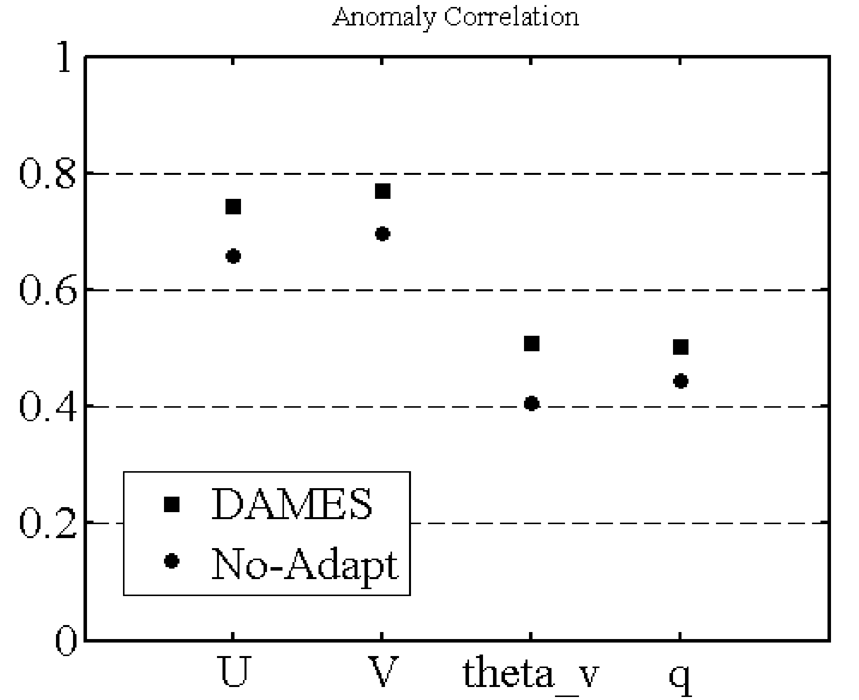
Anomaly Correlation

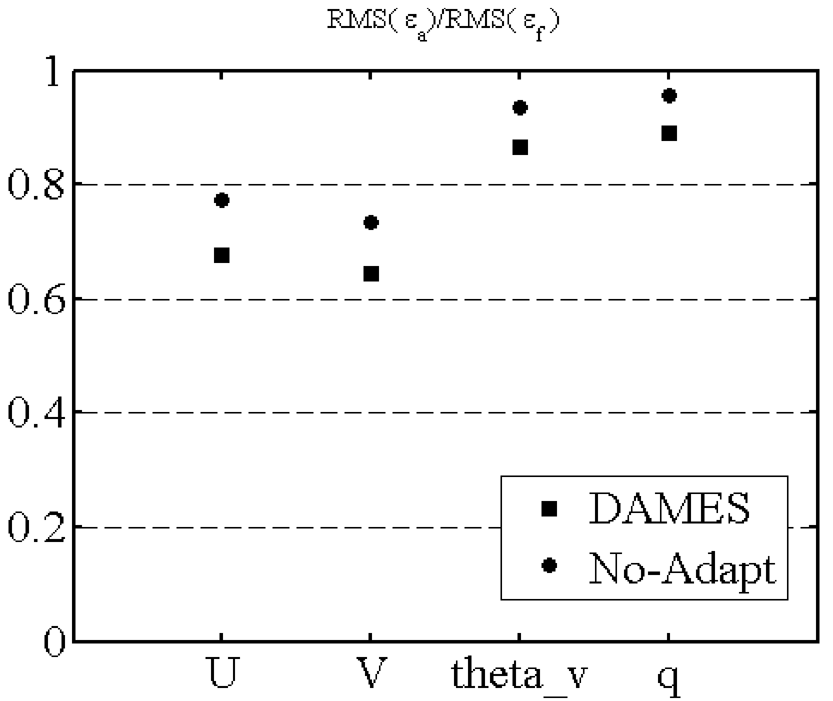


Anomaly correlation is between analysis correction and the "perfect" correction that would have eliminated all initial condition error.

Adaptively localized ensemble covariance produced smaller initial condition errors than covariance model used in operational 3D-PSAS/NAVDAS scheme





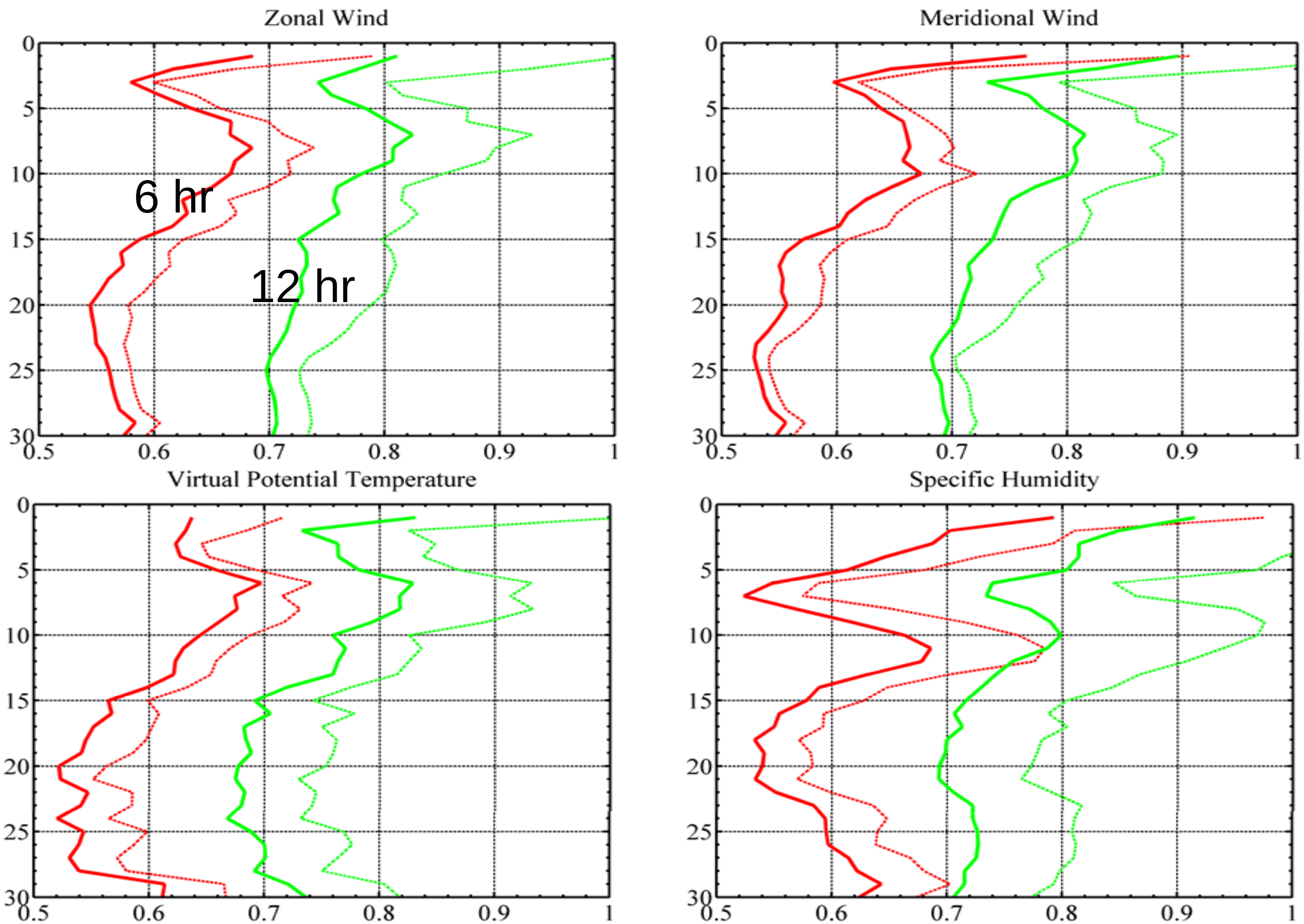


Adaptive localization from DAMES gives smaller analysis error than fixed horizontal localization (no vertical localization).

P^f_0 is OK. What about the TLM implied by 4D $\underline{\mathbf{P}}^f = \underline{\mathbf{P}}_K^f \odot \underline{\mathbf{C}}_s \odot \underline{\mathbf{C}}_s$?

M07 and Data Assimilation Using Modulated Ensembles

Adaptive localization enables ensemble TLM



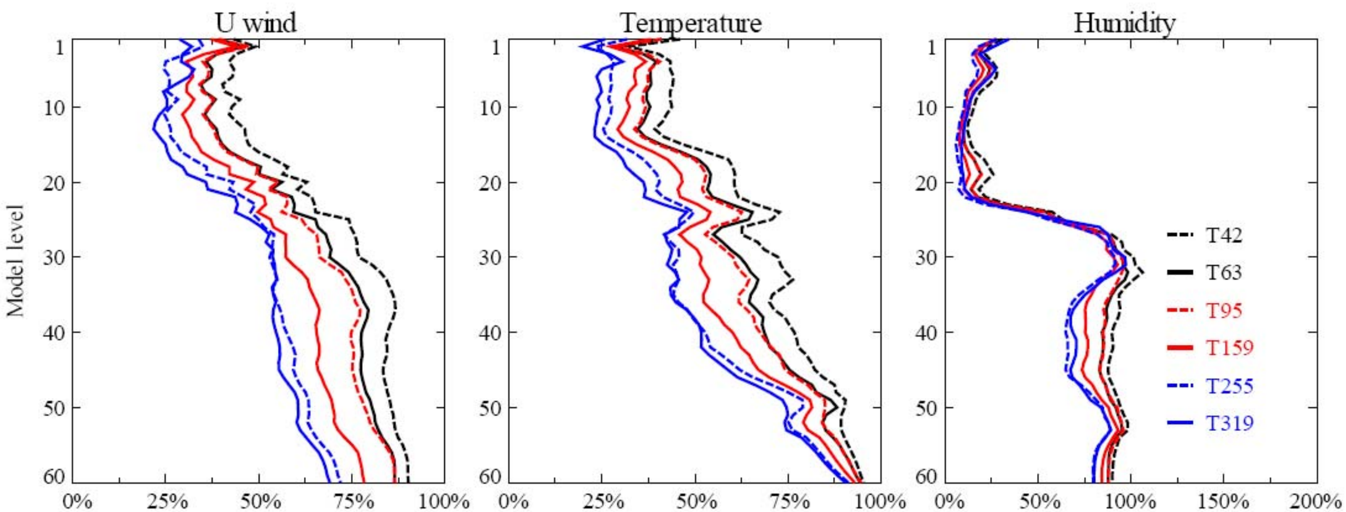
Solid – attenuation
Dashed – no attenuation

TLM implied by

$$\mathbf{P}^f = \mathbf{P}_K^f \odot \mathbf{C}_s \odot \mathbf{C}_s$$

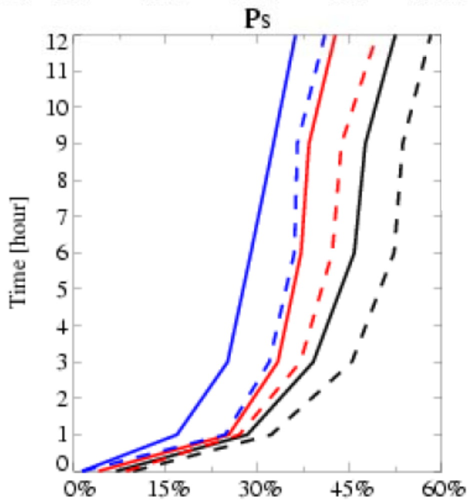
$$r = \frac{\left\| M(x_i + S^{-1}\delta x_i) - M(x_i) - \mathbf{M}_i(\delta x_i) \right\|}{\left\| M(x_i + S^{-1}\delta x_i) - M(x_i) \right\|}$$

Accuracy of ECMWF TLM



Will the 4D-Var approach be defeated by nonlinearity?

E. Andersson, M. Fisher, E. Hólm, L. Isaksen, G. Radnóti, and Y. Trémolet



$$r = \frac{\left\| M(x_i + S^{-1}\delta x_i) - M(x_i) - \mathbf{M}_i(\delta x_i) \right\|}{\left\| M(x_i + S^{-1}\delta x_i) - M(x_i) \right\|}$$

$$= \frac{noise}{signal} = \frac{n}{s}$$

Figure 3: Relative error of the tangent linear model for various resolutions with respect to T511 nonlinear, diabatic model after 12h for the 3 dimensional variables and for the whole 4D-Var window for surface pressure. Diagnostics are computed on the T255 resolution grid. From Radnóti et al. (2005).

Conclusions

- Adaptive localization should aim to account for propagation and scale variations of error distribution
- Proposed adaptive localization given by even powers of correlations of smoothed ensemble
- Huge modulated ensembles give square root of localized ensemble covariance matrix
- Errors can move over 1000 km in 12 hr window
- Modulated ensembles enable 4D-VAR global solve
- Adaptively localized covariance beats operational covariance model in idealized experiment with pseudo-obs
- Adaptive localization enables ensemble based TLMs