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# Intrinsic Middle Atmosphere Predictability

Keith Ngan and Giles Eperon  
Data Assimilation and Ensembles



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# Background



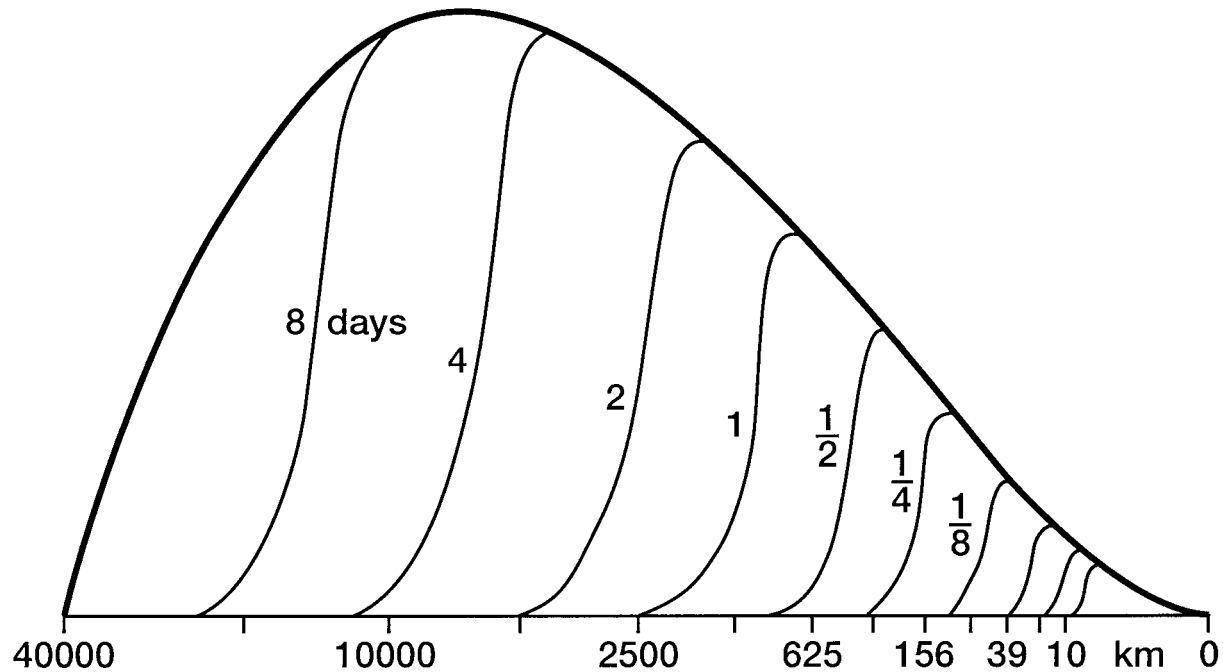
# Definitions

Many different aspects (and definitions) of predictability.

This talk is concerned with the **growth and propagation** of small-scale, small-amplitude errors in spectral space.

Approach is embodied in the **classical picture** of predictability due to Lorenz, Leith & Kraichnan.

# Lorenz (1969)



Predictability is measured by **relative errors** (i.e. relative KE)

There is an **inverse error cascade** from small to large scales

Key assumption: **homogeneous, isotropic turbulence**.



# Applicability to the real atmosphere

The real atmosphere cannot be described exactly by 2-D or quasi-geostrophic turbulence.

Conventional view is that the classical picture carries over straightforwardly to the real atmosphere.

Key quantity: **eddy turnover time**

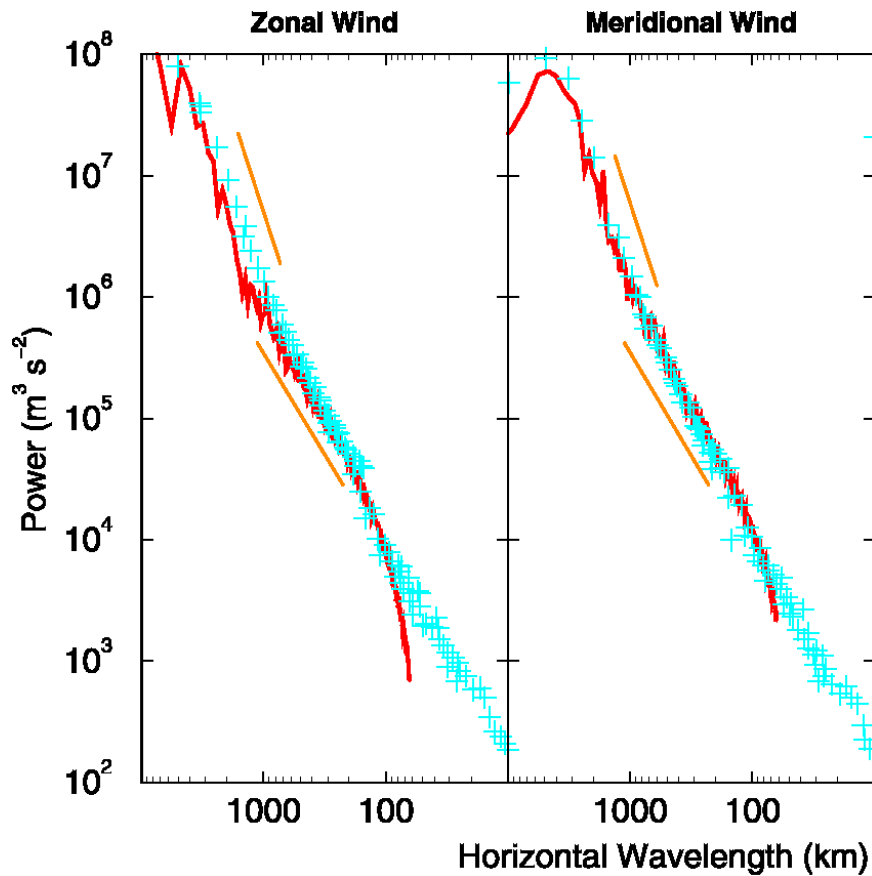
$$\tau(k; k_1) = \left( \int_{k_1}^k k'^2 E(k') dk' \right)^{-\frac{1}{2}}$$

For  $E(k) \sim k^{-p}$  and  $k \rightarrow \infty$

$$\tau(k; k_1) \sim \begin{cases} \text{constant}, & p > 3 \\ k^{(p-3)/2}, & p < 3 \end{cases}$$

**Upshot:** predictability behaviour depends crucially on the atmospheric energy spectrum.

# GCM simulations

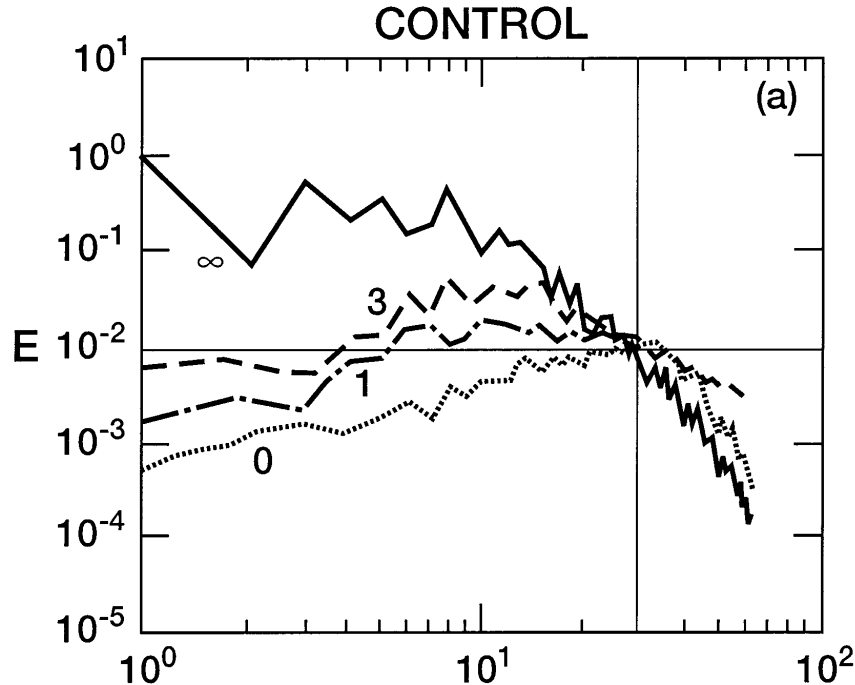


Hamilton et al. (2008)

Mesoscale energy spectrum remains controversial.

One possible explanation: downscale propagation of wave energy (e.g. Bartello 1995).

# NWP Predictability



Tribbia & Baumhefner(2004)

NWP predictability **departs from the classical picture:**

- Rapid saturation of small scales; exponential growth of synoptic scales.
- Don't have simple inverse error cascade: predictability regimes (Boer 1994).



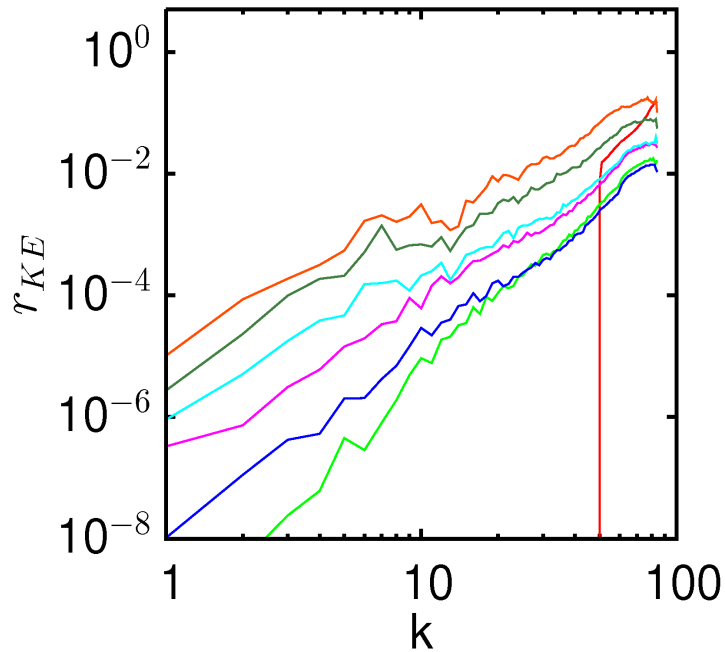


# Rotating stratified turbulence

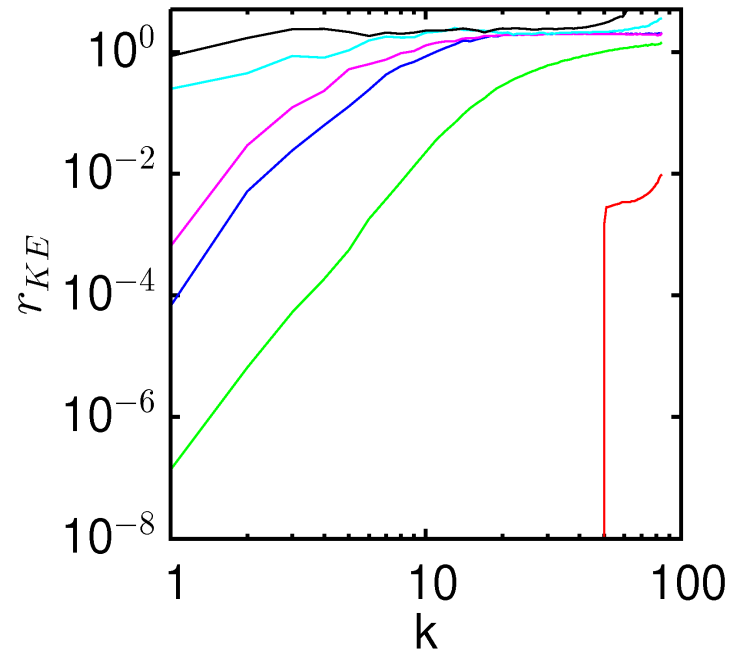
subsynoptic [ $L_d/L_0 = 10$ ]

(super)synoptic [ $L_d/L_0 = 0.1$ ]

$\tau_{KE}$  vs.  $k$ ; Run 1



$\tau_{KE}$  vs.  $k$ ; Run 4



Predictability decay for subsynoptic flow is significantly slower.

Implication: gravity waves *increase* predictability.

Reference: Ngan, Bartello & Straub, JAS 2009.



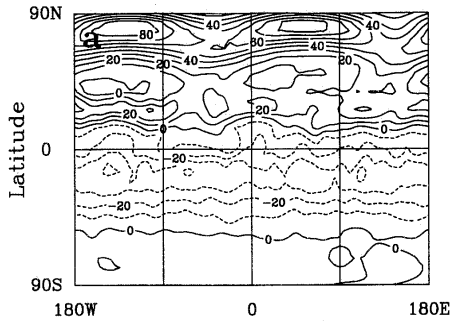
# Middle atmosphere predictability

# Middle atmosphere predictability

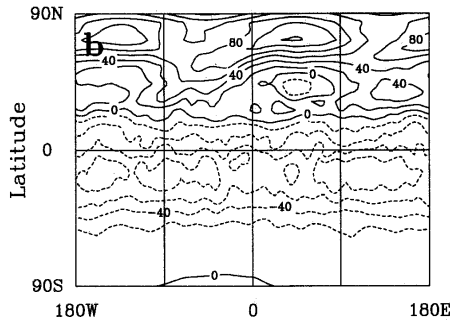
Most studies of NWP predictability have been restricted to the troposphere.

One might expect differences in the middle atmosphere, where small-scale gravity waves play an important role.

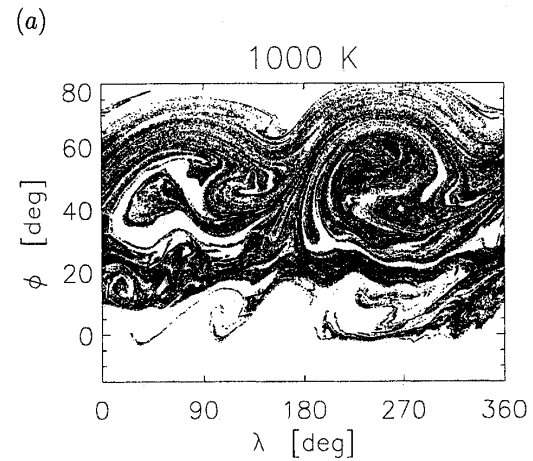
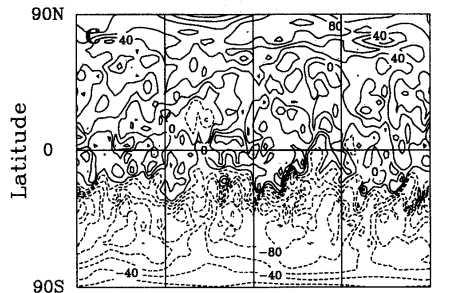
Troposphere



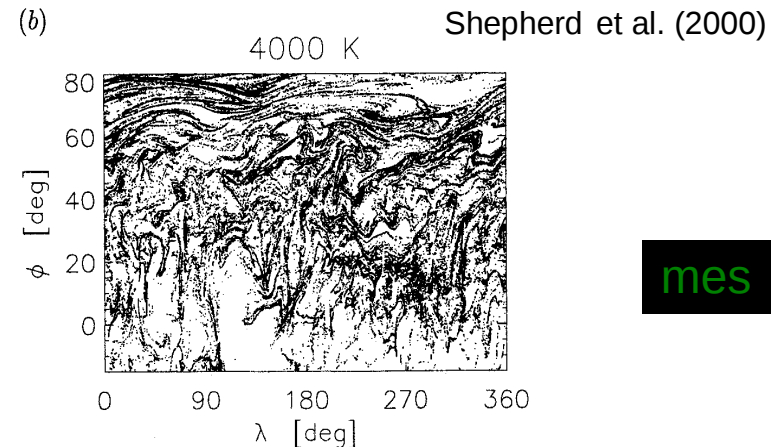
Stratosphere



Mesosphere



strat

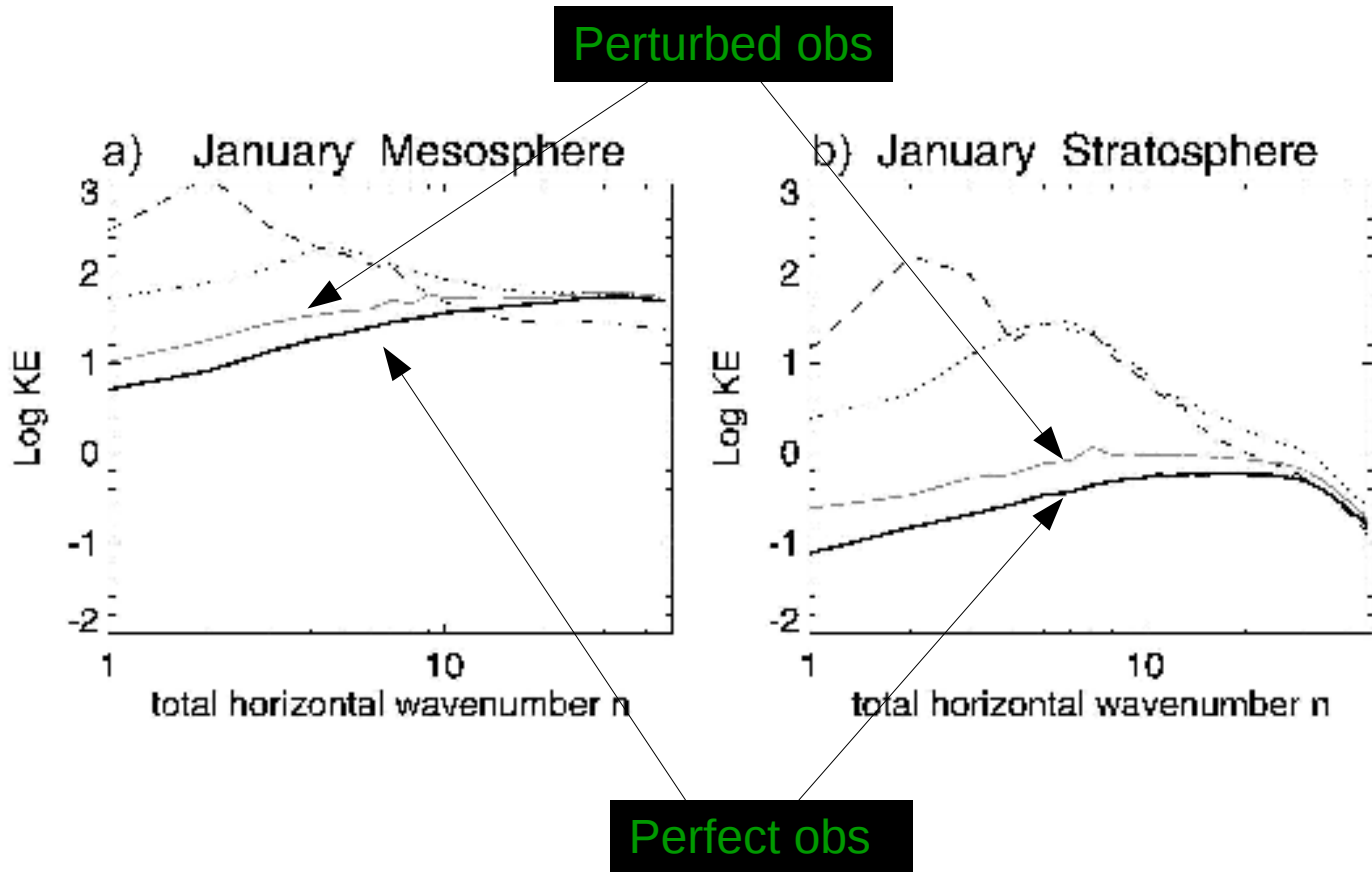


mes

# Nezlin et al. (2008)

Examined predictability of the CMAM DAS.

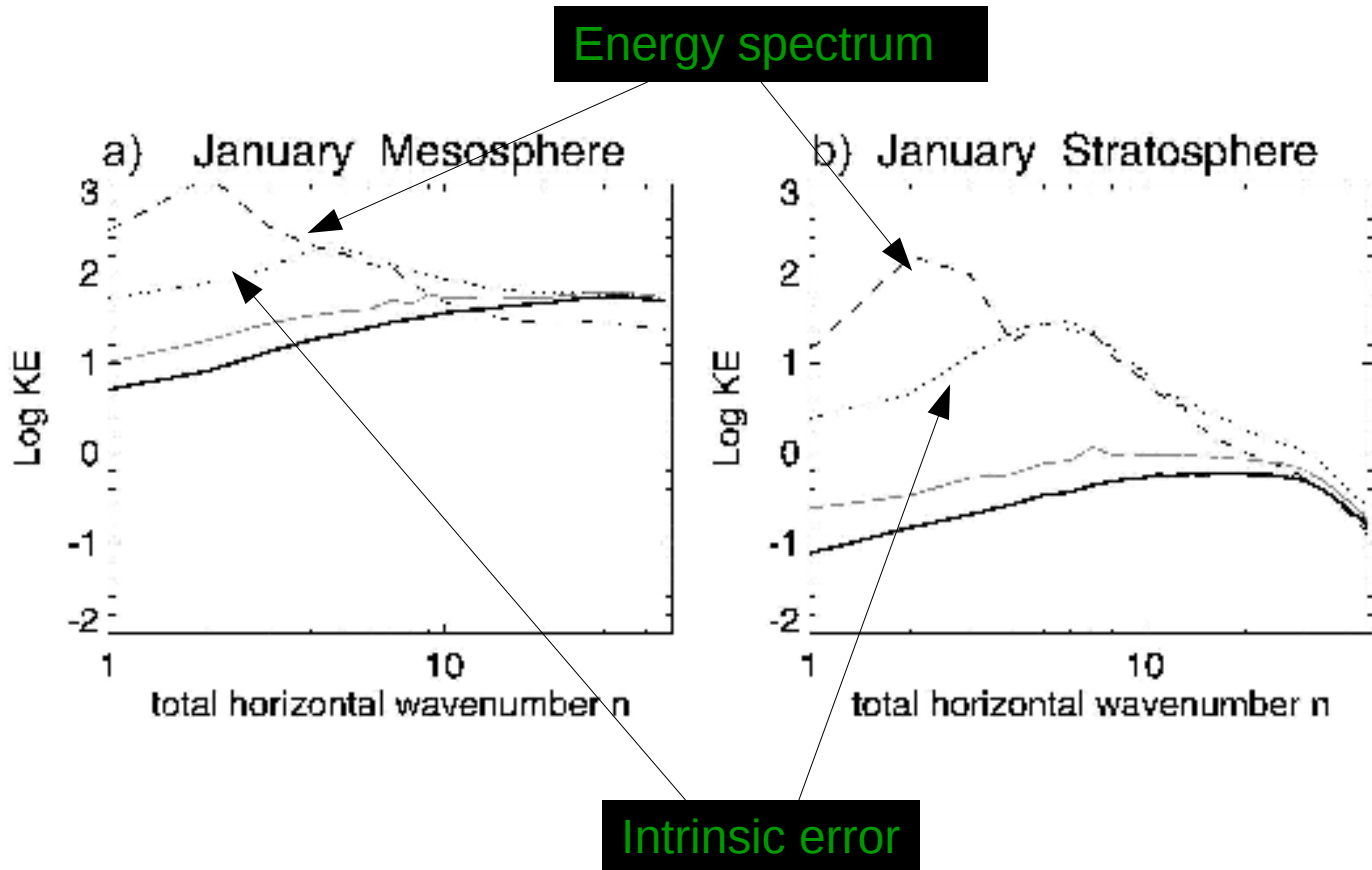
Key result: large-scale stratospheric information can be assimilated in the mesosphere.



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# Liu et al. (2009)

Examined predictability of a whole-atmosphere model (NCAR WACCM: lid at ~ 140 km).

Key result: vertical coupling due to gravity waves

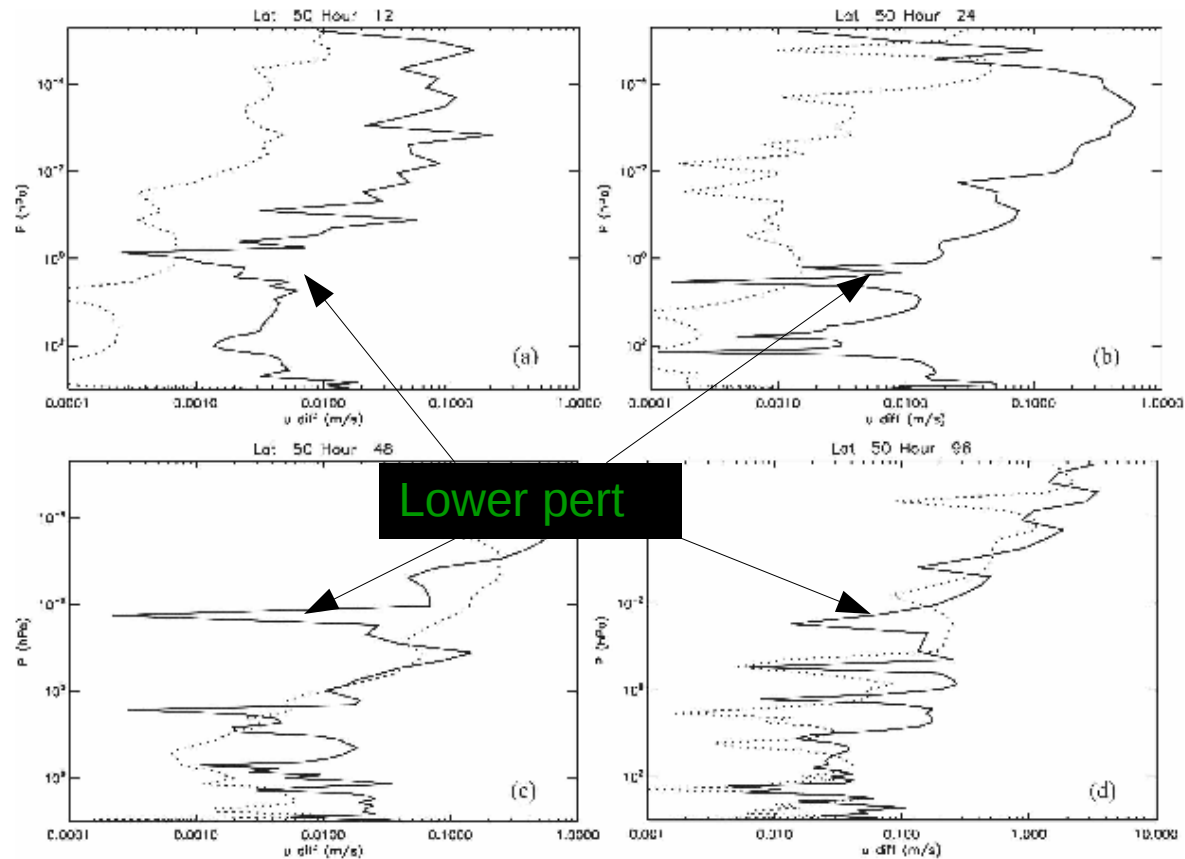


FIG. 7. Vertical profiles of rms zonal wind error in case L (solid line) and case U (dotted line) (a) 0.5 day, (b) 1 day, (c) 2 days, and (d) 4 days after the simulation starts.

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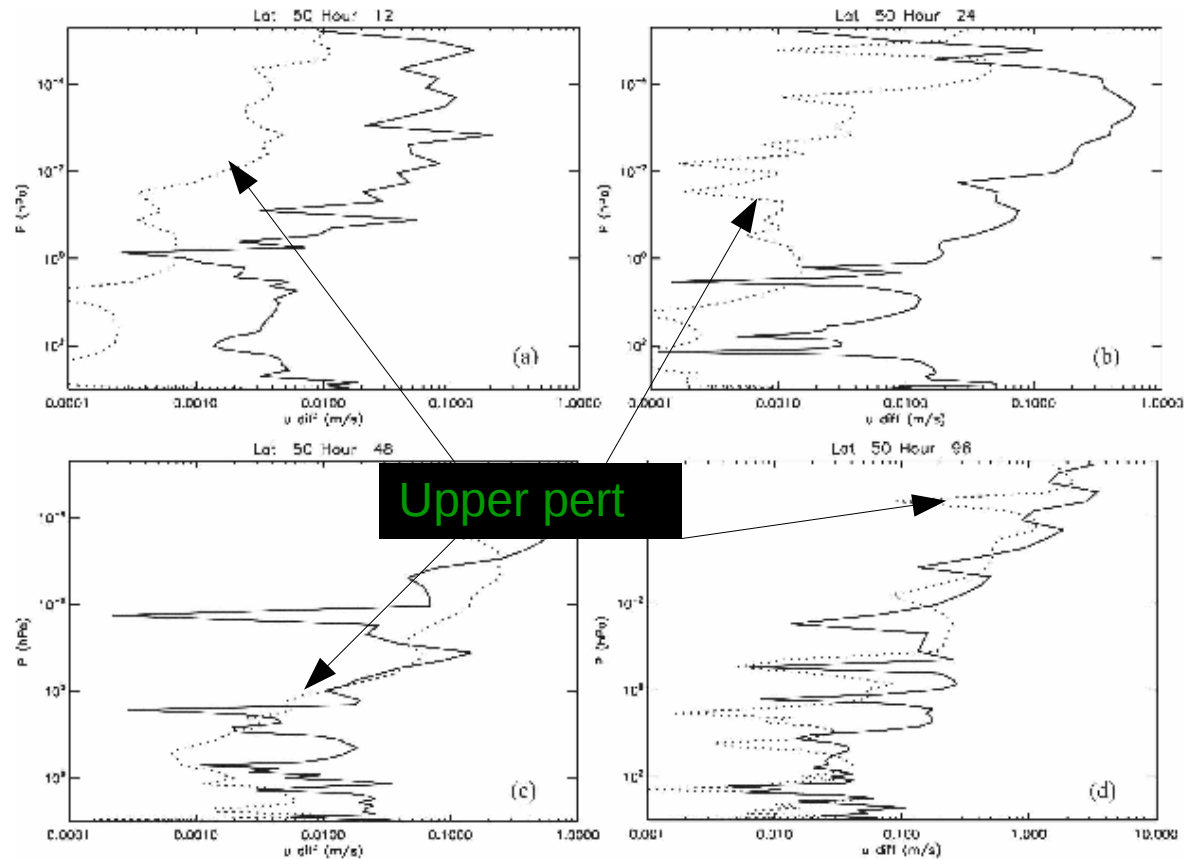


FIG. 7. Vertical profiles of rms zonal wind error in case L (solid line) and case U (dotted line) (a) 0.5 day, (b) 1 day, (c) 2 days, and (d) 4 days after the simulation starts.



# A few questions

Focusing on the intrinsic error:

- What is the role played by **gravity waves**? For a shallow spectrum,  $Ro$  and  $Fr$  increase towards small scales.
- What happens in the **stratosphere and mesosphere**? Expect them to be more predictable.
- Is rapid **vertical coupling** robust?





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# NWP model



# NWP Model (Met Office Unified Model)

Key features:

- Based on compressible Navier-Stokes
- Comprehensive model physics (e.g. convection, radiation, microphysics)
- Semi-Lagrangian/semi-implicit timestepping

Overview of simulations:

- 432 x325 grid (spacing ~ 50 km)
- 50 levels (lid at ~ 60 km)
- 30-day forecasts
- Initial conditions taken from winter 2006 analysis



# Procedure

Perturbations are constructed by decomposing horizontal velocity field into spherical harmonics and randomising phase.

Two cases

- Small scale:  $70 \leq n_f \leq 216$
- Large scale:  $1 \leq n_f \leq 3$

No initialisation (e.g. horizontal non-divergence or geostrophy) or data assimilation: “intrinsic predictability”.



# Diagnostics

Error spectra:

$$\Delta_{KE}(k) = \sum_{|\mathbf{k}|=k} \frac{(\Delta\omega)^2}{k^2}, \quad \Delta\omega(k) = \omega^{(p)} - \omega^{(c)}$$

Relative error:

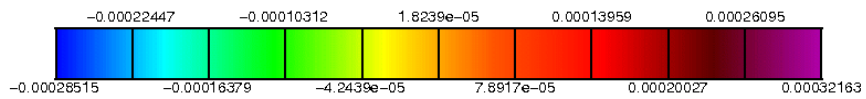
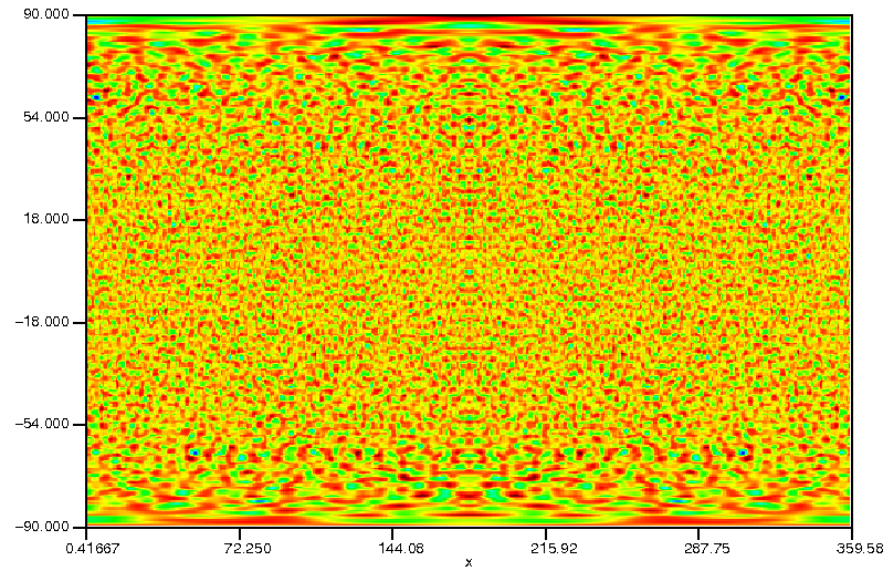
$$r_{KE}(k) = \frac{\Delta_{KE}(k)}{E_K(k)}$$

Mean relative error:

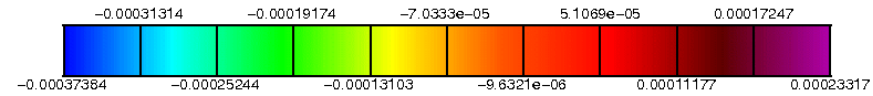
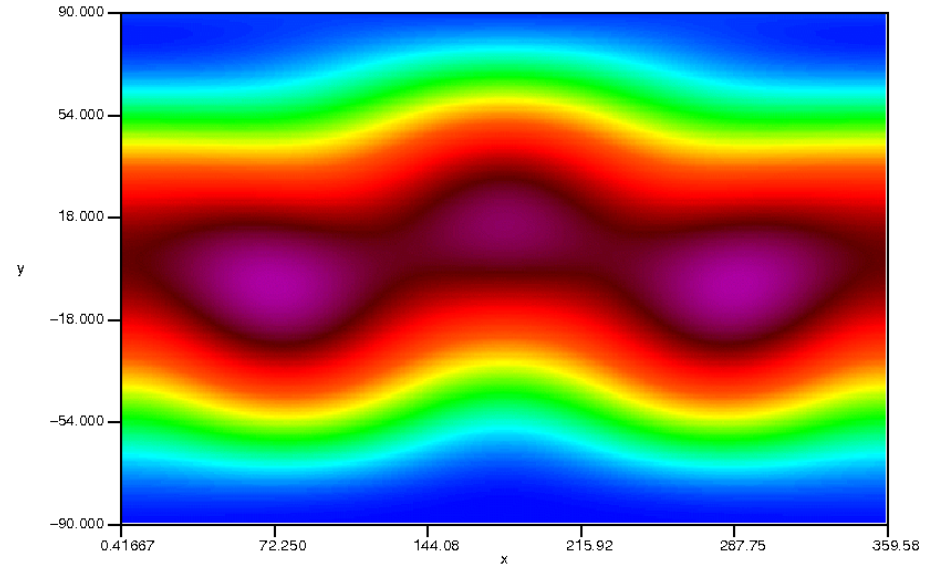
$$\overline{r_{KE}}(\tau) = \frac{\sum_k \Delta_{KE}(k)}{E_K}$$



# Perturbations



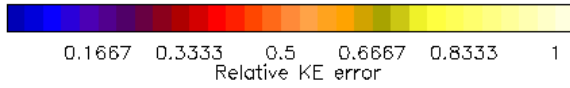
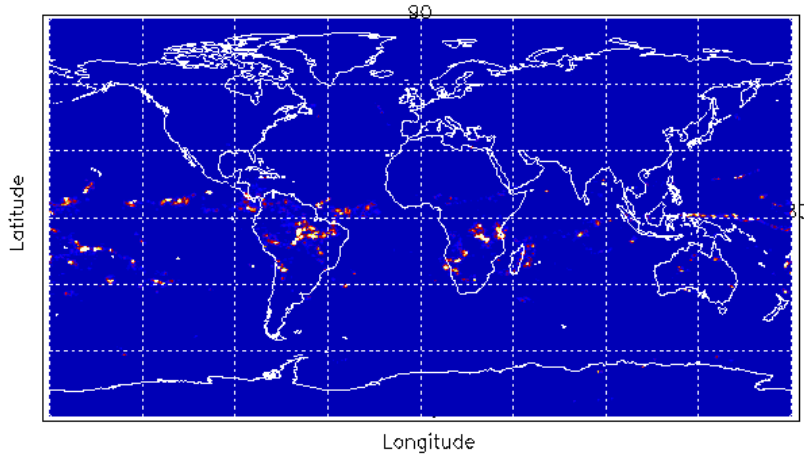
small scale



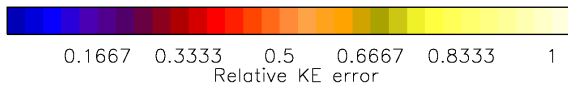
large scale



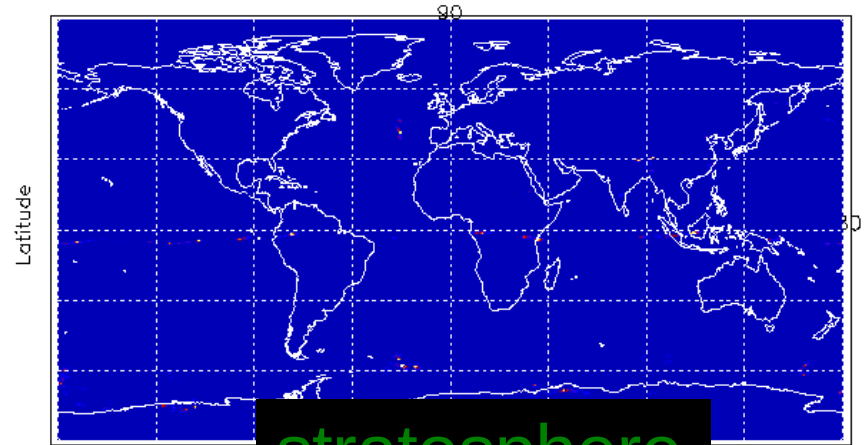
# Spatial error (t=24hours)



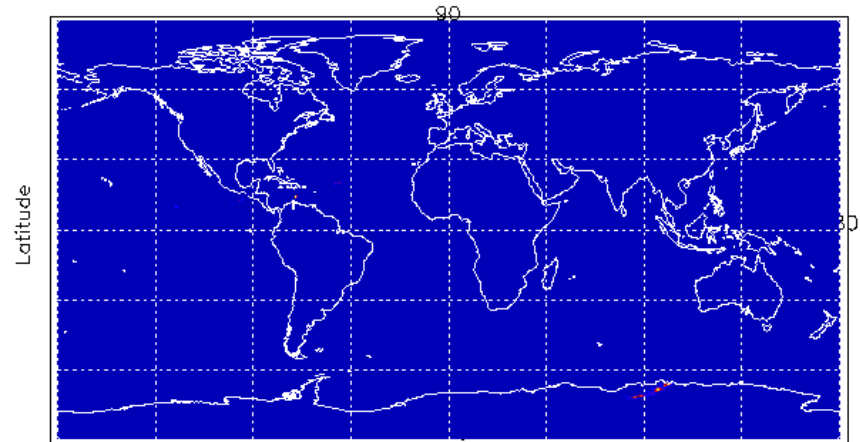
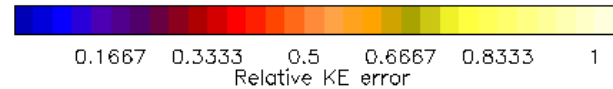
**Troposphere**



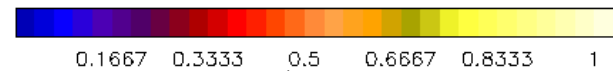
**Mesosphere**



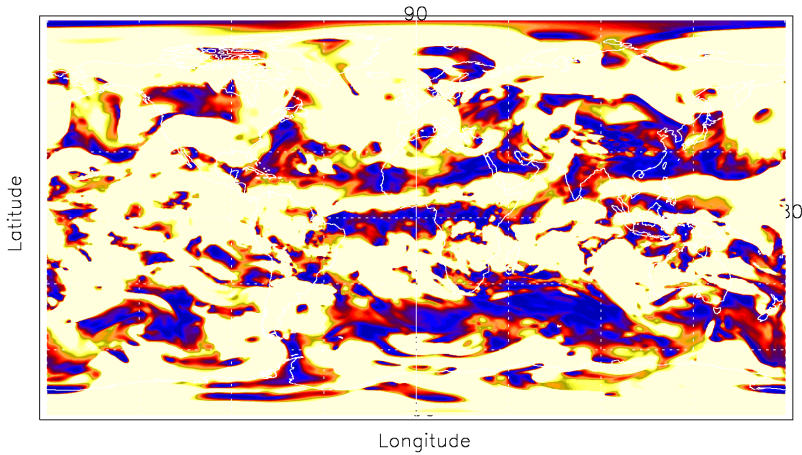
**stratosphere**



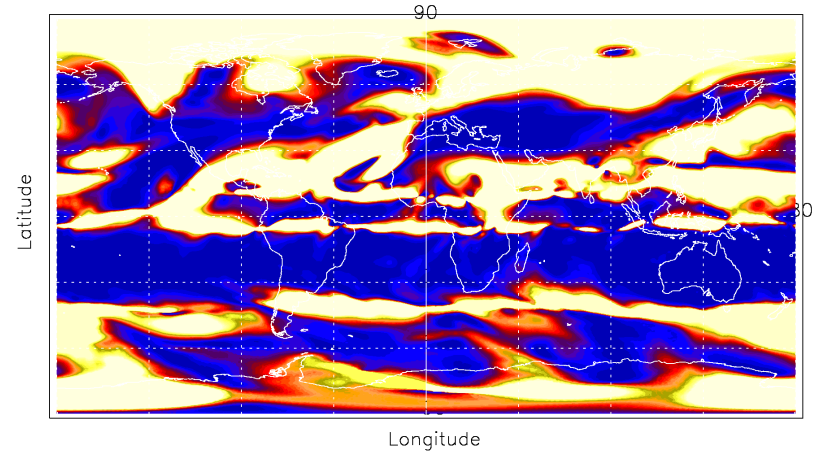
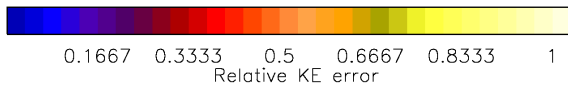
Longitude



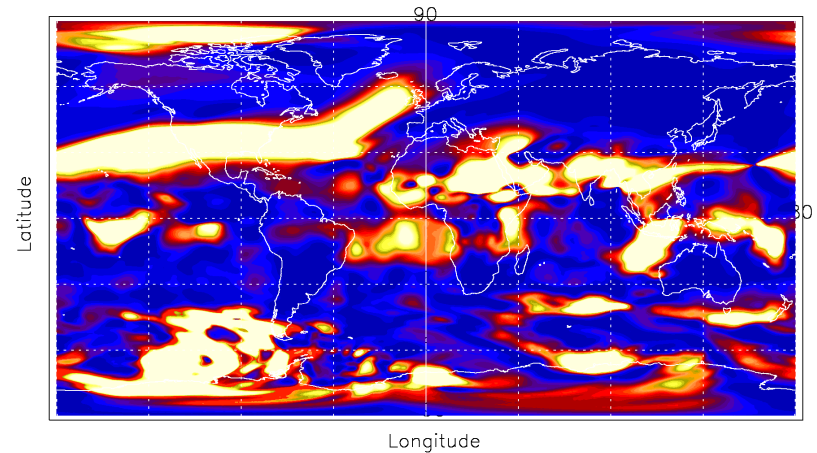
# Spatial error (t=720hours)



**Troposphere**



**Stratosphere**



**Mesosphere**

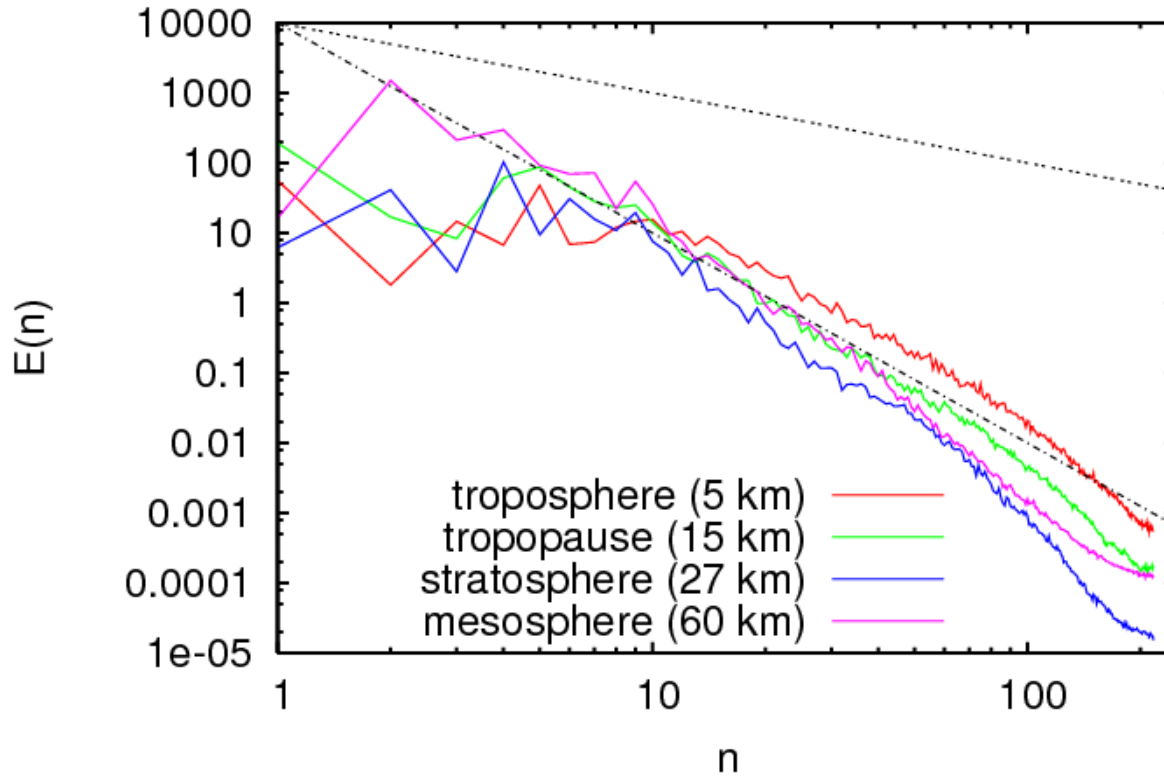
Errors are much smaller in the mesosphere.  
Slower error growth in tropics.



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# Energy Spectra

$E(n)$  vs  $n$



Mesospheric spectra of middle-atmosphere GCMs show evidence of shallowing (Koshyk et al. 1999).

Similar results are obtained with a 1-minute timestep.

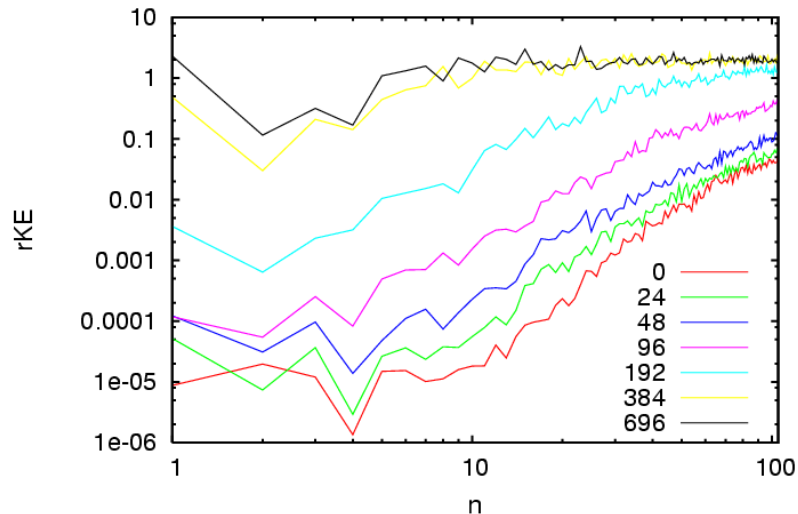




# Relative error spectra (small-scale pert)

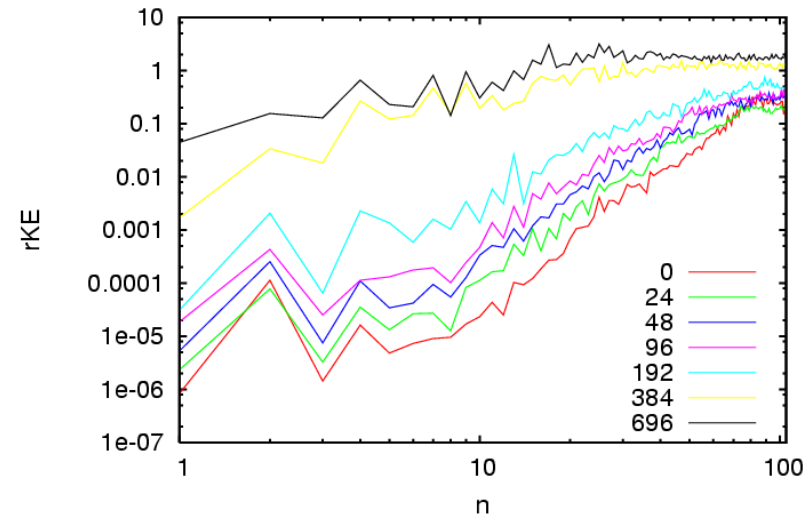
trop

rKE(n) vs n, lev=17



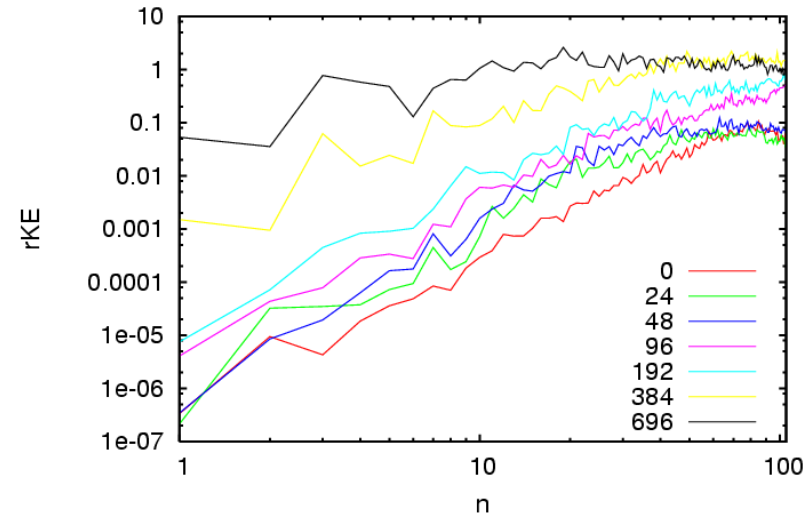
strat

rKE(n) vs n, lev=38



mes

rKE(n) vs n, lev=50



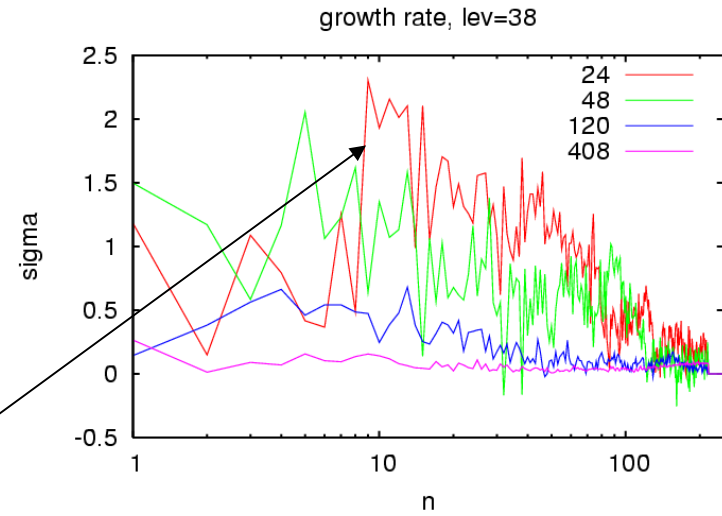
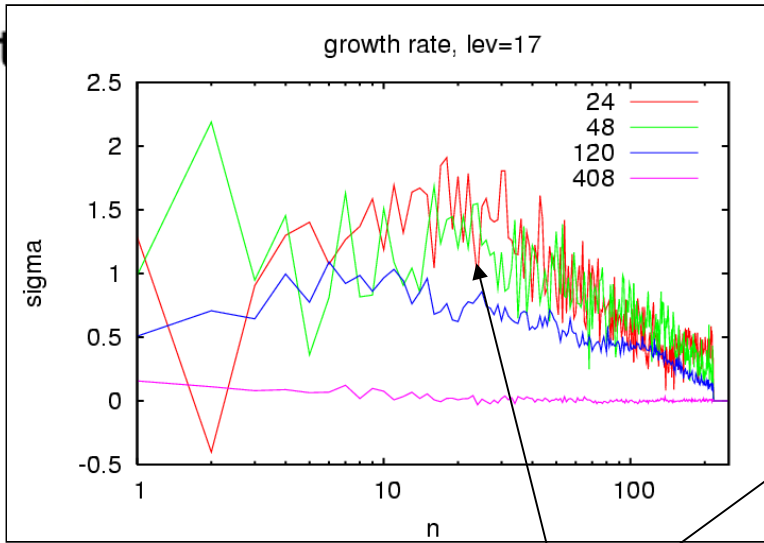
- Stratosphere and mesosphere are more predictable.
- Structure of evolution is broadly similar from troposphere to mesosphere.



# Spectral growth rates

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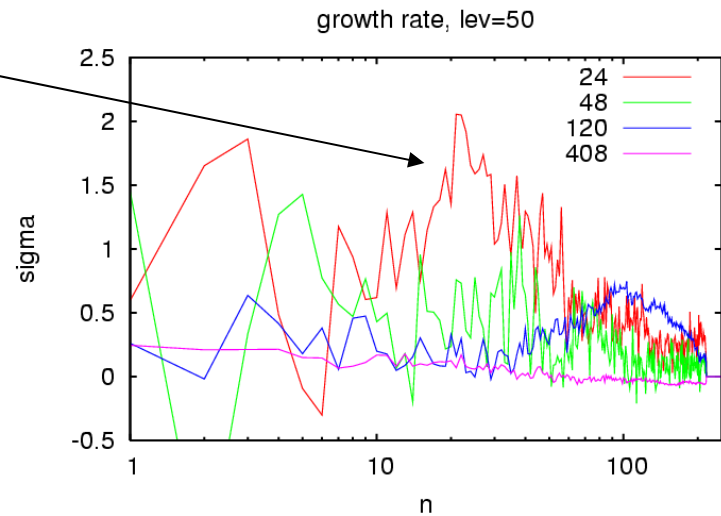
trop



strat

intermediate

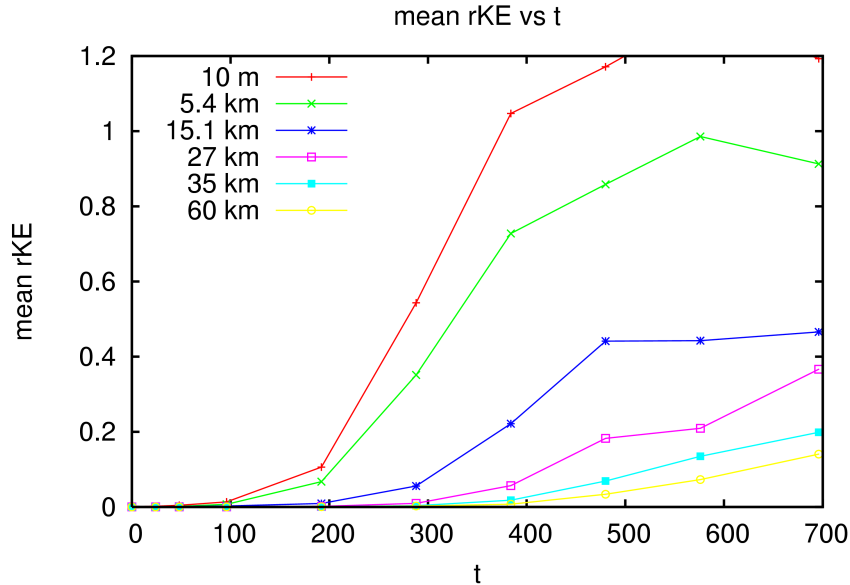
Initially there is preferential growth on intermediate, baroclinically-active scales (Tribbia & Baumhefner 2004).



mes



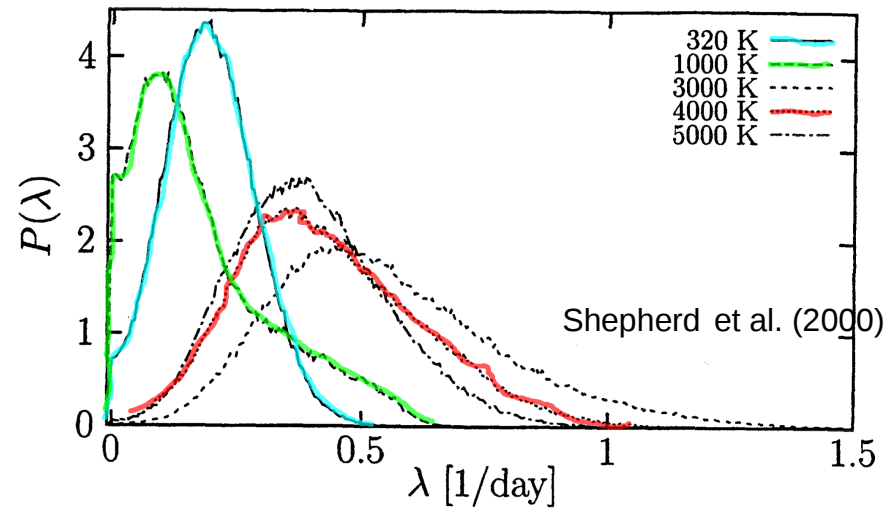
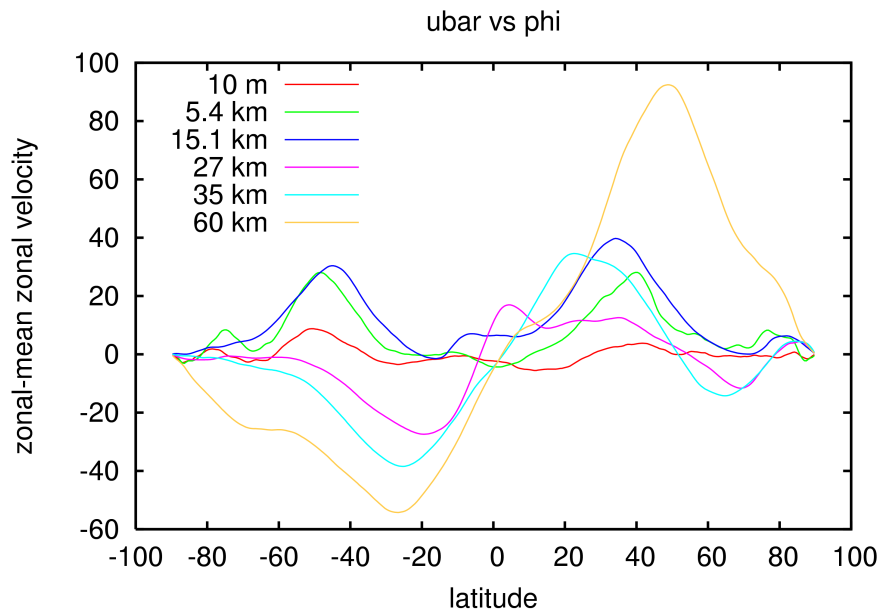
# Enhanced mesospheric predictability



mean relative error

zonal wind

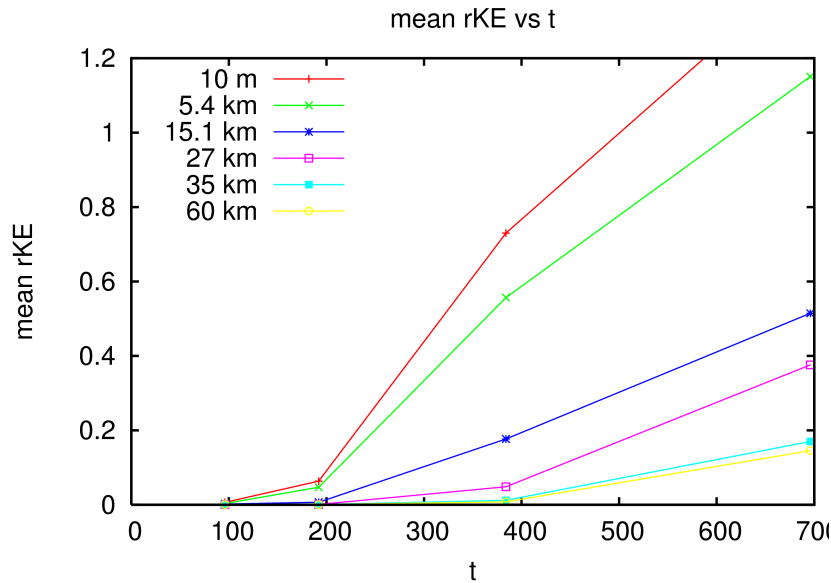
lyap exp's



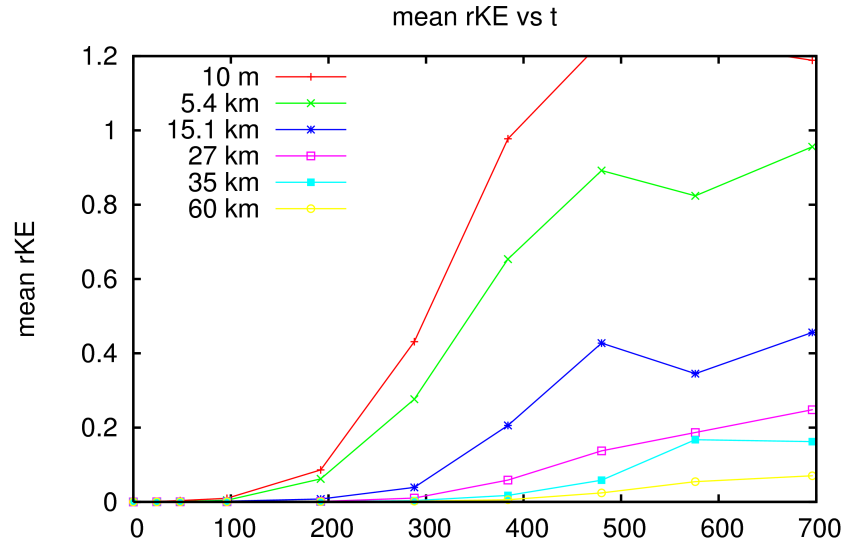


# Vertical localisation

trop



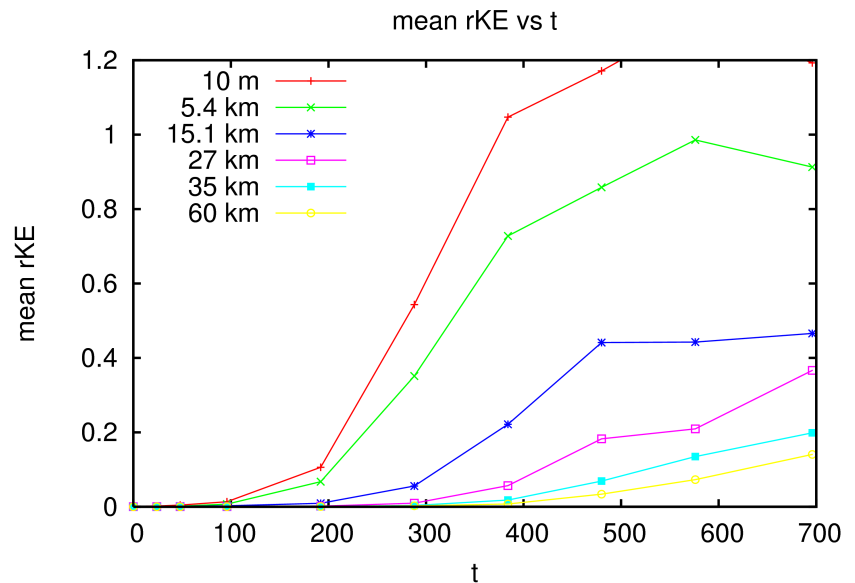
mes



- Results closely resemble those for whole-atmosphere perturbation (cf. Liu et al. 2009).

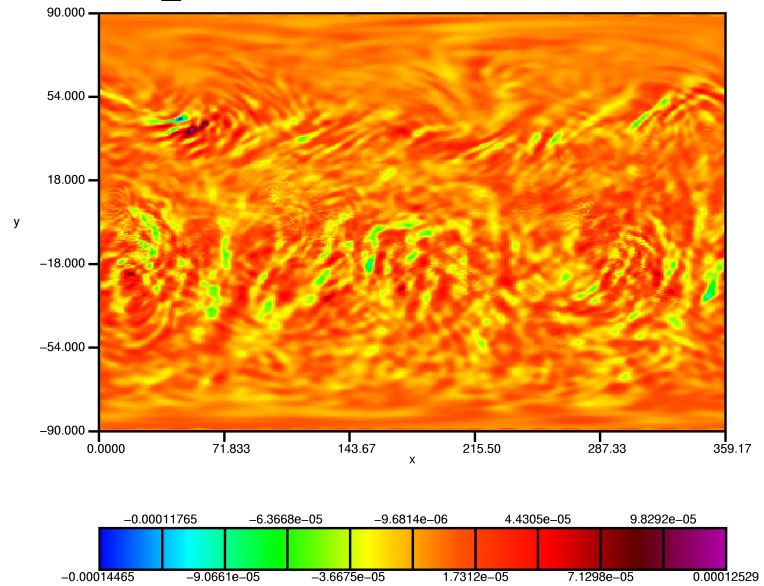
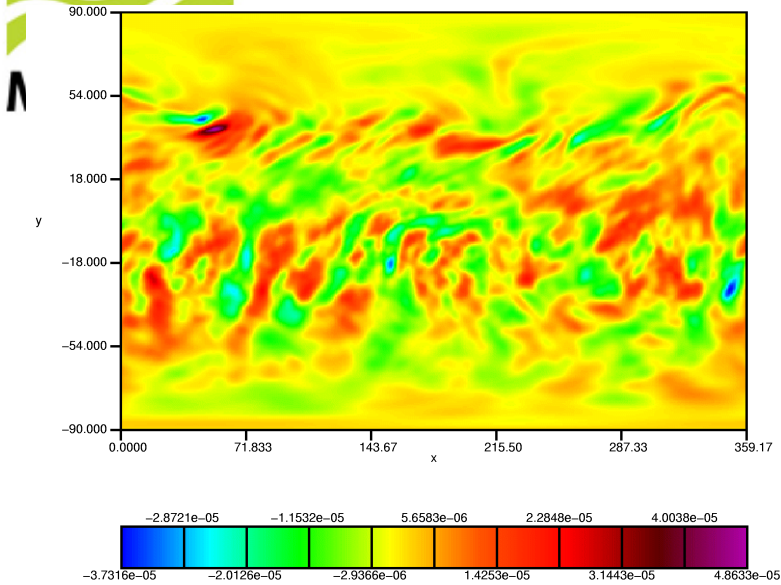
- Mechanism: vertically-propagating waves

strig

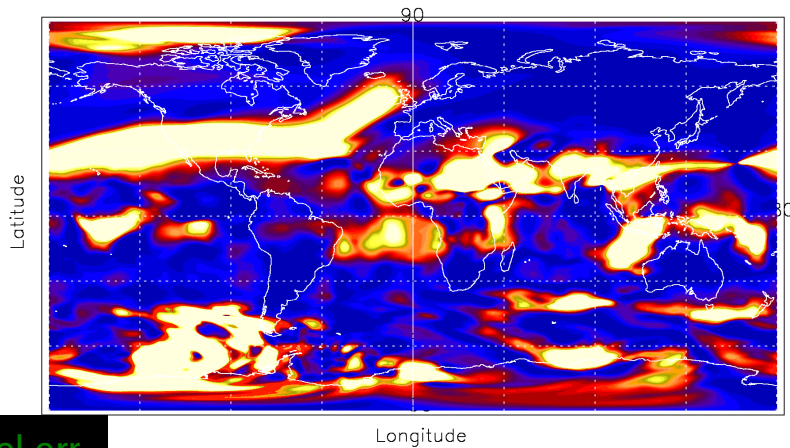




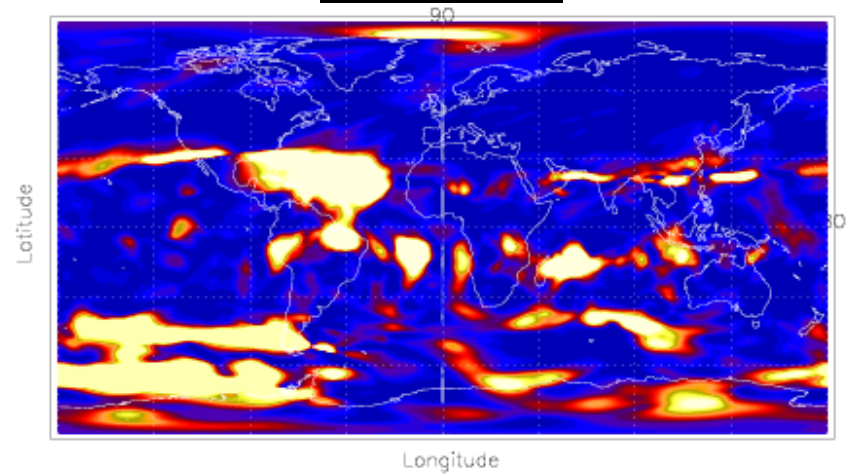
# Resolved gravity waves



$\Delta t = 20 \text{ min}$

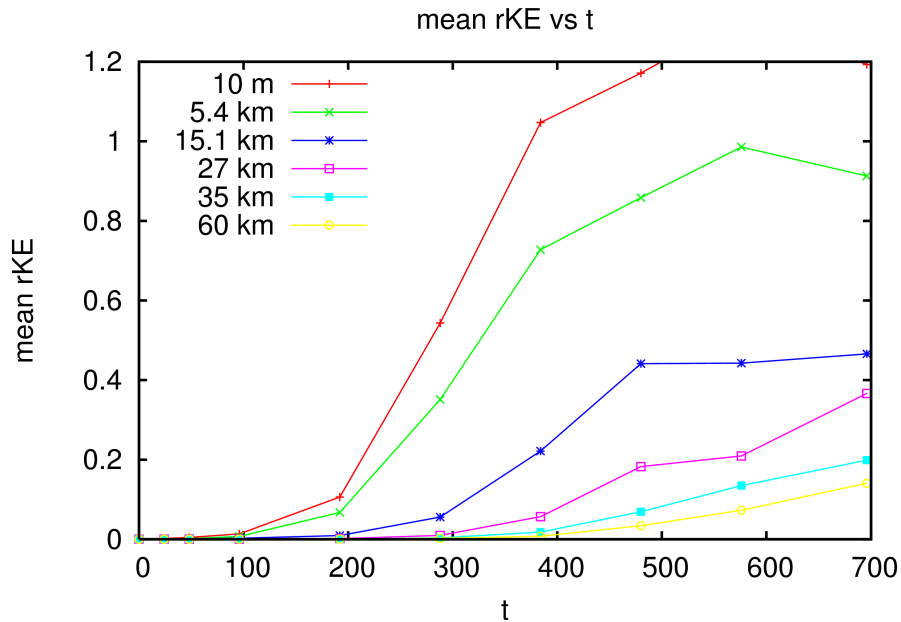


$\Delta t = 1 \text{ min}$

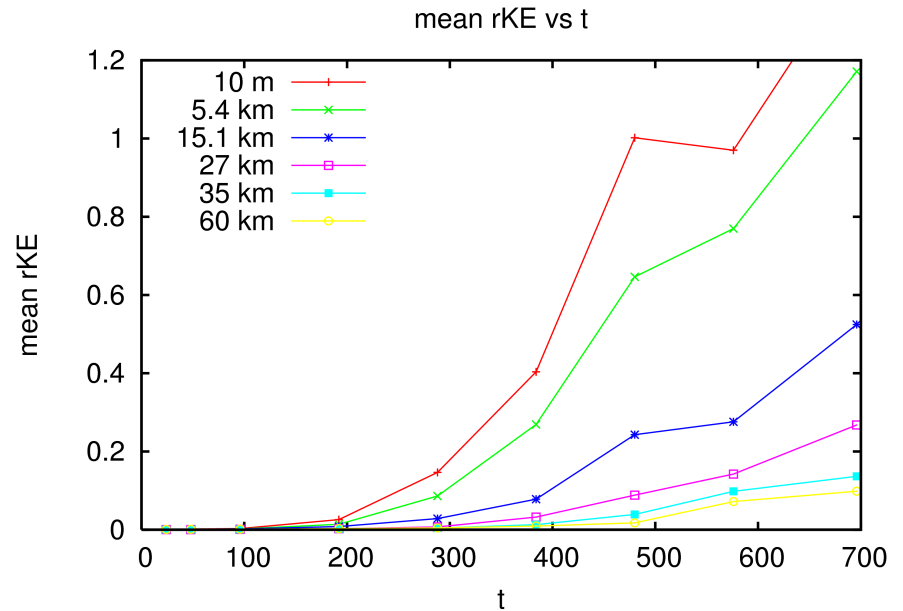




# 1-minute timestep



$\Delta t = 20 \text{ min}$



$\Delta t = 1 \text{ min}$

- Operational NWP model is run with a fairly large timestep. In the mesosphere, Courant number  $> 1$ .
- With  $\Delta t = 1 \text{ min}$ , advective Courant number  $< 1$ , but wave Courant number is large.
- Fast waves are not being resolved properly. Implications for predictability?



# Discussion



# Open questions

- Influence of increased resolution (more resolved gravity waves, shallower spectrum)
- Influence of initialisation/ data assimilation.





# Summary

- Predictability decay is significantly slower for small-scale, strongly stratified flow.
- Predictability decay is slower in the mesosphere than in the troposphere. But difference is modest: numerics (e.g. resolving gravity waves) could be an issue.



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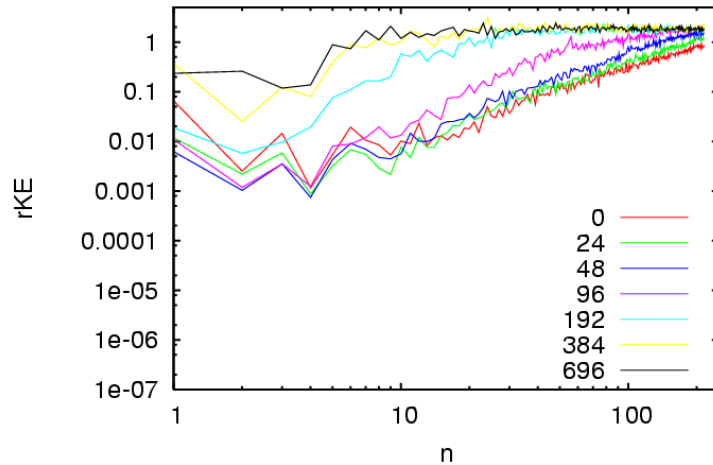
# Questions and answers



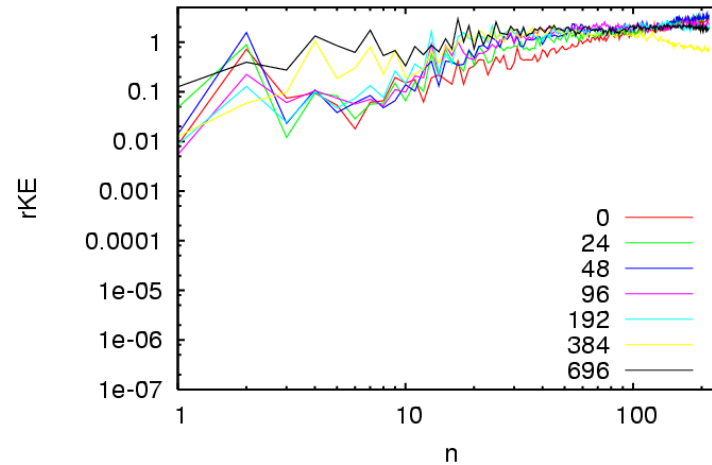
# Relative error spectra (O(1) large-scale pert)

trop

rKE(n) vs n, lev=17



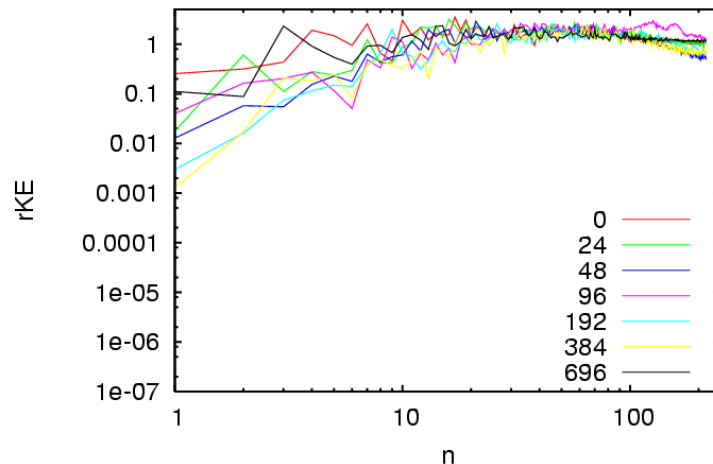
rKE(n) vs n, lev=38



strat

tion of a large-amplitude, large-scale perturbation  
ability loss at large scales.

rKE(n) vs n, lev=50



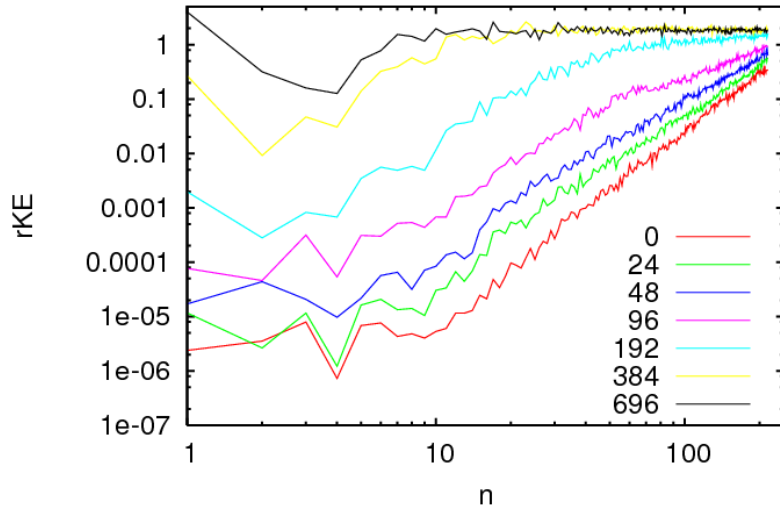
mes



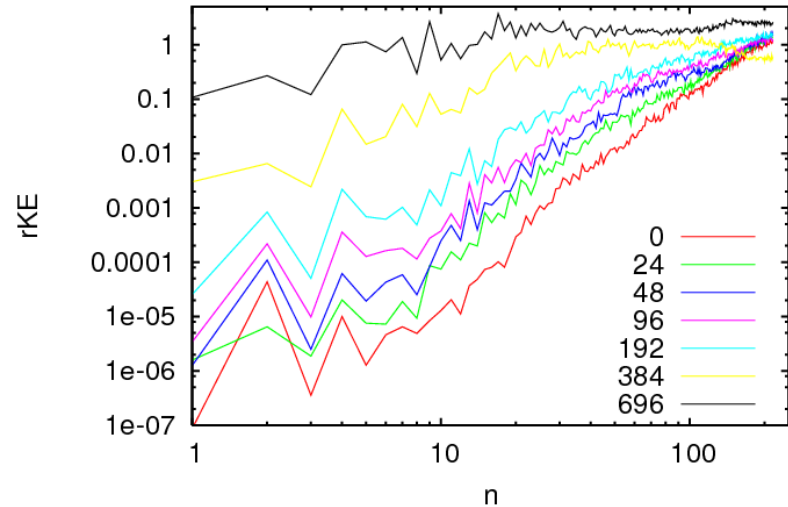
# Relative error spectra (large-scale pert)

trop

rKE(n) vs n, lev=17

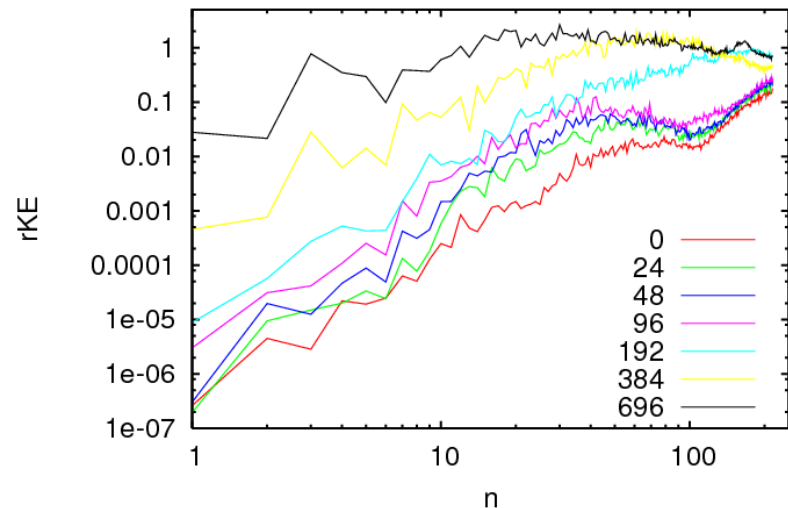


rKE(n) vs n, lev=38



strat

rKE(n) vs n, lev=50



mes

- Results are essentially identical to those for the small-scale perturbation.
- Spectral filtering is not exact: residual small-scale noise (energy is  $\sim 10^6$  smaller).



# Classical picture of predictability

Much of our intuition about atmospheric predictability derives from the pioneering work of Lorenz, Leith & Kraichnan.

Using a stochastic model of the barotropic vorticity equation, Lorenz (1969) showed that there is finite atmospheric predictability: infinitesimal small-scale errors contaminate the largest scales within a finite period of time.

Lorenz's analysis was later corrected and extended by Leith & Kraichnan (1972).

This is the so-called atmospheric **butterfly effect**.



# Dynamical regimes

Nature of the dynamics is controlled by  $Ro$  and  $Fr$ . Expand in  $Ro$ :

$O(1)$ : geostrophy

$O(Ro)$ ,  $Ro \sim Fr$ , quasi-geostrophy

Hierarchy of balanced models.

Want to examine contrast between (super)synoptic (large-scale) and subsynoptic (small-scale) flows.

# Energy spectra in turbulence

Navier-Stokes equations for a constant-density fluid:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0,$$

in  $T^2$  or  $T^3$ . Turbulence for  $Re = UL/\nu \gg 1$ .

Kinetic energy spectrum

$$E(k) = \sum_{\mathbf{k}=|k|} \frac{1}{2} \mathbf{u}(\mathbf{k}) \mathbf{u}(\mathbf{k})^*$$

3-D (Kolmogorov)

$$E(k) \sim k^{-5/3} \text{ [inertial range]}$$

2-D (Batchelor-Leith-Kraichnan)

$$E(k) \sim k^{-3} \text{ [small scales; enstrophy range]}$$

$$E(k) \sim k^{-5/3} \text{ [large scales; energy range]}$$

Quasi-geostrophic turbulence is essentially identical.



# Applicability to the real atmosphere

The real atmosphere cannot be described exactly by 2-D or quasi-geostrophic turbulence.

Conventional view is that the classical picture carries over straightforwardly to the real atmosphere.

Key quantity: **eddy turnover time**

$$\tau(k; k_1) = \left( \int_{k_1}^k k'^2 E(k') dk' \right)^{-\frac{1}{2}}$$

For  $E(k) \sim k^{-p}$  and  $k \rightarrow \infty$

$$\tau(k; k_1) \sim \begin{cases} \text{constant}, & p < 3 \\ k^{(p-3)/2}, & p > 3 \end{cases}$$

**Upshot:** predictability behaviour depends crucially on the atmospheric energy spectrum.