Jacobian mapping between vertical coordinate systems in data assimilation

Yves J. Rochon[1] , Louis Garand[1], D.S. Turner[1], and Saroja Polavarapu[1]

with contributions from Shuzhan Ren [2], Jacques Hallé[1] and Yulia Nezlin[2]

[1]*Atmospheric Science and Technology Directorate, Environment Canada,* [2]*Department of Physics, University of Toronto*

Contact: yves.rochon@ec.gc.ca

Environment Canada Environnement Canada

Abstract: In atmospheric data assimilation, radiances measured by remote sensing instruments form a significant component of the observation network. Radiance assimilation involves fast radiative transfer models (RTM) which project profiles provided by forecast models onto the observation space for direct comparison with the measurements. One of the features typically characterizing fast RTMs is the use of a fixed vertical coordinate. In the absence of a fast RTM for calculating radiances directly using the levels of the forecast model, an interpolation of forecast profiles to the RTM coordinate is necessary. In variational data assimilation, the mapping of the Jacobians of the observations from the radiative transfer model coordinate to the forecast model coordinate is therefore also necessary. This mapping of Jacobians is accomplished through the adjoint of the forecast profile interpolator. As shown here, the nearest neighbour log-linear interpolator commonly used operationally can lead to incorrect mapping of Jacobians and can potentially lead to incorrect assimilation. This problem has been previously masked in part through the smoothing effect of forecast error vertical correlations on the analysis increments. To solve this problem, an alternative interpolator relying on piecewise log-linear weighted averaging over the layers is proposed. This interpolator is found to satisfy design guidelines stipulated for ensuring acceptable Jacobian mappings.

Introduction

Context:

- ✓ Fast RTMs for assimilation of radiances from nadir sounders often rely on regression based models evaluated on fixed pressure levels (e.g. RTTOV).
- ✓ Numerical prediction (e.g. NWP) models often use different vertical levels and a different vertical coordinate (e.g. η-hybrid).
- ✓ In this circumstance, Jacobian mapping from RTM to model coordinate is required in data assimilation.
- ✓ Data assimilation requires explicit pairing of the vertical interpolator and Jacobian mapping.

a) profile x' on RTM levels ← profile x on model levels

$$x'(p_i) = x'_i = s_i(\mathbf{x}) = \sum_j W_{i,j} x_j \quad \text{or} \quad \mathbf{x'} = \mathbf{W}\mathbf{x}$$

b) Jacobian mapping:

model vertical coordinate ← RTM vertical coordinate

$$\left.\frac{\partial f}{\partial x_j}\right|_{\mathbf{x}} = \sum_i \left.\frac{\partial f}{\partial x'_i}\right|_{\mathbf{x'}} \left.\frac{\partial x'_i}{\partial x_j}\right|_{\mathbf{x}} = \sum_i \left.\frac{\partial f}{\partial x'_i}\right|_{\mathbf{x'}} W_{i,j} \quad \text{or} \quad \mathbf{h} = \mathbf{W}^T\mathbf{h'}$$

The Jacobian mapping matrix is the adjoint $\mathbf{W}^T$ of the forward model vertical interpolator matrix **W**.

Identification of problem:

- ✓ Model levels not participating in forward interpolation (blind levels) lead to improper Jacobian mapping.
- ✓ Blind levels can result when the model vert. resolution is higher than the RTM vert. resolution.
- ✓ Improper mapping **heavily masked** by vert. correlations of background covariances.

Remainder of poster:

- ✓ Identify an appropriate design for the vertical interpolator and its adjoint for use with fast RTMs in data assimilation when required.
- ✓ Investigate sensitivity to choice of interpolator and representativeness quality of mapped Jacobians.

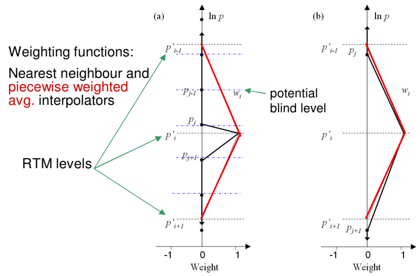
Interpolators

Interpolators for data assimilation:

- ➢ Nearest neighbour log-linear interpolator (operationally applied at EC for example)
- ➢ Proposed alternative: piecewise weighted averaging log-linear interpolator

$$x'_i = \frac{\int_i^{i+1} w_i\, x \cdot d\ln p + \int_{i-1}^{i} w_i\, x \cdot d\ln p}{\int_i^{i+1} w_i \cdot d\ln p + \int_{i-1}^{i} w_i \cdot d\ln p}$$

evaluated using the trapezoidal rule with weights w.



Mapping comparisons

Jacobian mappings via adjoint of:

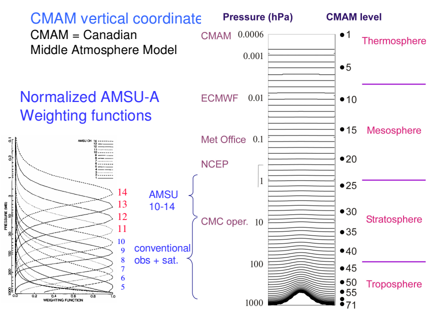
- ➢ Nearest neighbour interpolator
- ➢ Proposed interpolator

Compared to

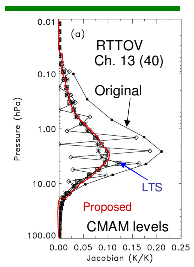
- ➢ Layer Thickness Scaling (LTS) interpolation for Jacobian mapping (no forward interpolator and adjoint pairing – not applicable to DA)
- ➢ RTM calculations on model levels (D.S. Turner)

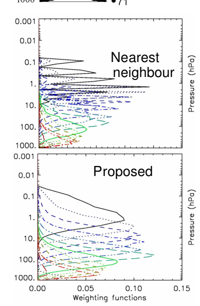
using AMSU-A channels 1 to 14 and GLFBL (D.S. Turner) Jacobian calculations for AIRS (5) and HIRS (5) channels.

LTS mapping method was used in Saunders et al. (2006) Garand et al. (2001) RTM intercomparisons (JGR)

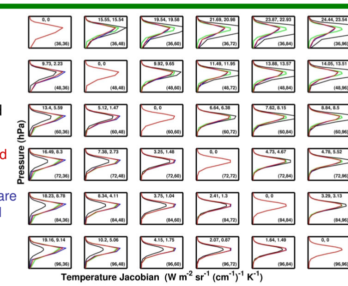


Mapping of AMSU-A Jacobians





Jacobian mappings for HIRS channel 12 for various (M,N)



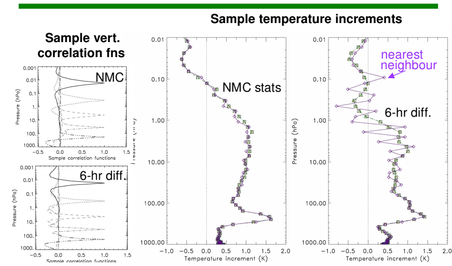
LIST OF AIRS and HIRS CHANNELS FOR WHICH SIMULATIONS WERE PERFORMED. HWHM STANDS FOR THE HALF-WIDTH AT HALF-MAXIMUM OF THE JACOBIAN PROFILE

Channel	Frequency (cm^{-1})	Pressure (hPa) at			Related atmospheric variable(s)
		peak	lower HWHM	higher HWHM	
AIRS 305	737.1	750	440	96	Temperature
AIRS 453	793.1	900	670	1010	temperature and water vapour
AIRS 1090	1040.1	25	12	80	ozone
AIRS 1766	1544.3	340	260	400	water vapour
AIRS 2197	2500.3	920	670	1010	temperature
HIRS 1	668.9	2	0.3	50	temperature
HIRS 7	749.6	800	490	980	temperature
HIRS 8	898.7	820	620	970	surface temperature and cloud detection
HIRS 9	1028.3	25	1	90	ozone
HIRS 12	1481.0	400	280	550	water vapour

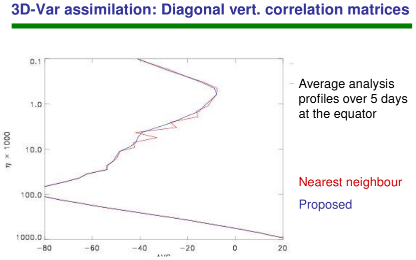
Profile relative error measure (%) from proposed interpolator mappings over AIRS and HIRS channels and various model and RTM vertical resolutions:

71% of cases with <5% 90% of cases with <15% for 17 280 cases

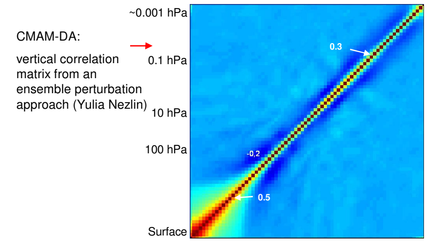
1D assimilation: Impact of vert. correl. & vert. interpolators



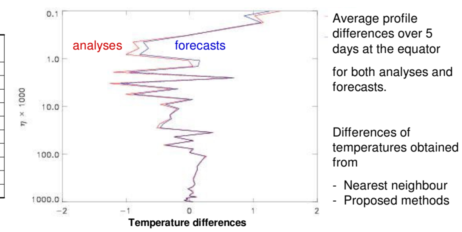
3D-Var assimilation: Diagonal vert. correlation matrices



3D-Var assimilation: Ensemble perturbation scheme vert. correlation matrix (global average)

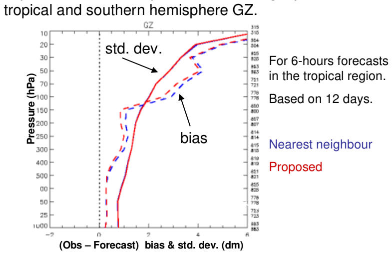


3D-Var assimilation: Differences in analyses/forecasts with ensemble perturbation scheme vert. correlation matrices



3D-Var assimilation: Impact on geopotential height GZ (GEM model and NMC statistics: preliminary results)

Impact in lower atmosphere is weak. Slightly visible on tropical and southern hemisphere GZ.



Summary and comments

- ✓ Proposed vertical interpolator satisfies Jacobian mapping requirements.
- ✓ P.S.: The forward vertical interpolator and its adjoint can account for surface pressure dependency of model coordinate when required.
- ✓ Level of benefit depends at least on vertical resolutions and width of vertical correlation functions.
- ✓ Stand-alone code to be made available shortly (contact: yves.rochon@ec.gc.ca and louis.garand@ec.gc.ca)
- ✓ Manuscript to QJRMS conditionally accepted.