EXAMPLE 2.3

Consider a time signal (e.g., a voltage) that is generated according to the following rules: (a) The waveform is generated with a sample-and-hold arrangement where the "hold" interval is 1 sec; (b) the successive amplitudes are independent samples taken from a set of random numbers with uniform distribution from -1 to $+1$; and (c) the first switching time after $t = 0$ is a random variable with uniform distribution from 0 to 1. (This is equivalent to saying the time origin is chosen at random.) A typical sample realization of this process is shown in Fig. 2.3. Note that the process mean is zero and its mean-square value works out to be one-third. [This is obtained from item (b) of the description.] ■

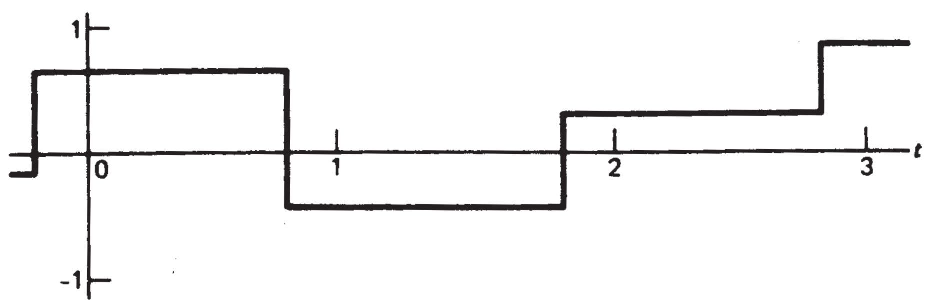


Figure 2.3 Sample signal for Example 2.3.
(Brown and Hwang 1997)

EXAMPLE 2.4

Consider another time function generated with a sample-and-hold arrangement with these properties: (a) The "hold" interval is 0.2 sec, (b) the successive amplitudes are independent samples obtained from a zero-mean normal distribution with a variance of one-third, and (c) the switching points occur at multiples of .2 units of time; that is, the time origin is not chosen at random in this case. A sketch of a typical waveform for this process is shown in Fig. 2.4. ■

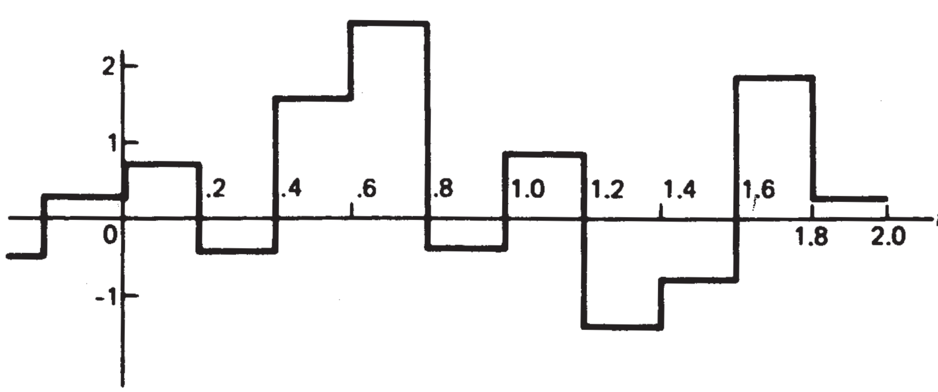


Figure 2.4 Typical waveform for Example 2.4.
(Brown and Hwang 1997)

EXAMPLE 4.1 Consider two scalar zero-mean processes $x(\cdot)$ and $y(\cdot)$ with

$$\Psi_{xx}(t_1, t_2) = P_{xx}(t_1, t_2) = \sigma^2 e^{-|t_1 - t_2|/T}, \qquad \Psi_{yy}(t_1, t_2) = P_{yy}(t_1, t_2) = \sigma^2 e^{-|t_1 - t_2|/10T}$$

where these two correlations are plotted as a function of the time difference $(t_1 - t_2)$ in Fig. 4.4. For a given value of $(t_1 - t_2) \neq 0$, there is a higher correlation between the values of $y(t_1)$ and $y(t_2)$ than between $x(t_1)$ and $x(t_2)$. Physically one would then expect a typical sample $x(\cdot, \omega_i)$ to exhibit more rapid variations in magnitude than $y(\cdot, \omega_i)$, as also depicted in Fig. 4.4. Note that such information is not contained in $P_{xx}(t)$ and $P_{yy}(t)$, or $\Psi_{xx}(t)$ and $\Psi_{yy}(t)$, all of which are the same value for this example, σ^2, as seen by evaluating the preceding expressions for $t_1 = t_2$. ■

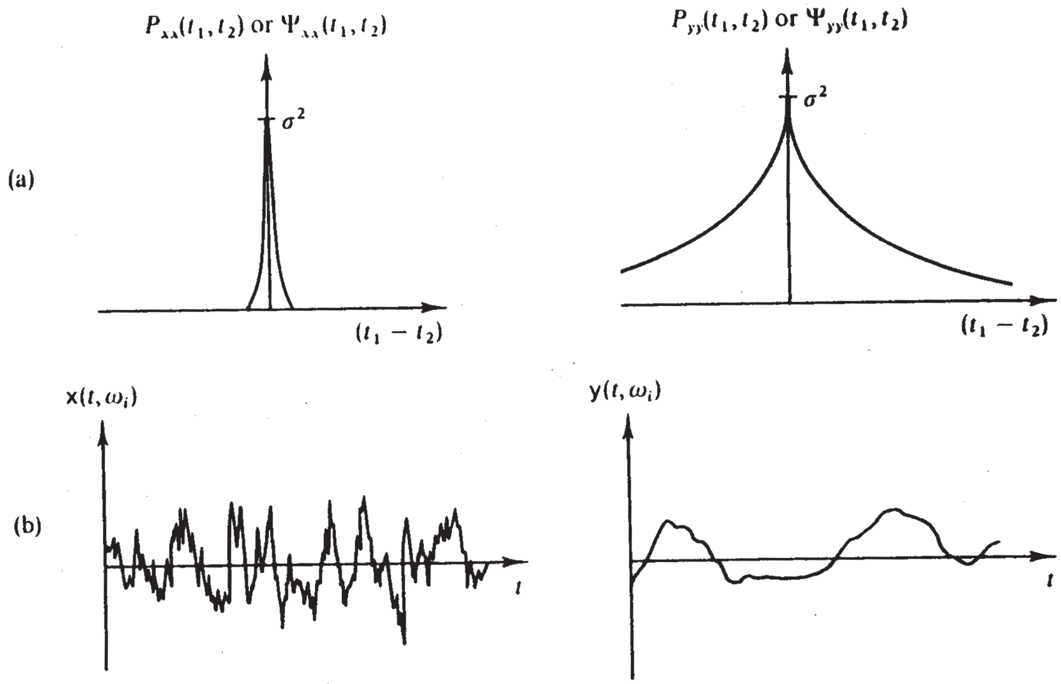


FIG. 4.4 Second moment information about stochastic processes. (a) Correlation or variance kernels. (b) Typical samples from the stochastic processes.

Maybeck (1979)

EXAMPLE 4.2 Figure 4.6 depicts the autocorrelation functions and power spectral density functions (using the most common convention of definition) of a white process, an exponentially time-correlated process, and a random bias. Note that a white noise is uncorrelated in time, yielding an impulse at $\tau = 0$ in Fig. 4.6a; the corresponding power spectral density is flat over all ω—equal power content over all frequencies. Figure 4.6b corresponds to an exponentially time-correlated process with correlation time T, as discussed in Example 4.1. Heuristically, these

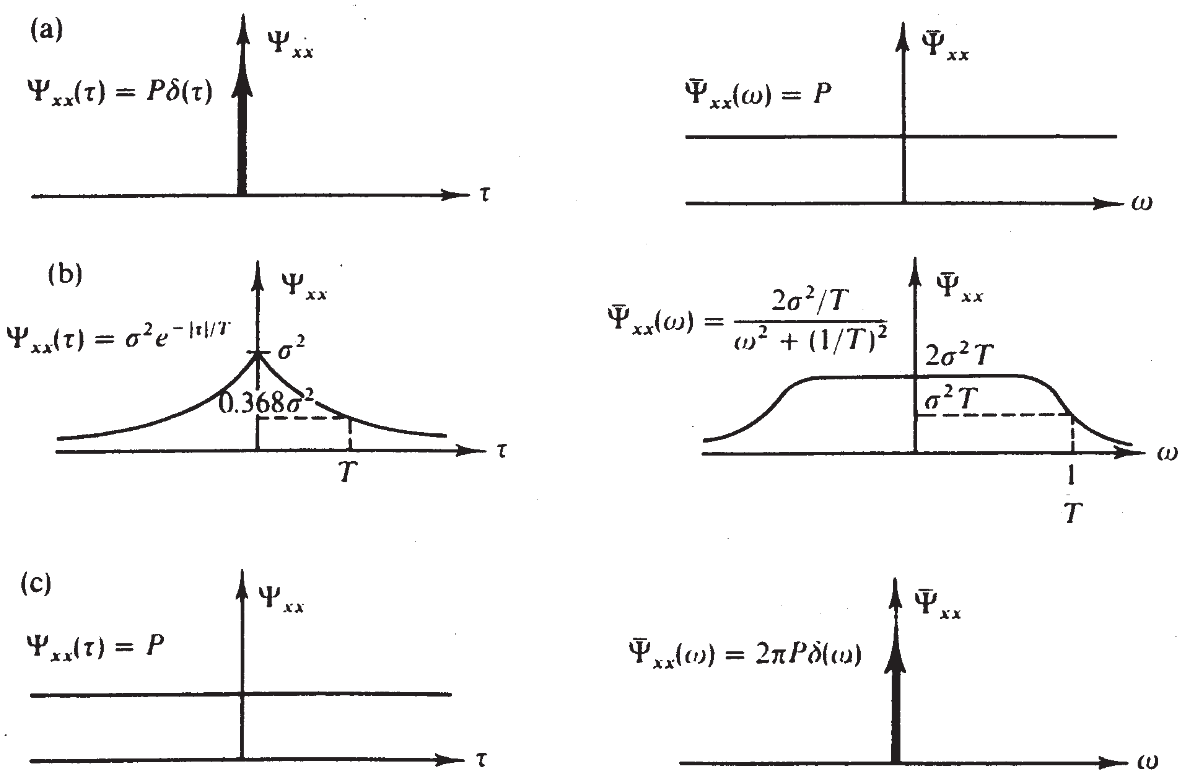


FIG. 4.6 Typical autocorrelations and power spectral densities. (a) White process. (b) Exponentially time-correlated process. (c) Random bias. (Maybeck 1979)

Bandlimited white noise

Bandlimited white noise is a random process whose spectral amplitude is constant over a finite range of frequencies, and zero outside that range. If the bandwidth includes the origin (sometimes called baseband), we then have

$$S_{bwn}(j\omega) = \begin{cases} A, & |\omega| \le 2\pi W \\ 0, & |\omega| > 2\pi W \end{cases} \tag{2.9.3}$$

where W is the physical bandwidth in hertz. The corresponding autocorrelation function is

$$R_{bwn}(\tau) = 2WA\,\frac{\sin(2\pi W\tau)}{2\pi W\tau} \tag{2.9.4}$$

Both the autocorrelation and spectral density functions for baseband bandlimited white noise are sketched in Fig. 2.13. It is of interest to note that the autocorrelation function for baseband bandlimited white noise is zero for $\tau = 1/2W$, $2/2W$, $3/2W$, etc. From this we see that if the process is sampled at a rate of $2W$ samples/second (sometimes called the Nyquist rate), the resulting set of random variables are uncorrelated. Since this usually simplifies the analysis, the white bandlimited assumption is frequently made in bandlimited situations.

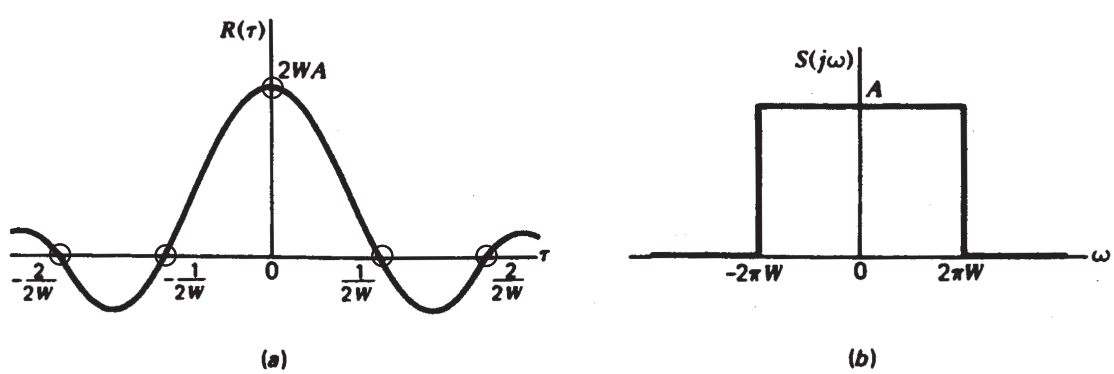
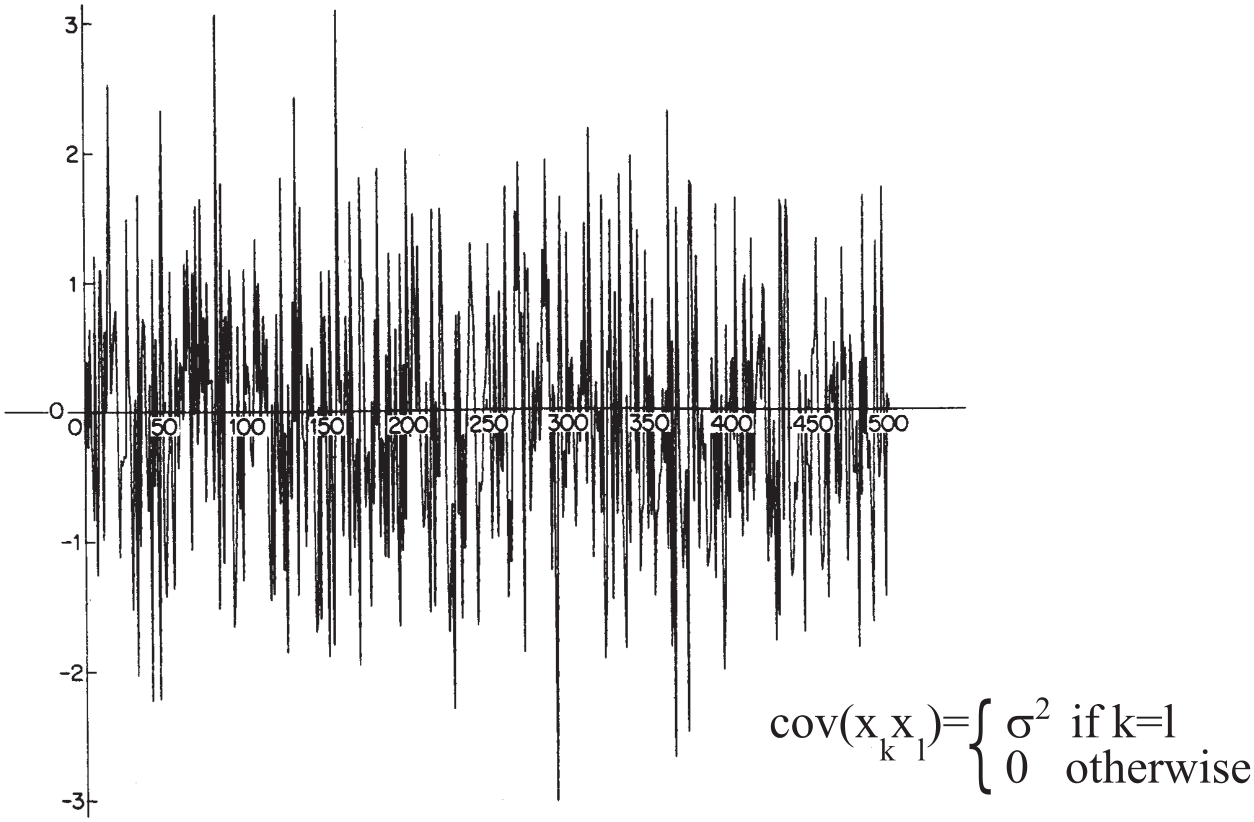
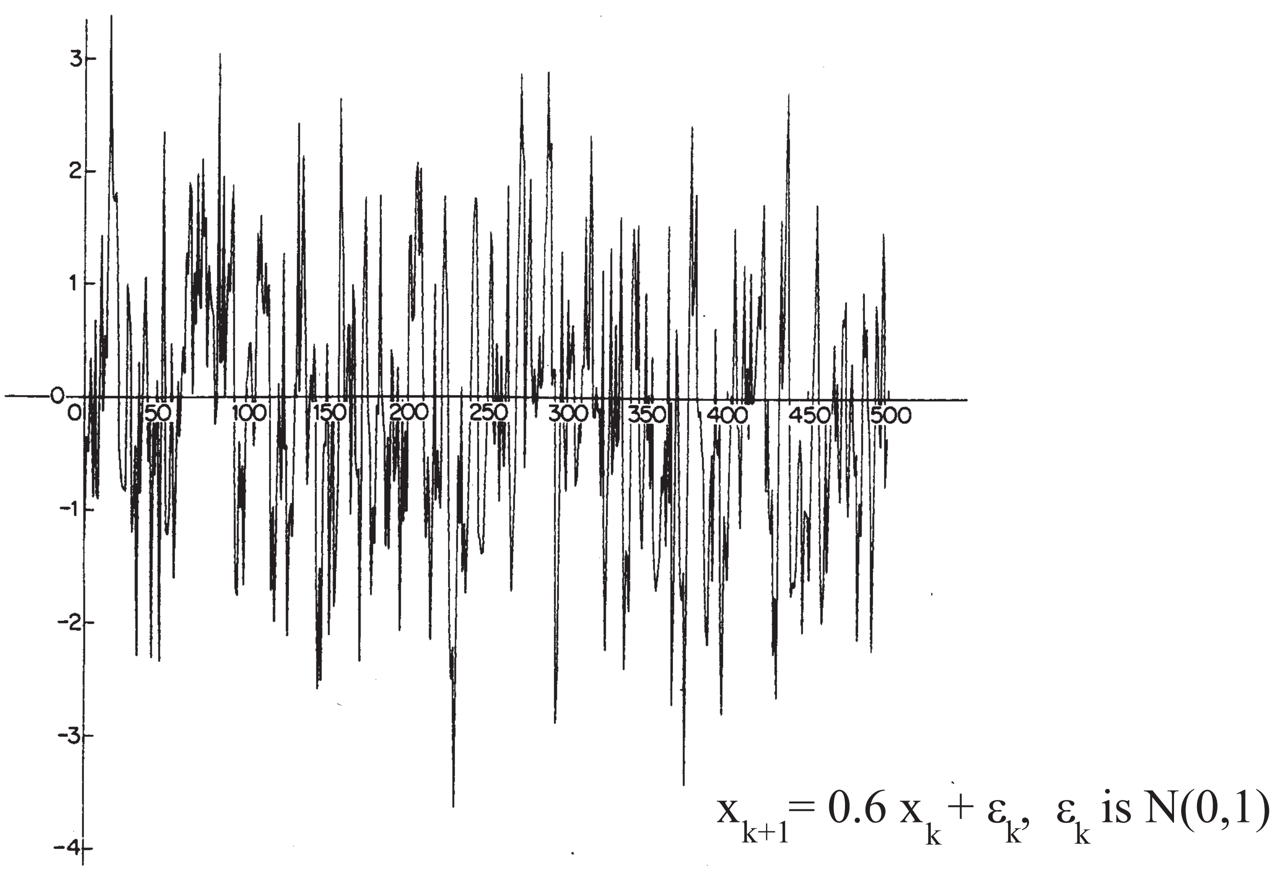


Figure 2.13 Baseband bandlimited white noise. (*a*) Autocorrelation function. (*b*) Spectral density function.

Brown and Hwang (1997)



Graph 3.1. 500 observations from a white noise process. $N(0,1)$
(Priestley 1981)



Graph 3.2. 500 observations from an AR(1) process.
Priestley (1981)