Lecture #2
Planetary Wave Models

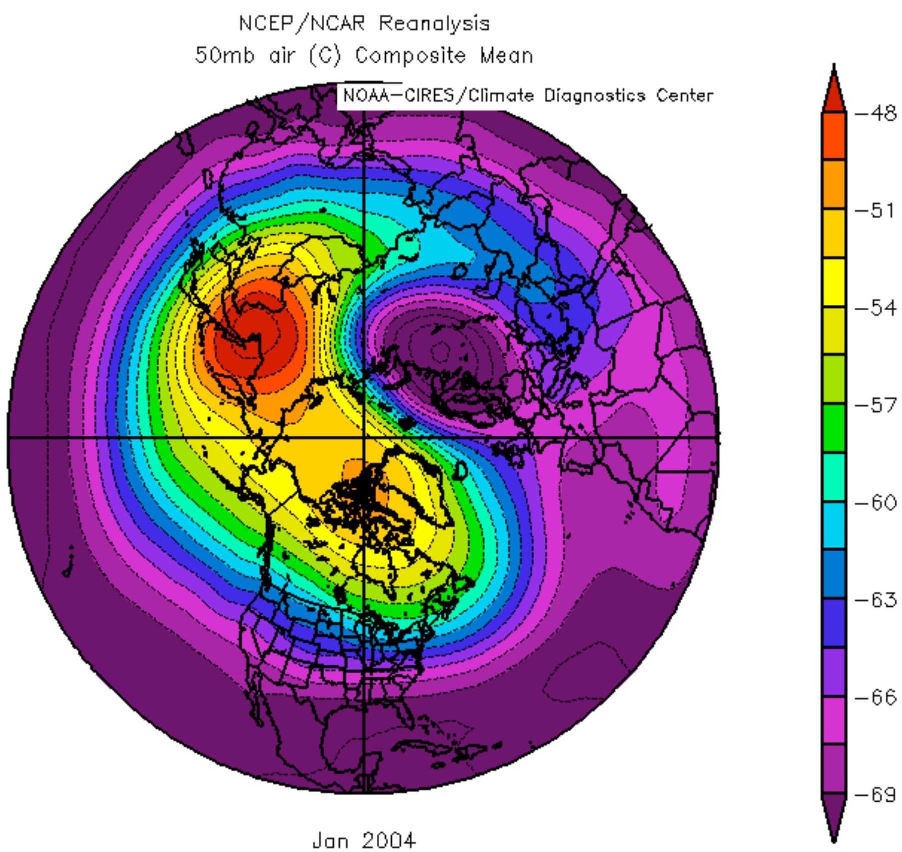
Charles McLandress (Banff Summer School 7-13 May 2005)

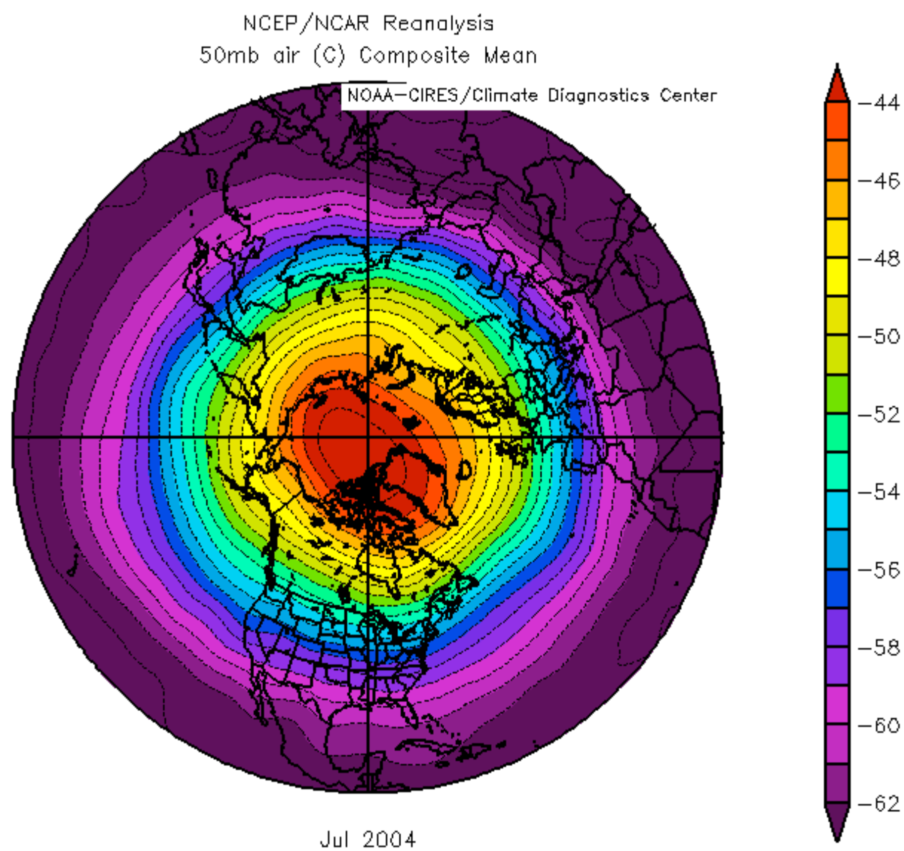
Outline of Lecture

1. Observational motivation
2. Forced planetary waves in the stratosphere
3. Traveling planetary waves in the mesosphere (the 2-day wave)

Part 1: Observational Motivation

50 hPa Temperatures from NCEP/NCAR Reanalysis





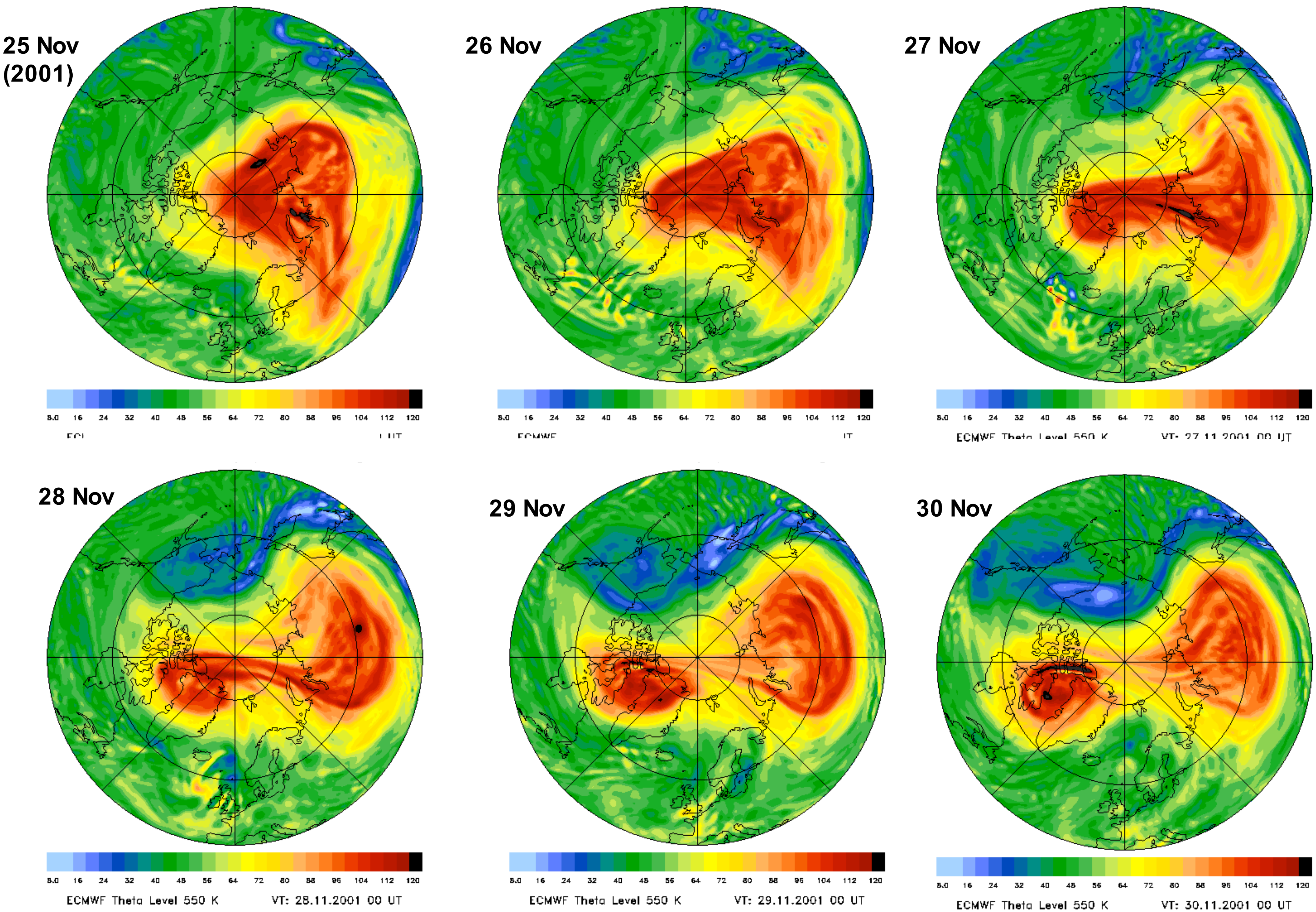
Northern Hemisphere winter

⇒ strong longitudinal variation (due to quasi-stationary planetary Rossby waves)

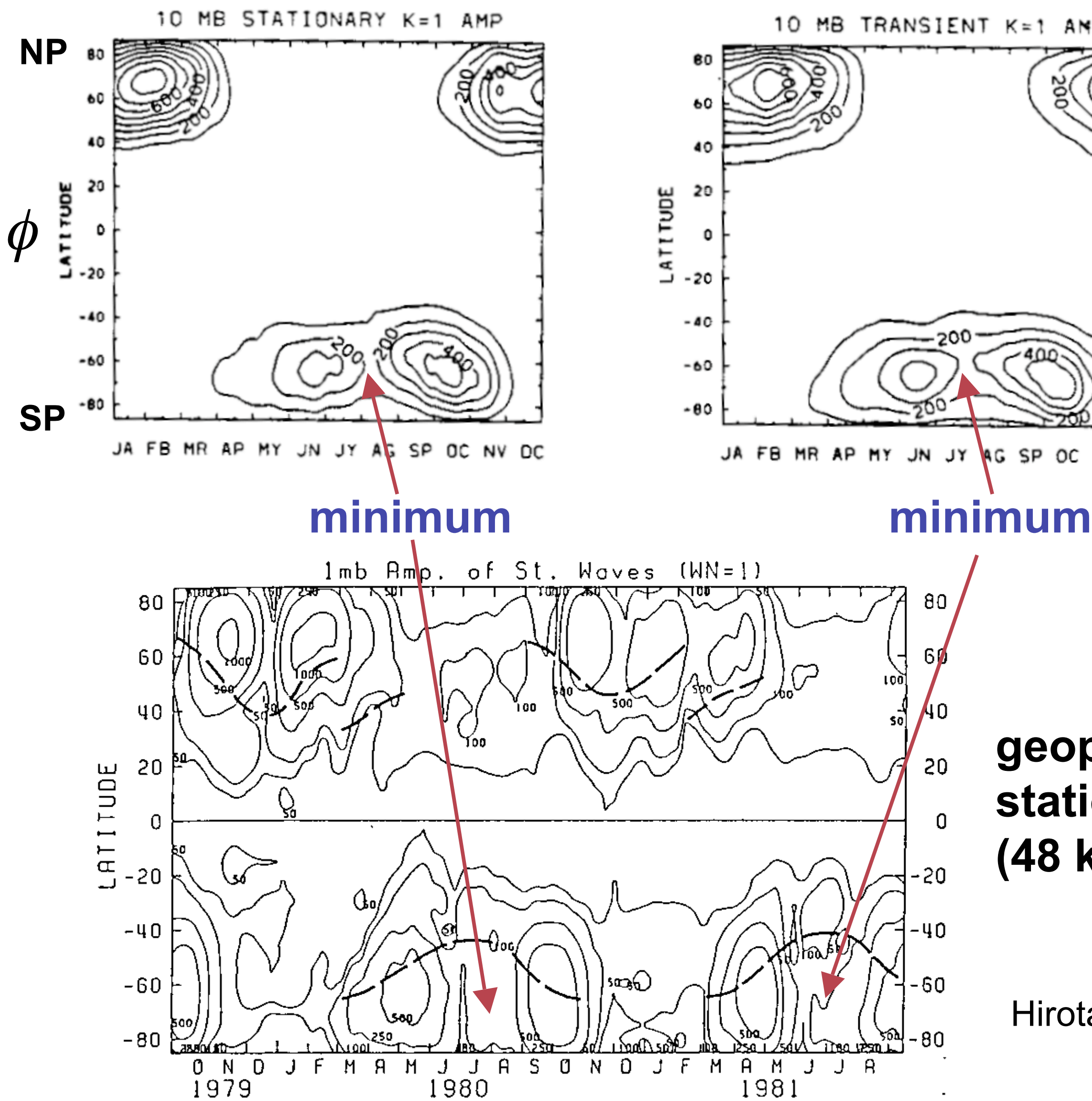
Northern Hemisphere summer

⇒ weak longitudinal variation (no quasi-stationary planetary Rossby waves)

Breaking planetary waves (PV on 550K surface ≈ 20 km)



Midwinter minimum of wave 1 in Southern Hemisphere



geopotential height of stationary & transient wave 1 at 10 hPa (32 km)

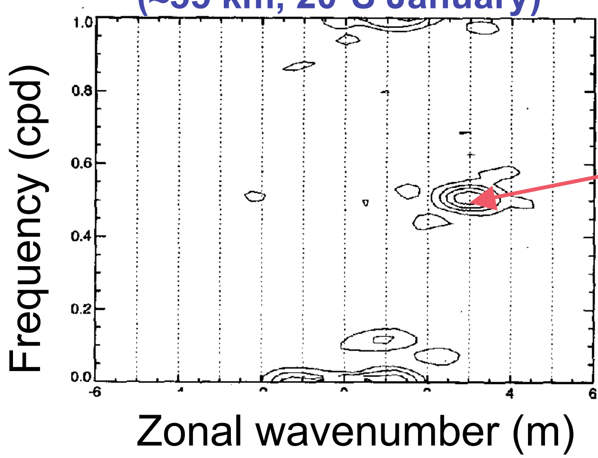
Randel (QJRMS 1988)

geopotential height of stationary wave 1 at 1 hPa (48 km)

Hirota et al (QJRMS 1983)

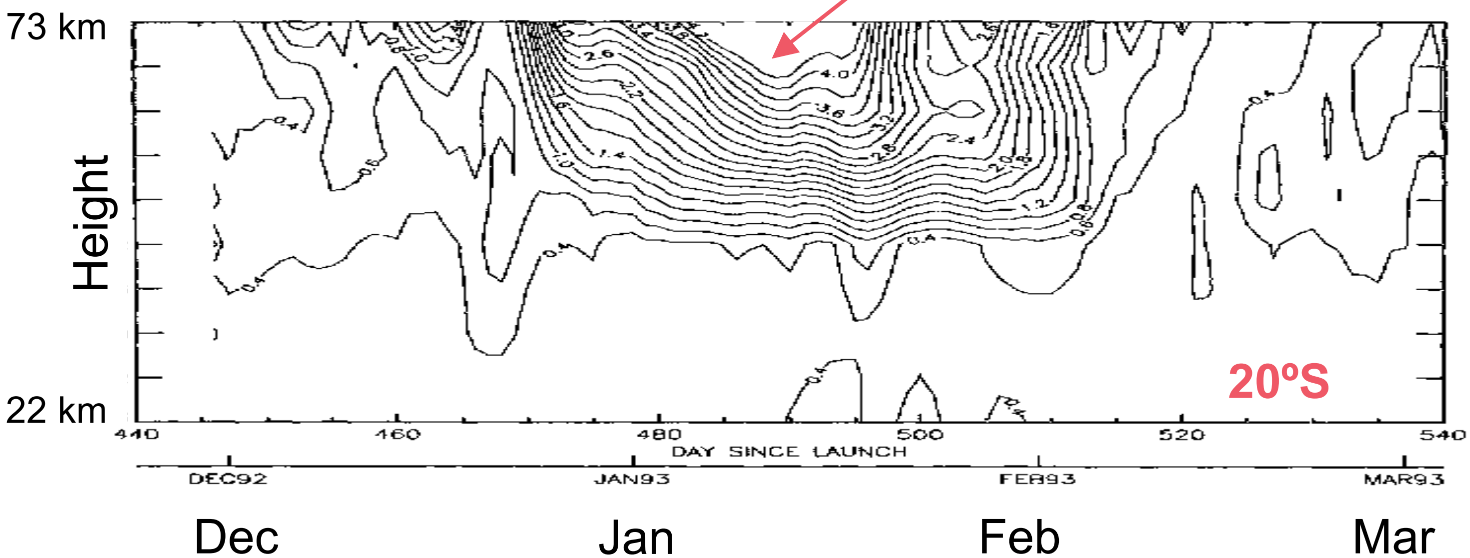
Quasi two-day wave seen in satellite temperature data

(≈55 km, 20ºS January)



Power at m = 3 & freq ≈ 1/(2 days)

quasi 2-day wave amplitude is maximum in mid-summer mesosphere



Wu et al (JAS, 1996)

Part 2: Forced planetary waves in the stratosphere

Characteristics of planetary Rossby waves

- planetary waves are disturbances having zonal wavelengths of the scale of the earth's radius.
- PWs in extratropics are in approximate geostrophic balance (referred to as ***planetary Rossby waves***).
- forced in the troposphere by topography, land-sea temperature contrasts, and synoptic eddies.
- restoring force is latitudinal gradient of background PV.
- horizontal propagation is westward with respect to the background zonal wind.
- vertical propagation into the stratosphere occurs for the longest spatial scales.

PW Models to be discussed

1. Barotropic model on the β-*plane*
2. Linear quasi-geostrophic model on the β-*plane*
3. Linear quasi-geostrophic model on the sphere
4. Quasi-linear models

1. Barotropic PW model on the β-plane

- incompressible fluid with purely horizontal flow.
- Newton's laws results in two equations for the zonal and meridional wind components.
- these two equations can be combined to form the vorticity equation.
- further simplification is made by replacing the spherical geometry with Cartesian geometry and by writing the Coriolis parameter $f = 2\Omega\sin\phi = f_o + \beta y$ (the β-plane approximation).

Rossby wave propagation mechanism

$$\frac{Dq}{Dt} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right) q = 0$$

$$q = \zeta + f = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f_o + \beta y$$

where q is the ***barotropic vorticity***.

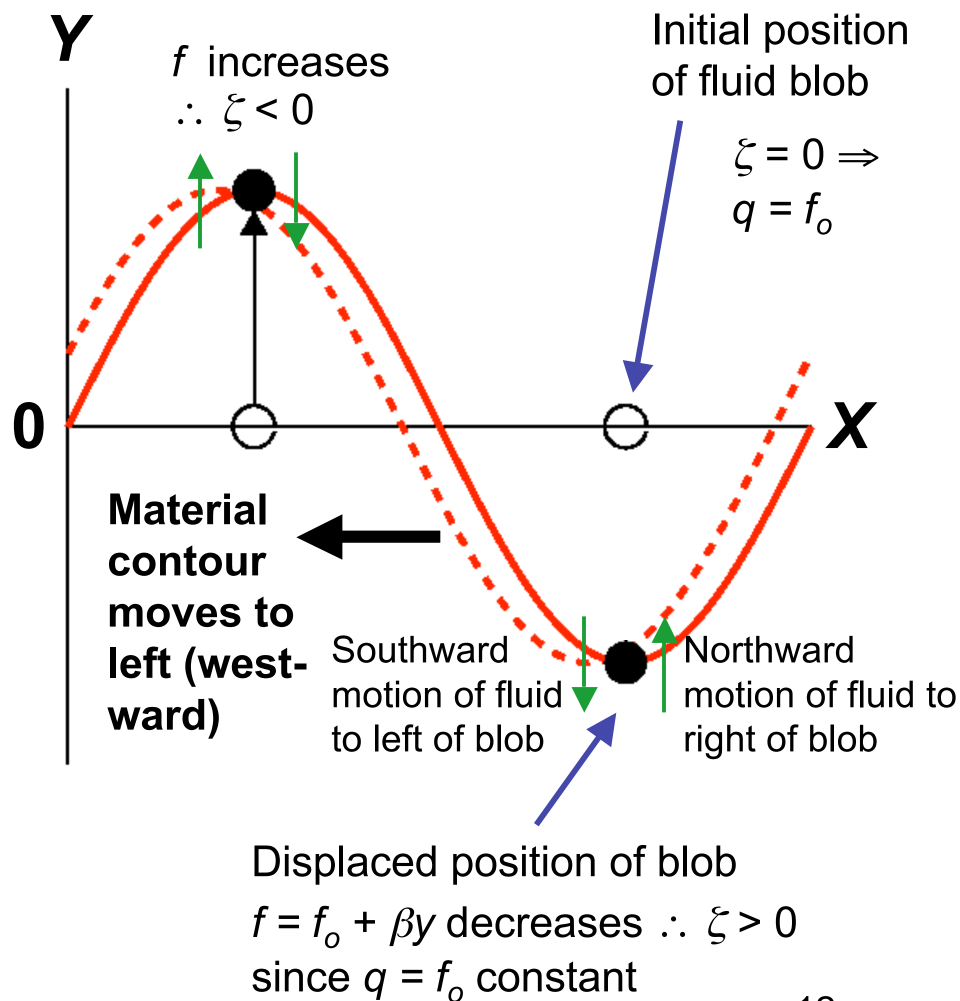
For a small amplitude disturbance on a constant zonal mean flow we get:

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right) q' + \beta \frac{\partial \psi'}{\partial x} = 0$$

$$\psi'(x, y, t) = \mathrm{Re}\left\{\tilde{\psi}\, e^{i(kx+ly-kct)}\right\}$$

$$\Rightarrow \quad c = \bar{u} - \frac{\beta}{k^2 + l^2}$$

⇒ Rossby wave propagates westward with respect to the mean flow.



2. Linear quasi-geostrophic PW model on the β-plane

- governing equations are:
  - equations of motion for the horizontal winds
  - hydrostatic equation
  - thermodynamic equation
  - mass continuity equation
- combined into a single equation called the ***quasi-geostrophic potential vorticity equation***.
- Cartesian geometry simplifies the problem.
- β-plane approximation is employed to retain the latitudinal gradient of the planetary vorticity.

Quasi-geostrophic-PV equation

$$\frac{Dq}{Dt} = 0$$ where

linearize

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right) q' + \frac{\partial \bar{q}}{\partial y} v' = 0$$

$$q = \zeta + f_o + \beta y + \frac{1}{\rho_o}\frac{\partial}{\partial z}\left(\rho_o \frac{f_o^2}{N^2}\frac{\partial \psi}{\partial z}\right)$$

$$\frac{\partial \bar{q}}{\partial y} = \beta - \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{1}{\rho_o}\frac{\partial}{\partial z}\left(\rho_o \frac{f_o^2}{N^2}\frac{\partial \bar{u}}{\partial z}\right)$$

Simple analytical solution for constant background zonal wind*:*

$$\psi'(x,y,z,t) = \mathbf{Re}\left\{\tilde{\psi}(z)\, e^{i(kx+ly-kct)}\right\} e^{z/(2H)}$$

$$\frac{d^2\tilde{\psi}}{dz^2} + m^2\tilde{\psi} = 0$$

$$m^2 = \frac{N^2}{f_o^2}\left[\frac{\beta}{\bar{u}-c} - (k^2+l^2)\right] - \frac{1}{4H^2}$$

- vertical propagation means $m^2 > 0$
- for a stationary wave (c = 0) we get:

$$0 < \bar{u} < U_c$$

$$U_c \equiv \beta\left[(k^2+l^2) + f_o^2/(2HN)^2\right]^{-1}$$

Charney-Drazin criterion

Requirements for vertical propagation:

1. **eastward background winds that are not too strong,**
2. **long horizontal wavelengths.**

3. Linear quasi-geostrophic PW model on the sphere

- quasi-geostrophic potential vorticity equation on the sphere
- stationary waves
- examines impact of latitudinal and vertical shear of background wind.
- PW structure computed numerically.
- Matsuno (JAS, 1970)

Matsuno's equations

$$\left(\frac{\partial}{\partial t} + \frac{\bar{u}}{a\cos\phi}\frac{\partial}{\partial\lambda}\right) q' + \frac{1}{a}\frac{\partial\bar{q}}{\partial\phi} v' = -\alpha\, q'$$

linear qg PV equation

$$q' = \zeta' + \frac{1}{\rho_o}\frac{\partial}{\partial z}\left(\rho_o \frac{f}{N^2}\frac{\partial\Phi'}{\partial z}\right)$$

qg PV of waves

$$v' = \frac{1}{f\, a\cos\phi}\frac{\partial\Phi'}{\partial\lambda}$$

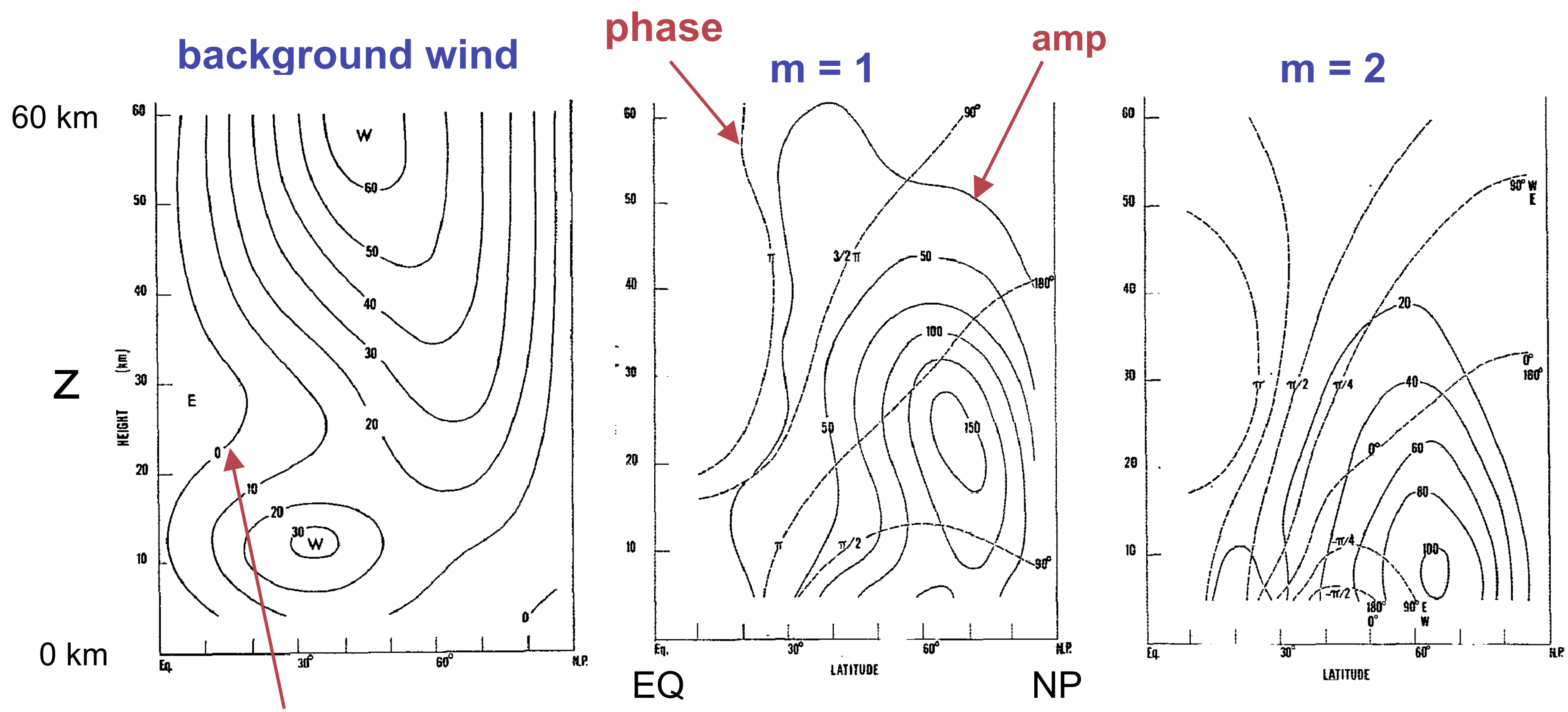
$$f = 2\Omega\sin\phi$$

Coriolis parameter now has full latitudinal dependence

as before write: $\Phi'(\lambda, \phi, z) = \mathbf{Re}\left\{\tilde{\Phi}_m(\phi, z)\, e^{im\lambda)}\right\}$

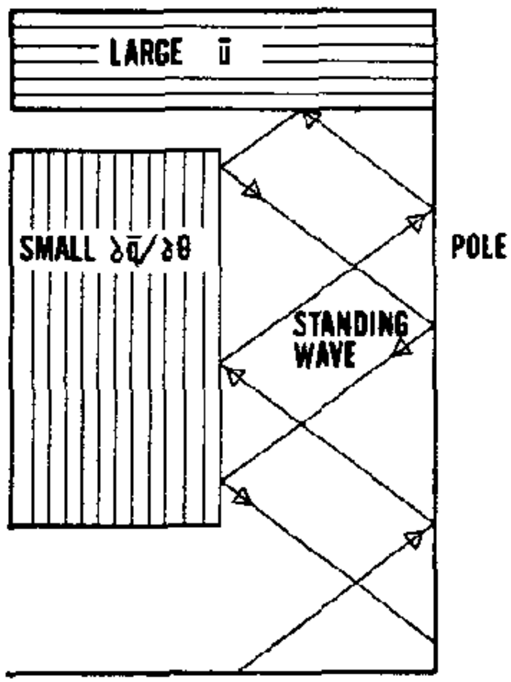
- background zonal wind must be specified.
- PDE for complex-valued wave amplitude is then solved numerically.

Results from Matsuno's model



zero-wind line is a critical line for stationary PW $\Rightarrow$ in linear case with dissipation PW is absorbed here.

Wavenumbers 1 and 2 are forced by specifying their amp & phase at 5 km.



Summary of Matsuno's results:

- wave 1 structure in good agreement with observations.
- region of weak latitudinal gradients of background PV inhibit wave propagation and confines PW to polar region.
- internal reflections result in amplitude maximum in middle stratosphere.
- wave 2 amplitude too weak.

4. Quasi-linear PW models

- planetary Rossby waves break in the stratosphere and interact strongly with the zonal mean flow.
- here we discuss several models which allow for this interaction but use only a single zonal wavenumber.
- referred to as ***quasi-linear*** because the wave can interact with the zonal mean flow but not with itself.
- validity of quasi-linear models was demonstrated by Haynes & McIntyre (1987) in context of barotropic model.
- we will use these models to try to explain the mid-winter minimum in PW 1 amplitude in the SH.

Plumb (1989) Model

- quasi-linear quasi-geostrophic β-plane model
- zonal mean wind is relaxed toward a prescribed ``radiative equilibrium" value u_r at a rate α.

$$\left(\frac{\partial}{\partial t} + \bar{u}\,\frac{\partial}{\partial x}\right) q' + \frac{\partial \bar{q}}{\partial y}\, v' = Z'$$

PW equation

$$\frac{\partial}{\partial t}\left(\frac{\partial \bar{q}}{\partial y}\right) = -\frac{\partial^2}{\partial y^2}\left(\overline{v'q'}\right) + \frac{1}{\rho_o}\frac{\partial}{\partial z}\left[\alpha \rho_o \frac{f_o^2}{N^2}\frac{\partial}{\partial z}\left(\bar{u} - \bar{u}_r\right)\right]$$

Zonal mean equation

$$\bar{u}(y,z,t) = U(z,t)\sin\left(\frac{\pi y}{L}\right)$$

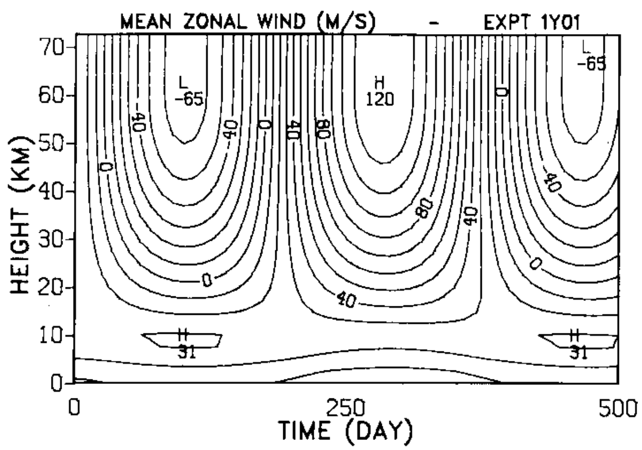
$$\psi'(x,y,z,t) = \mathrm{Re}\left\{\tilde{\psi}(z,t)\, e^{ikx}\sin\left(\frac{\pi y}{L}\right)\right\}$$

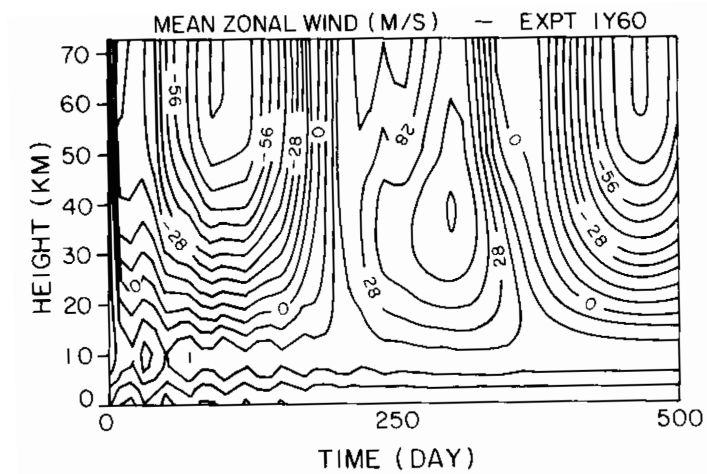
Substitute these forms in for zonal mean and PW and solve numerically

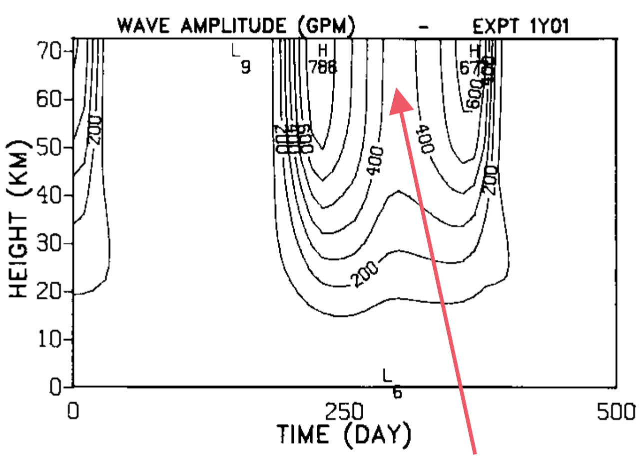
Results from Plumb's model

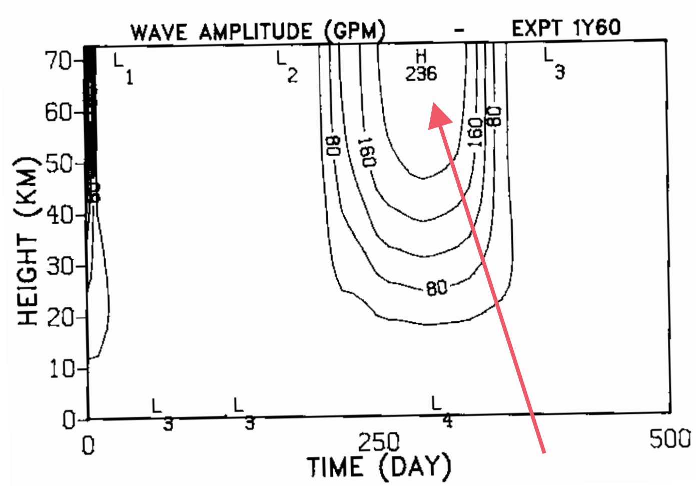
1) weak wave forcing:

2) strong wave forcing:









- wave 1 is forced at lower boundary.
- wave forcing is held fixed in time.

midwinter minimum

midwinter maximum

Summary of Plumb's results

- ***mid-winter minimum*** in wave 1 occurs when forcing at lower boundary is weak (like SH case):
  - response is nearly linear; wave has little impact on zonal mean.
  - zonal mean winds are close to the prescribed radiative equilibrium values which are strongest in mid winter.
  - the strong zonal mean winds in mid winter inhibit wave propagation (Charney-Drazin criterion) $\Rightarrow$ this results in early and late winter wave amplitude maxima.
- ***mid-winter maximum*** in wave 1 occurs when forcing at lower boundary is strong (like NH case):
  - wave interacts strongly with zonal mean flow and prevents westerlies from getting too strong.
  - westerlies that are not too strong permit wave propagation $\Rightarrow$ this results in a single wave amplitude maximum.

Some weaknesses of Plumb’s model

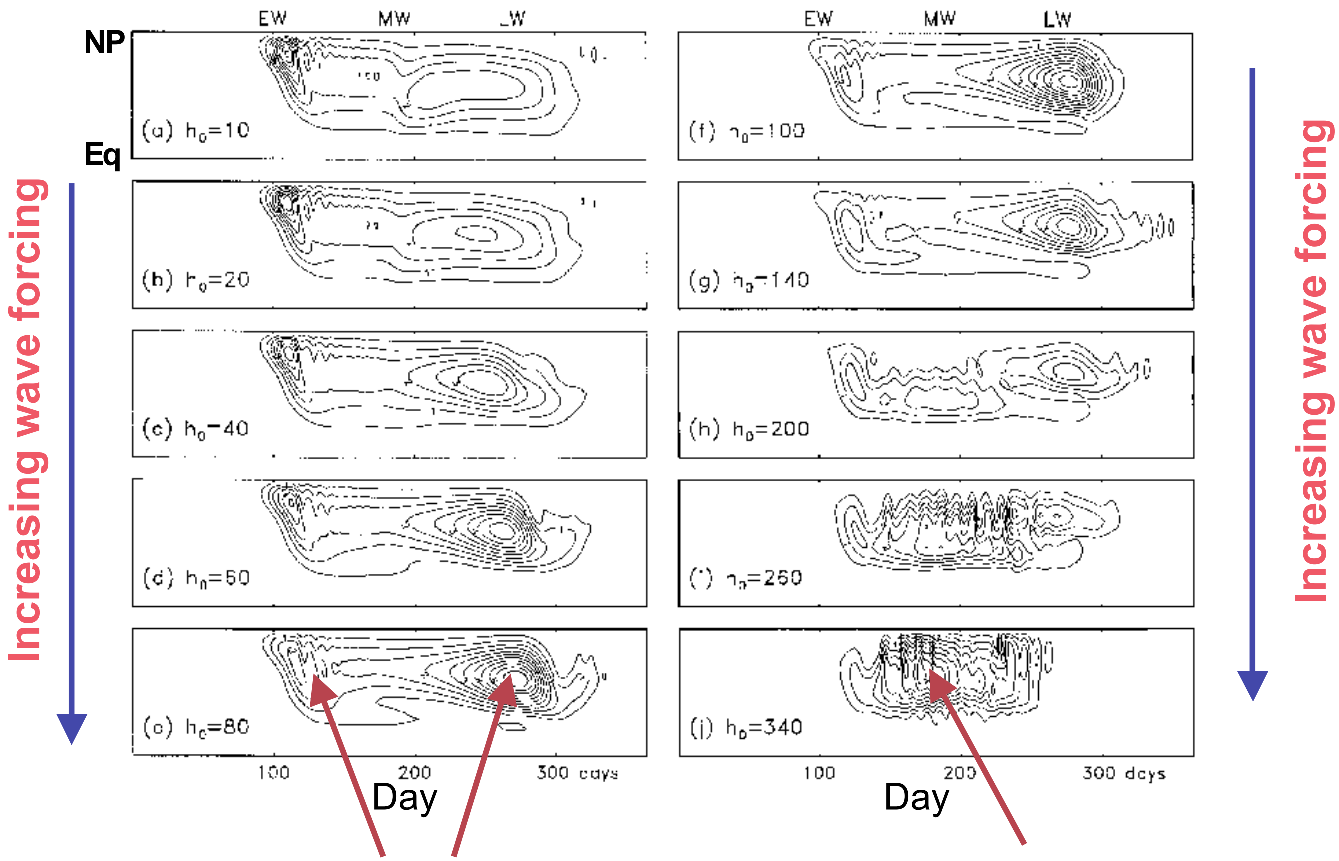
- Latitudinal propagation of waves not considered.
- Impact of latitudinal shear in zonal mean wind (i.e., latitudinal gradients of zonal mean PV) not considered.

Mechanistic primitive equation PW model

- Scott and Haynes (JAS, 2002)
- primitive equations on the sphere
- stratosphere-only model
- single stationary PW is forced at the lower boundary which is at 100 hPa.
- seasonal cycle is included by thermal relaxation to a seasonally varying temperature field.
- importance of latitudinal propagation can be examined which was not possible with the Plumb model.

Results from Scott & Haynes model

Normalized geopotential wave 1 amplitude at 33 km:

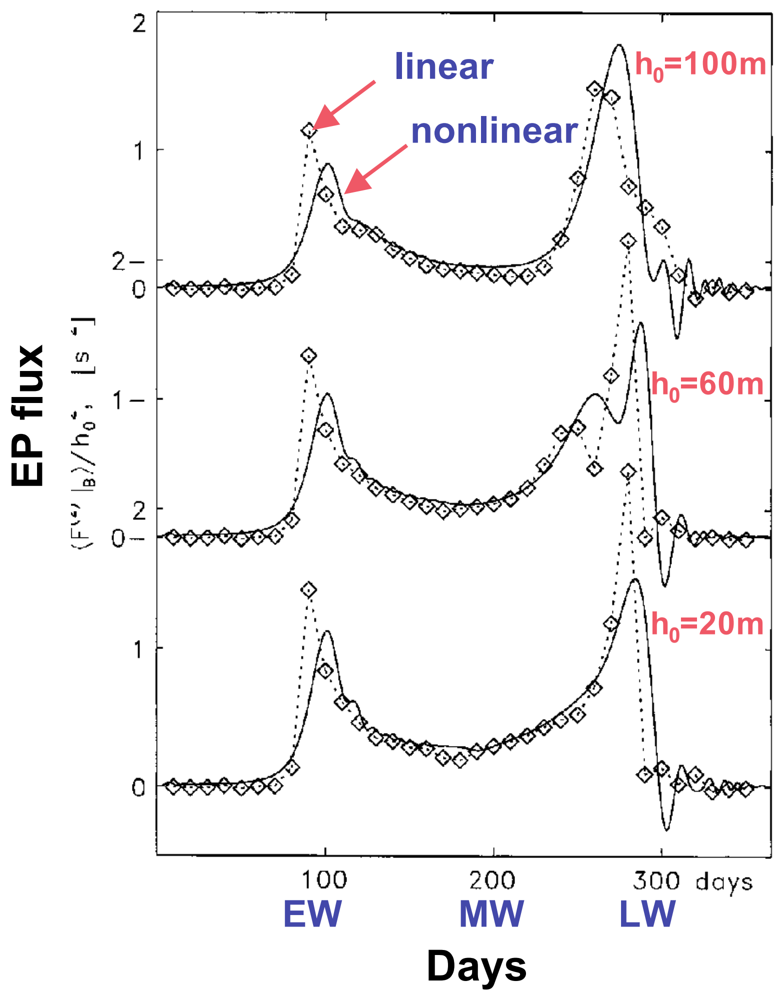


Early and late winter maxima for weak to moderate forcing

Single mid-winter maximum for strong forcing

Results from Scott & Haynes model

Normalized latitudinal average of F_z at lower boundary:

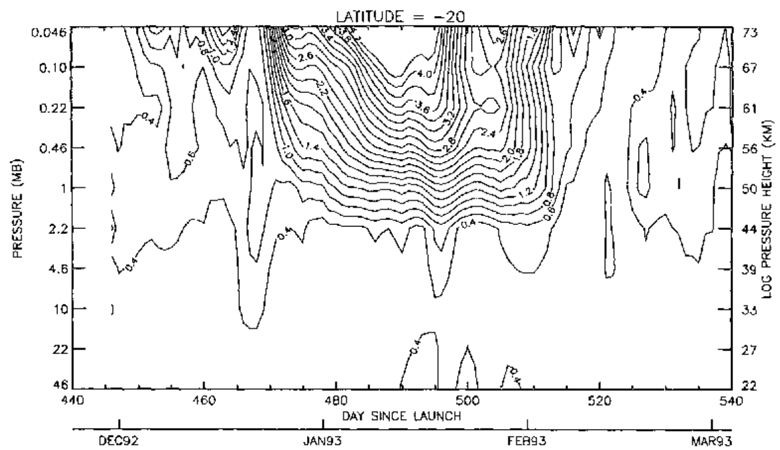


- linear steady-state results are computed using zonal mean winds for that day.
- maxima in linear solution correspond to resonances.
- similarity of nonlinear and linear solutions indicate that a ***resonance is what causes the early and late winter amplitude maxima***.
- conditions of wave transmission are most favourable to upward wave propagation in early winter, not mid winter.
- these results demonstrate the importance of latitudinal structure of the mean winds and are to be contrasted Plumb's where strong winds produced the mid-winter maximum.

Part 3: Traveling planetary waves in the mesosphere (the 2-day wave)

Possible interpretations of the two-day wave in the mesosphere

1. neutral normal mode
2. baroclinically unstable wave
3. combination of two



Wu et al (1993)

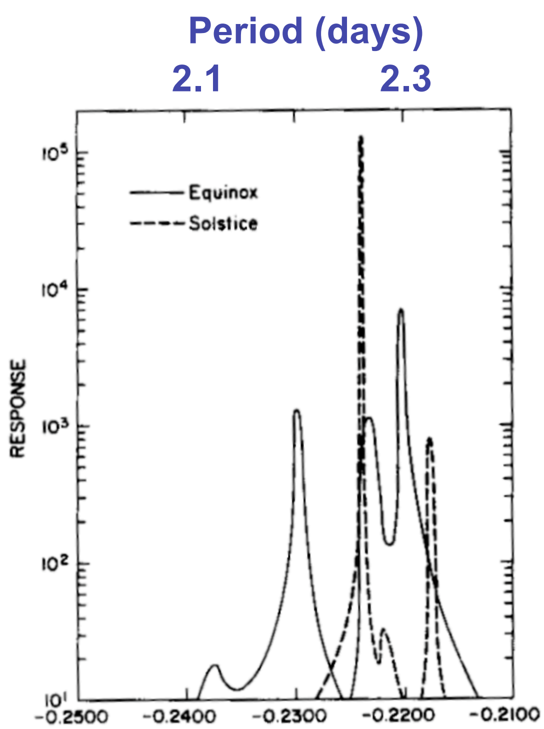
Models described here to study 2-day wave

1. a linear 2D (latitude by height) primitive equations model where wave frequency and zonal wavenumber are specified
2. 3D middle atmosphere GCM

Neutral Normal Modes

- ***normal modes*** of the atmosphere are free (unforced) disturbances.
- ***neutral*** means that the frequency is real-valued (i.e., not exponentially growing or decaying).
- analytical solutions are obtained by solving the linear primitive equations on the sphere for a windless background atmosphere without dissipation.
- for each zonal wavenumber there is a discrete set of normal modes each with a different frequency and meridional structure.
- if a disturbance is forced at this frequency the response is ***resonant*** $\Rightarrow$ it is the normal mode that grows most rapidly in time and dominates the overall response.

Calculation of neutral normal modes in the presence of background winds and dissipation

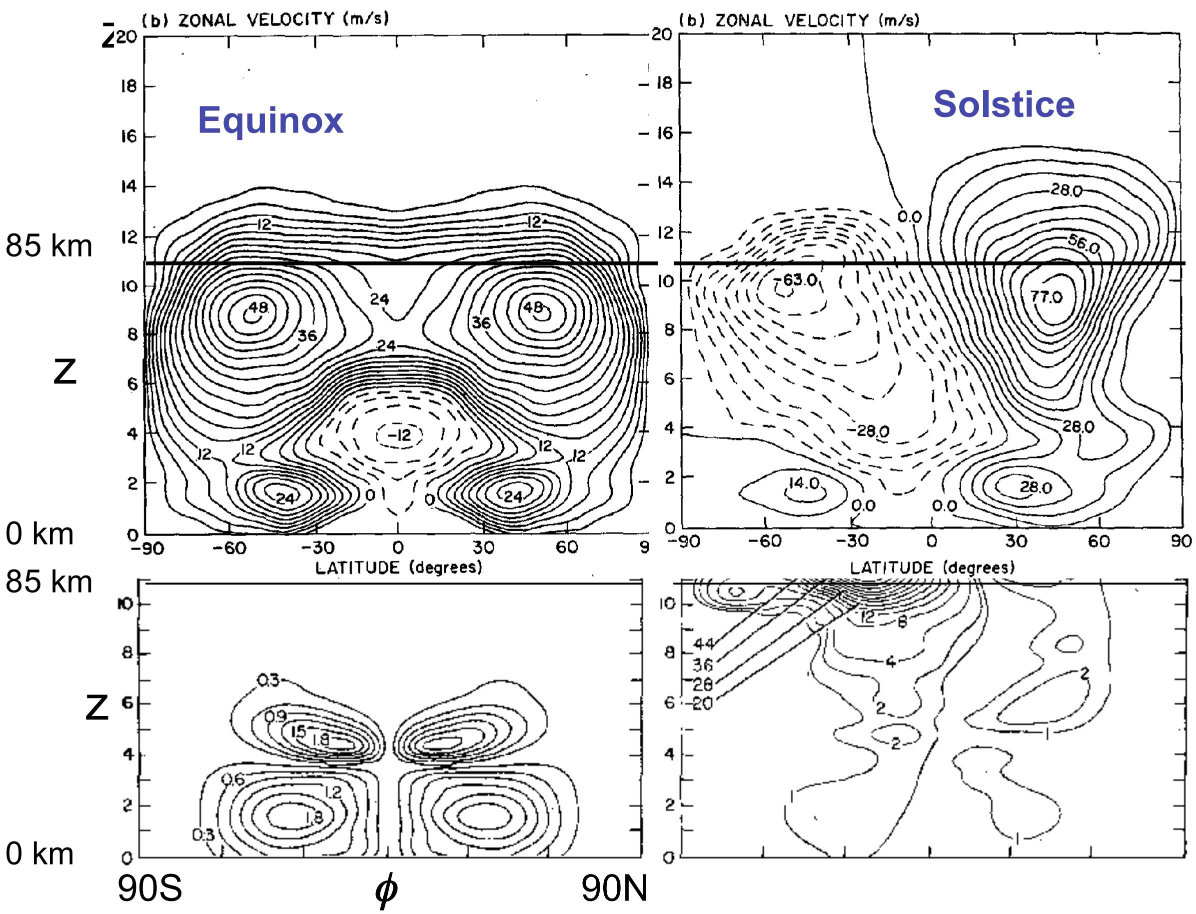


Salby (JGR 1981)

Computed numerically using a 2D primitive equations model:

- specify background zonal wind and temperature, wave dissipation, and boundary conditions
- specify zonal wavenumber (m=3)
- force a wave at lower boundary
- vary frequency of forcing until a resonant response is obtained ⇒ this is the neutral normal mode.

Salby's (1981) Results



Zonal mean wind (specified)

Geopotential amplitude (computed)

amplitude enhancement in summer mesosphere

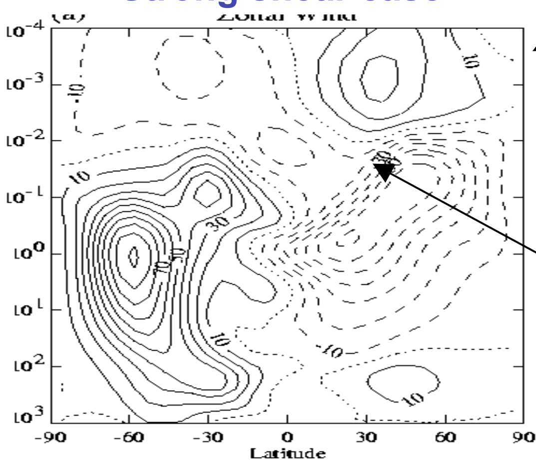
Instability mechanism of the two-day wave

- proposed by Plumb (1983)
- zonal mean easterlies near solstice may be baroclinically unstable.
- simulations using middle atmosphere GCMs reveal that 2-day wave amplification is related to baroclinic and barotropic instability of zonal mean state.

GCM results of Norton and Thuburn (1997)

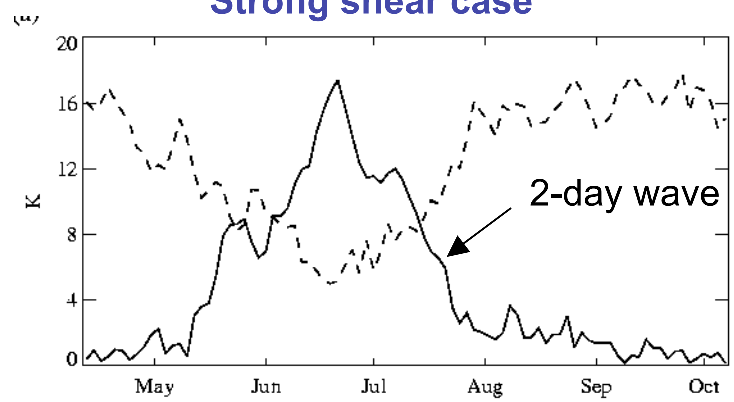
Zonal mean winds

Strong shear case



Strong shear zone

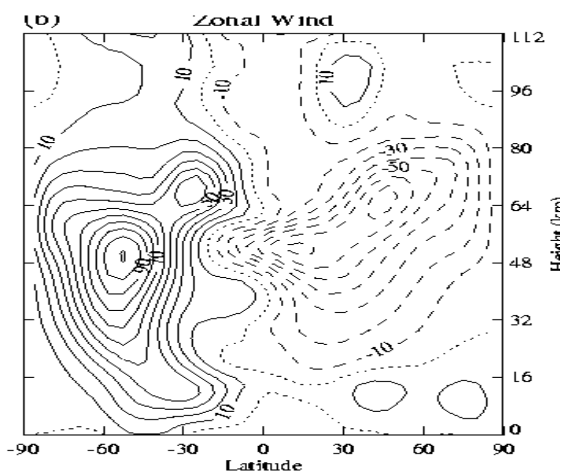
Strong shear case



2-day wave

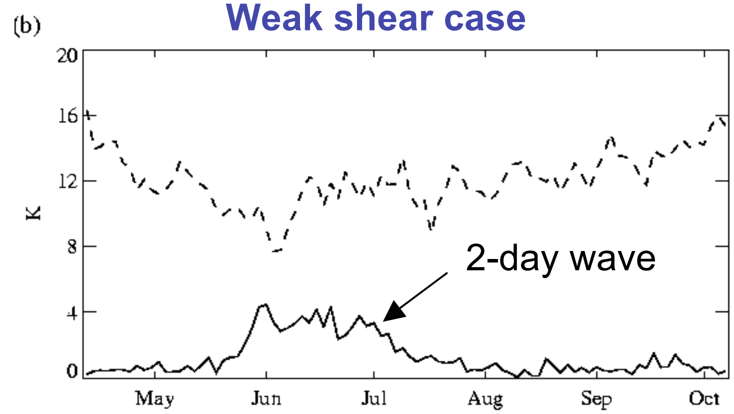
Weak shear case

Z



ϕ

Weak shear case



2-day wave

Month

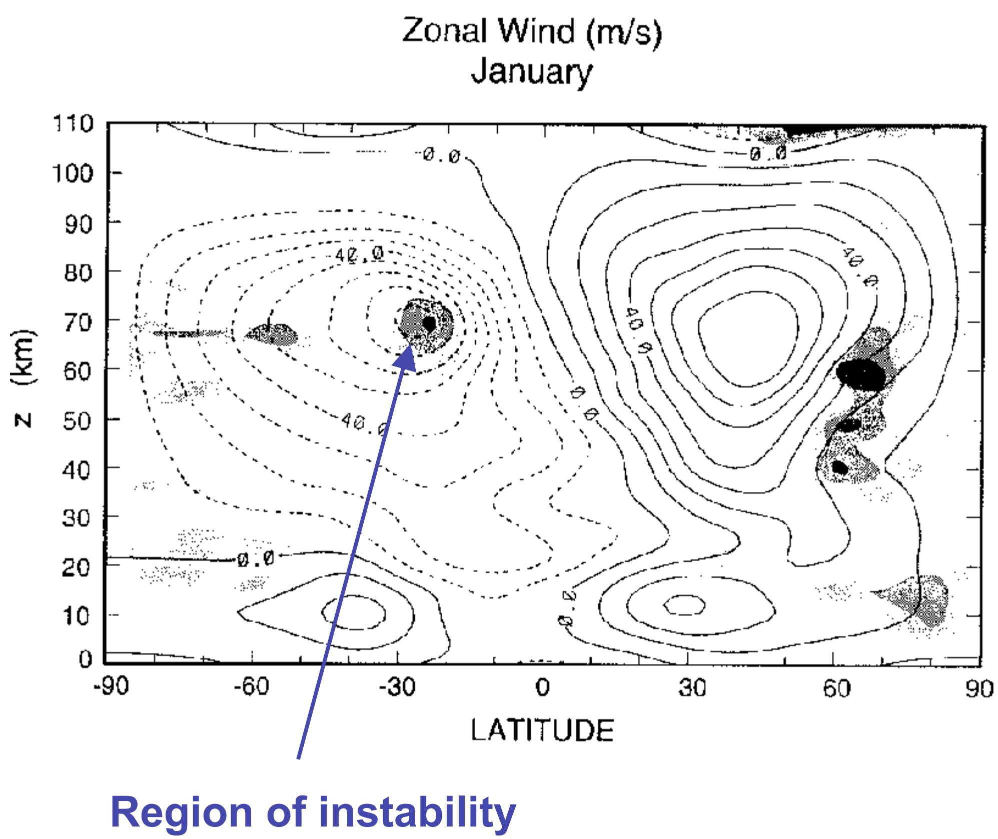
Normal Modes in an unstable mean flow

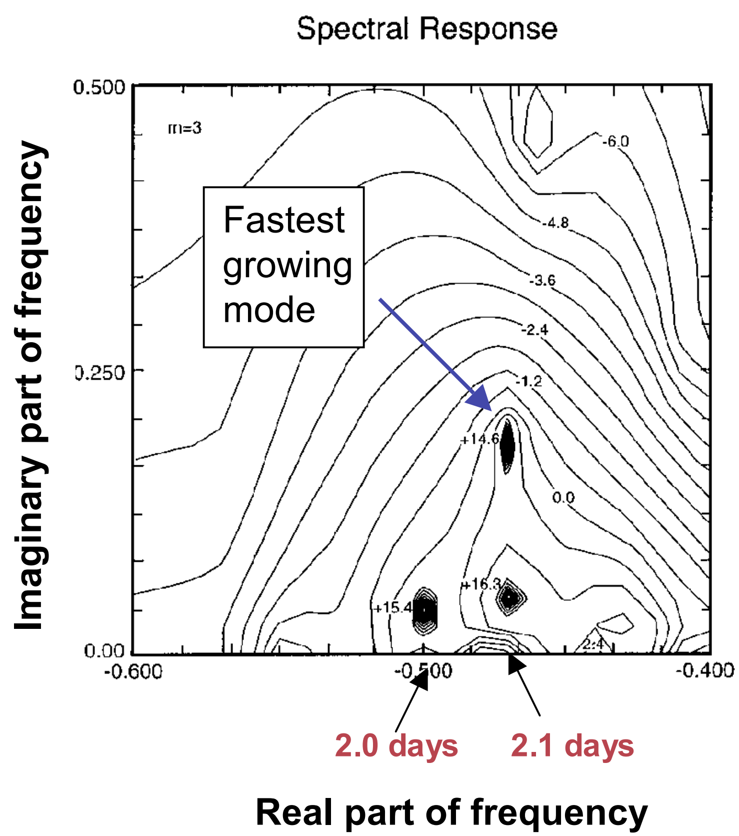
• the relationship between neutral normal modes and instability were examined by Salby and Callaghan (JAS 2002).

• they used Salby's (1981) 2D primitive equations model but considered unstable zonal mean background states and computed normal modes for complex-valued frequencies.

$\Rightarrow$ positive imaginary frequencies indicate exponential wave growth.

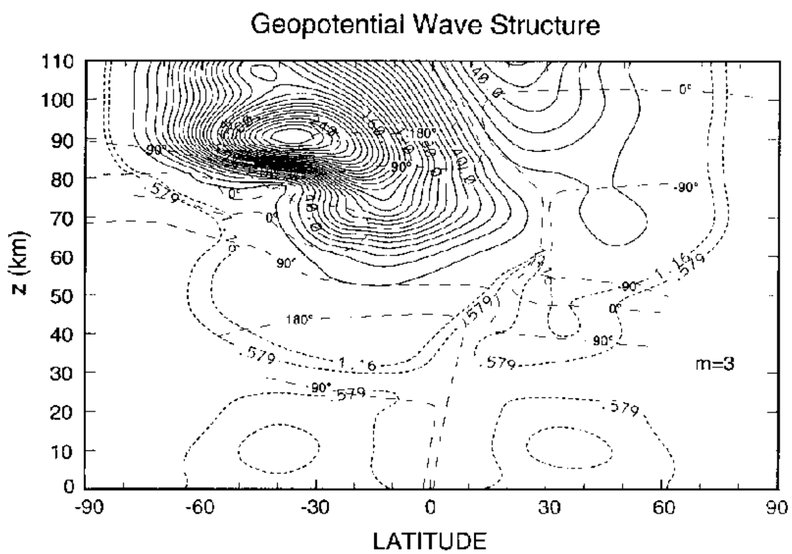
Salby and Callaghan (2002) results



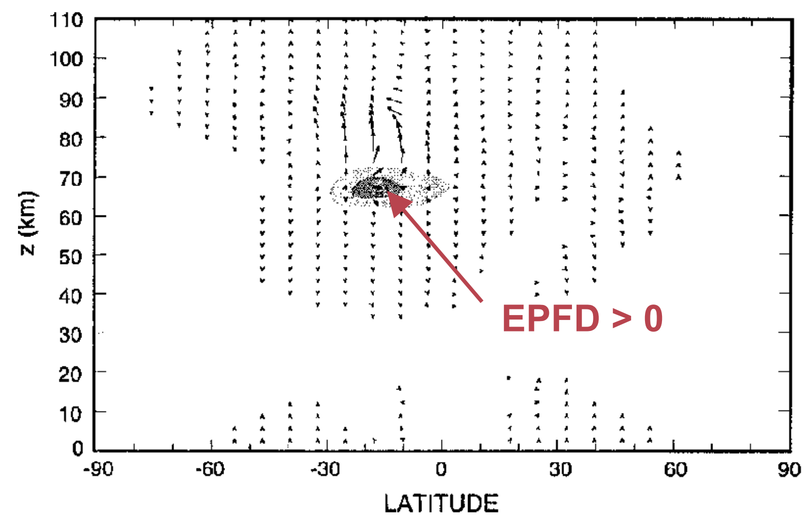


Salby and Callaghan (2002) results

Structure of most unstable mode:



EP flux and divergence of most unstable mode:



• Region of wave instability acts as a source of energy for the normal mode (.e, the 2-day wave grows by extracting energy from the zonal mean flow).

Summary of 2-day wave results

- The two opposing interpretations of the 2-day wave (neutral normal mode vs instability) now appear to be reconciled. Hurrah.
  - instability of the background state generates unstable normal modes that grow in time.
  - real part of the frequency and spatial structure of the most unstable mode is in good agreement with the observed 2-day wave.

The End