

**Gravity Waves**  
**in the Middle Atmosphere**  
**(a modeler's perspective)**

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# Outline of Lecture

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  2. Basic theory (simple)
  3. Parameterization in GCMs
  4. Results from the CMAM
- Part 2 (Observations)
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  2. Sampling issues
  3. What modelers need
- Summary

# Importance of small-scale GWs

## 1. GWs keep the mesosphere far from radiative equilibrium.

- radiative equilibrium:
  - ⇒ temperature highest where heating strongest & vice versa.
  - ⇒ summer pole should be warm, winter pole should be cold.
- observations show the opposite:
  - summer pole is cold
  - winter pole is warm.

Figure 1

- Why?

→ GWs deposit zonal momentum in the mesosphere.

→ this GW force (or drag) is negative in the winter hemisphere and positive in the summer hemisphere.

→ drag must be balanced by Coriolis force ( $f\bar{v}$ )  $\Rightarrow$  flow from summer pole to winter pole.

→ meridional wind ( $\bar{v}$ ) generates a vertical wind ( $\bar{w}$ ) by mass continuity.

→ downwelling ( $\bar{w} < 0$ )  $\Rightarrow$  adiabatic compression  $\Rightarrow$  warming (winter pole).

→ upwelling ( $\bar{w} > 0$ )  $\Rightarrow$  adiabatic expansion  $\Rightarrow$  cooling (summer pole).

→ this is dynamical heating and cooling.

- zonal winds computed from radiative-equilibrium temperature:
  - are much stronger than observed winds
  - do not reverse in mesosphere.

Figure 2

- strong radiative-equilibrium wind is understood from thermal wind equation:

$$\frac{\partial \bar{U}_r}{\partial z} \propto -\frac{1}{f} \frac{\partial \bar{T}_r}{\partial \phi}$$

→ in NH winter  $\partial \bar{T}_r / \partial \phi < 0$  and  $f > 0$

⇒  $U_r$  increases with height.

## 2. GWs warm winter polar stratosphere.

- downwelling ( $\bar{w}^*$ ) produced by breaking GWs in the mesosphere extends into the stratosphere.
    - downwelling  $\Rightarrow$  adiabatic compression  $\Rightarrow$  warming.
    - polar winter stratosphere is cold and dark.
      - $\Rightarrow$  long radiative relaxation time scale ( $1/\alpha$ ).
- $\therefore \bar{w}^* < 0$  and small  $\alpha \Rightarrow$  large departure from radiative equilibrium, since from thermodynamic equation have that

$$(\bar{T} - \bar{T}_{rad}) \approx -\alpha^{-1} N^2 \bar{w}^* .$$

Figure 3

- “downward-control” impact of GWs is stronger in SH winter stratosphere since planetary wave drag is weaker.
- absence of drag in GCMs in SH winter is referred to as the “cold-pole problem.”
  - alleviated using GW drag parameterizations
  - or higher horizontal resolution.

Figure 4

### 3. GWs help drive the equatorial QBO.

Figure 5

- in the presence of tropical upwelling, planetary waves are unable to drive QBO.  
→ small-scale GWs provide the required extra drag (Dunkerton, 1997).

Figure 6

- most GCMs do not naturally produce a QBO from resolved waves  $\Rightarrow$  and must use parameterized GWs to get one.

Figure 7



# Basic theory (simple)

## 1. Hydrostatic GWs without rotation

(see Andrews et al., 1982)

- use hydrostatic approximation  $\Rightarrow$  pressure gradient equals gravitational force.
- neglect the rotation of the Earth.
- assume background atmosphere with constant zonal wind ( $U$ ) and buoyancy frequency ( $N$ ); allow only density ( $\rho_0$ ) to vary with height.
- consider small-amplitude case  
 $\Rightarrow$  governing equations can be linearized.
- assume wavelike perturbations proportional to  $\exp[i(kx + mz - \omega t)]$ , where
  - $k$  is the horizontal wavenumber  
(i.e., only east-west propagation),
  - $m$  is the vertical wavenumber,
  - $\omega$  is the (ground-based) frequency.

- dispersion relation for the resulting set of equations\* is

$$m^2 = \frac{N^2 k^2}{\hat{\omega}^2}.$$

- $\hat{\omega} \equiv \omega - kU$  is the intrinsic frequency.

→ It is the frequency of the wave measured in a reference frame moving with the background wind (or Doppler-shifted frequency).

- vertical propagation:

– since  $m^2$  is positive,  $m$  is real and the wave is vertically propagating.

– for upward energy propagation (i.e., upward group velocity) phase lines tilt:

→ eastward with height for  $\hat{\omega} > 0$ ,

→ westward with height for  $\hat{\omega} < 0$ .

\*It has been assumed that the vertical scale of the wave is much less than the density scale height, i.e., the Boussinesq approximation.

- amplitude growth with height:
  - to conserve energy the amplitude of a vertically propagating wave increases exponentially with height<sup>†</sup>.
- momentum flux:
  - a vertically propagating wave transports horizontal momentum upward.
  - this can be seen by computing the momentum flux ( $\tau$ ).
    - it is easy to show that for our simple example  $\tau = \rho_0 \overline{u'w'}$  is constant.
  - It is important to note that the sign of the momentum flux depends on the direction of horizontal propagation:<sup>‡</sup>

$$\begin{aligned} \Rightarrow \tau &> 0 \text{ for } \hat{\omega} > 0, \\ \tau &< 0 \text{ for } \hat{\omega} < 0. \end{aligned}$$

<sup>†</sup>This follows from the exponential decrease of background density.

<sup>‡</sup>The continuity equation in pressure coordinates indicates that  $u'$  and  $w'$  are approximately in phase for  $\hat{\omega} > 0$  and out of phase for  $\hat{\omega} < 0$ .

## 2. Rotation and nonhydrostatic effects

(see Gill, 1982)

- the previous example was for hydrostatic GWs and did not include the effects of the Earth's rotation.
- when these effects are included, the dispersion relation becomes

$$m^2 = \frac{(N^2 - \hat{\omega}^2)k^2}{\hat{\omega}^2 - f^2}$$

where  $f$  is the Coriolis parameter.

- whether a wave is vertically propagating now depends on the intrinsic frequency, since  $m^2 > 0$  requires

$$|f| < |\hat{\omega}| < N.$$

- for  $m^2 < 0$ , the wave is vertically trapped.
  - amplitude decreases exponentially with height.
  - momentum flux is zero.

- example: mountain waves ( $\omega = 0$ ) propagate vertically into the stratosphere only if  $\lambda > (2\pi U)/N \approx 10$  km for typical values of  $U$  and  $N$ .  
 $\Rightarrow$  small-scale mountain waves, where  $\lambda < (2\pi U)/N$ , are vertically trapped (lee waves).

Figure 8

### 3. Gravity wave drag

- in the above examples (where the waves are steady, linear, and undamped), the momentum flux is independent of height.
- since the flux divergence is zero, such waves do not alter the background flow:

$$\frac{\partial U}{\partial t} + \dots = -\frac{1}{\rho_0} \frac{\partial \tau}{\partial z} = 0.$$

- when the GWs are damped they deposit their momentum, which exerts a force on the background flow.

→ This is called gravity wave drag.

- wave dissipation occurs when:
  - (1)  $\hat{\omega} \rightarrow 0$  (a critical line), where the vertical wavelength gets very small, or
  - (2) amplitude gets large enough that the wave breaks (like a wave on a beach).

# Parameterization of GWs in GCMs

## 1. Resolved vs unresolved GWs

- a GCM cannot resolve horizontal scales smaller than a model grid box.  
→  $\sim 300\text{-}500$  km for most GCMs.
- neglecting unresolved GWs leads to erroneous results as seen in Figure 4.
- simulations using high-resolution GCMs show that the momentum flux spectrum in the mesosphere is very shallow.

Figure 9

⇒ extremely high resolution is required to resolve most of the wave momentum flux → this is not currently possible.

⇒ these GWs must be parameterized.

## 2. GWD parameterizations

- GW quantities are expressed in terms of the background (i.e., resolved) winds and temperatures.
- simplifying assumptions are made to make the problem computationally feasible.
- features common to all parameterizations:
  - GW propagation is governed by a linear dispersion relation and conservation of (pseudo-) momentum.
  - only vertical propagation of energy is allowed  $\Rightarrow$  a GW is assumed to remain in a grid box.
  - background quantities are assumed to vary only in height, and to vary more slowly in height than wave quantities.
  - a GW is removed at a critical level (i.e., critical level filtering).



- a GW source is specified in the troposphere for a discrete or continuous spectrum of waves:
  - momentum flux as a function of horizontal wavenumber and frequency.
- features which differentiate all parameterizations:
  - dissipation mechanism when wave amplitudes are large (saturation criterion).
  - specific details of source spectrum (i.e., momentum flux distribution).

### 3. Summary of GWD parameterizations

- Lindzen (1981):
  - saturation criterion: when the amplitude of a GW is large enough that the total (i.e., background plus GW) temperature lapse rate is convectively unstable.
    - above the ‘breaking height’, turbulence damps the GW so as to maintain neutral stability  $\Rightarrow$  momentum is deposited over a range of heights.
- Hines (1997):
  - saturation criterion: when a GW meets a critical level induced by the combination of the background wind and the wind induced by the entire spectrum of GWs.
    - at this height, the GW is obliterated  $\Rightarrow$  all the wave’s momentum is deposited at once.

- Warner-McIntyre (1996):
  - saturation criterion: when a GW exceeds a prescribed amplitude threshold that is based empirically on the observed GW spectrum in the middle atmosphere.
    - above this height, the wave amplitude is constrained to remain at the prescribed saturated value.
- Alexander-Dunkerton (1999):
  - saturation criterion: when a GW exceeds convective instability threshold.
    - at this height, the GW is obliterated, and all of its momentum is deposited.

# Results from the CMAM

## 1. Description of the CMAM:

- a middle atmosphere GCM, which extends from the ground to  $\sim 100$  km.
- a whole slew of physical processes (radiation, chemistry, clouds, hydrologic cycle, etc.).
- details in Beagley et al. (1997).
- T32 horizontal resolution  $\Rightarrow \sim 600$  km.
- 65 vertical levels  $\Rightarrow \Delta z \sim 1.5$  km above troposphere.

## 2. Orographic GWD experiments:

- momentum deposition by unresolved GWs generated by flow over topography → parameterization of McFarlane (1987).
- spatial pattern of the GW source is well known → it's the topography.
- separate Earth's topography into resolved ( $\lambda_x > 600$  km) and unresolved ( $\lambda_x < 600$  km) parts ⇒ for a T32 model.  
→ use standard deviation of unresolved topography to specify the amplitude of the GW.
- single GW, with zero frequency and propagation direction opposite to horizontal wind vector at each GCM grid point.
- saturation criterion of Lindzen (1981).

Figure 10

Figure 11

- summary of orographic GWD experiments:
  - no impact in the summer stratosphere or mesosphere. Why?
    - because of critical-level filtering of the stationary GW by the stratospheric easterlies (i.e., where  $U = 0$ ).
  - greater impact in NH winter than in SH winter.
    - because there are more mountains in the NH.
  - no reversal of the zonal mean zonal wind in the mesosphere.
    - because a stationary GW can only decelerate winds to zero, not reverse them.

### 3. Non-orographic GWD experiments:

- non-orographic GWs are generated by mechanisms other than topography:
  - convection, wind shear, etc.
- sources are difficult to characterize:
  - highly variable in space and time.
  - generation mechanisms poorly understood.
- because of these uncertainties, modellers assume the simplest form:
  - a source that is uniform in space and time.
- we will examine 2 non-orographic GWD parameterizations described earlier:
  - (1) Hines (1997)
  - (2) Warner-McIntyre (1996).

- the GW source for both is identical:
  - “isotropic” in intrinsic frequency  
(momentum flux is distributed symmetrically about  $\hat{\omega} = 0$ ).
- aspects we will examine:

(1) saturation mechanism

Figure 12

(2) GW launch level

Figure 13

Figure 14



- summary of non-oro GWD experiments:
  - details of saturation mechanism not important to the time mean flow. Why?
    - because by rescaling the saturation criteria, different parameterizations can produce similar results.
    - the simulation using only critical-level filtering makes this point most clearly.
  - this means that the heated debate over “whose parameterization is right?” is irrelevant to modellers.
  - what is important is:
    - (1) the height at which the GWs deposit their momentum,
    - (2) the momentum flux distribution at the source height.

#### 4. Resolved GWs in GCMs:

- GCMs generate a broad spectrum of resolved GWs as a result of tropospheric processes like convection, flow over (resolved) topography.
- these disturbances can propagate vertically, grow in amplitude, dissipate, and deposit their momentum.
- we have seen evidence of resolved waves in the CMAM simulation with only critical-level filtering (Figure 12).  
→ Are they GWs?
- to answer this a more quantitative analysis is required:

Figure 15

Figure 16

Figure 17

⇒ Koshyk et al. (1999) paper demonstrates that GCMs with upper boundaries above  $\sim 80$  km have enhanced KE spectra in mesosphere:

→ divergent (i.e., gravity wave) component of KE as large as rotational.

→ power at wide range of spatial scales.

→ CMAM is the most energetic model.

- Which of the GCMs is most realistic?

→ We don't know → no global observations of these horizontal scales.

- resolved GWs in the tropics:

- deep convection in the tropics causes condensation of water  $\Rightarrow$  release of latent heat.
  - $\rightarrow$  heating generates waves with a wide range of spatial and temporal scales.
- more waves are able to propagate vertically since the Coriolis force is small.
  - $\rightarrow$  see dispersion relation.
- waves generated by convective heating propagate upward and interact with the zonal mean flow.
  - $\rightarrow$  dissipating waves drive zonal wind oscillations like the QBO in the lower stratosphere and the SAO in the stratosphere and mesosphere.

- deep convection cannot be resolved in GCMs  $\Rightarrow$  must be parameterized.
- there are various convective parameterizations used in models: moist convective adjustment, mass-flux schemes, etc. (details unimportant here).
- the ability of GCMs to simulate the QBO and SAO depend crucially on these resolved equatorial waves and consequently on the convection parameterizations.
- a careful GCM intercomparison of convective parameterizations and corresponding wave spectra was performed by Horinouchi et al. (2003).

Figure 18

- their results explain why a GCM like SKYHI can generate a QBO-like oscillation without parameterized GWs, while the CMAM cannot:
  - SHYHI uses a convection scheme which is very “active” and generates a broad spectrum of frequencies.
  - CMAM uses a parameterization that does not.
  
- unclear which is more realistic:
  - lack of global measurements.
  - uncertainty in interpreting satellite-derived proxies of tropical latent heating (Horinouchi, 2002).

# **Part 2 (Observations)**

- 1. Measurement techniques**
- 2. Sampling issues**
- 3. What modelers need**

# 1. Measurement techniques

- overview of several techniques:

- airglow imagers:

- measure emission from photo-chemically excited species in the upper mesosphere and lower thermosphere.

- sensitive only to GWs with long  $\lambda_z$  and relatively short  $\lambda_x \Rightarrow$  nonhydrostatic GWs → often vertically trapped or ducted waves.

Figure 19

- lidars:

- measure backscattered laser radiation  $\Rightarrow$  air density ( $\sim$  30-60 km).

- ideal gas law and hydrostatic equilibrium  $\Rightarrow$  temperature profile.

- high vertical resolution.

Figure 20



– radars:

→ winds in middle atmosphere.

→ high temporal frequency.

→ comparison of MF radar to CMAM:

Figure 21

→ CMAM has a much steeper KE spectrum than does radar.

→ due to numerical damping of high frequency waves in CMAM.

– pressurized balloons:

→ drift along in prevailing wind.

→ measure horizontal and vertical wind components, temperature, etc.

⇒ able to measure momentum fluxes and intrinsic frequency → very important.

Figure 22

– satellites:

→ optically thick limb radiances from Microwave Limb Sounder on UARS ⇒ proxy for temperature along satellite track.

→ high horizontal resolution.

→ technique sensitive to long  $\lambda_z$ .

→ also sensitive to direction of GW propagation (orientation of wave phase lines with instrument line of sight).

→ global information about convective sources in tropics:

Figure 23

## 2. Sampling issues

- each measurement technique is sensitive to a portion of the GW spectrum.
  - ⇒ no one technique can measure the entire spectrum!
  - in order to compare observations and model results, must first apply an “observational filter” to the simulated spectrum. → this mimics the instrument sensitivity (Alexander, 1998).
- Alexander (1998) also explains that GW intermittency complicates the interpretation of the measurements:
  - intermittent sources will generate wave packets.
  - packets with long  $\lambda_z$  will propagate rapidly upward through the atmosphere.
  - ⇒ these waves will be difficult to see.
  - packets with short  $\lambda_z$  propagate slowly upward and so are much more likely to be observed.

### 3. **What modelers need**

- observational constraints on the source spectra for GWD parameterizations.
- observational constraints on the resolved GWs in the mesosphere.