Lecture #3:
Gravity Waves in GCMs

Charles McLandress (Banff Summer School 7-13 May 2005)

Outline of Lecture

1. Role of GWs in the middle atmosphere
2. Background theory
3. Resolved GWs in GCMs
4. Parameterized GWs in GCMs

Part 1: The Role of Gravity Waves in the Middle Atmosphere

- keep mesosphere far from radiative equilibrium (e.g., cold summer mesopause and reversal of the mesospheric jets).
- alleviate cold bias in Southern Hemisphere winter polar stratosphere.
- help drive quasi-biennial oscillation (QBO) in tropical lower stratosphere.

Gravity waves seen in noctilucent clouds in mesosphere

Cold Summer Mesopause

Cooling is due to adiabatic ascent in the summer pole that results from an eastward force exerted by **breaking gravity waves**:

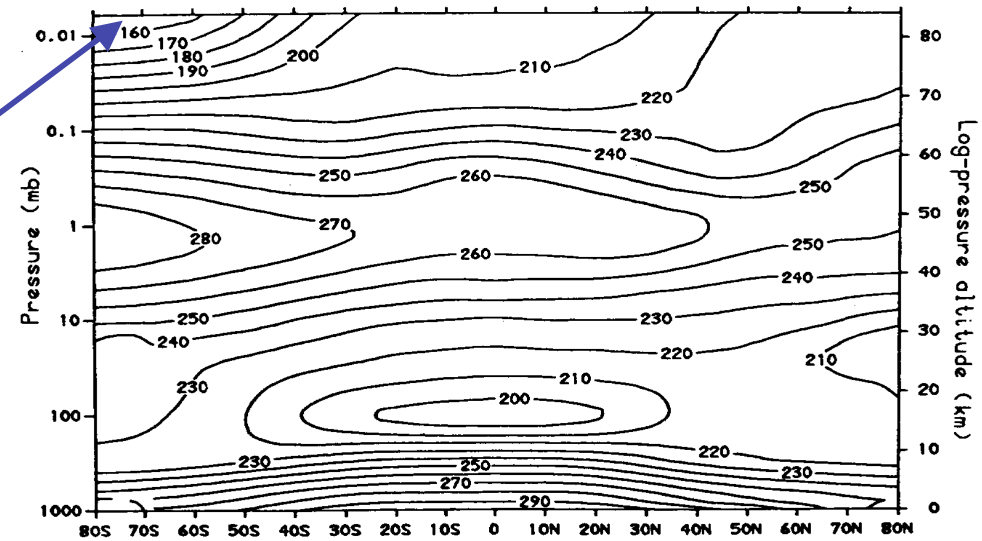
- the eastward force generates an eastward wind that is deflected toward the winter pole by the Coriolis force.

- conservation of mass then implies ascent at the summer pole.

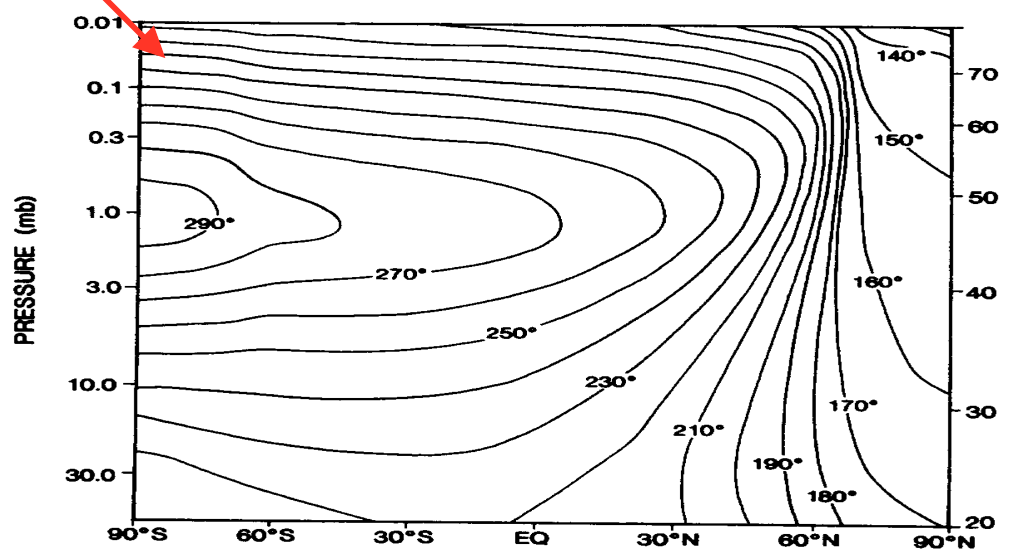
150K

220K

Observed Temperature (January)



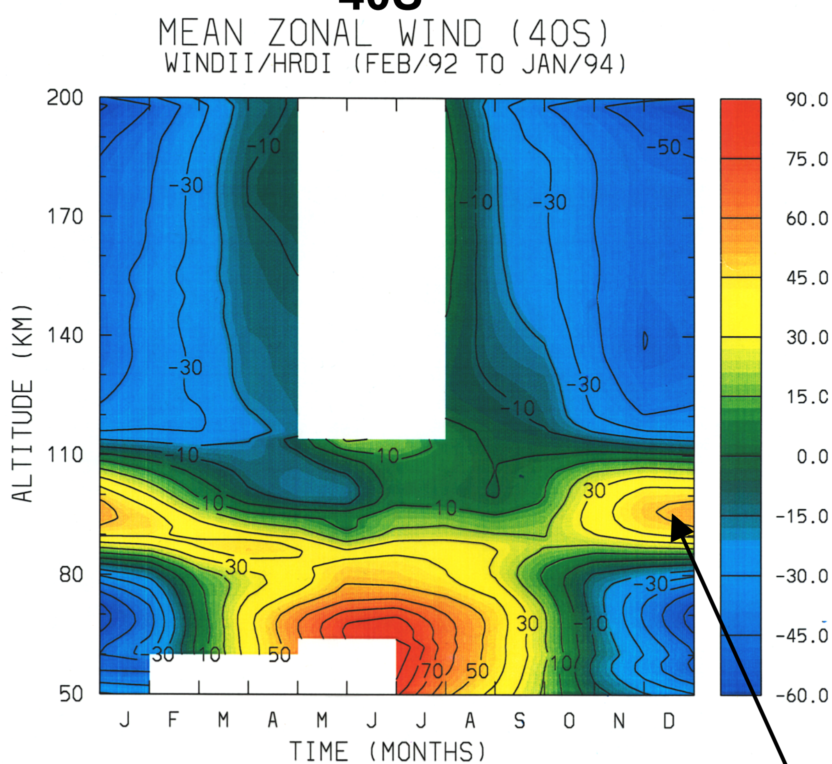
“Radiative” Temperature (January)



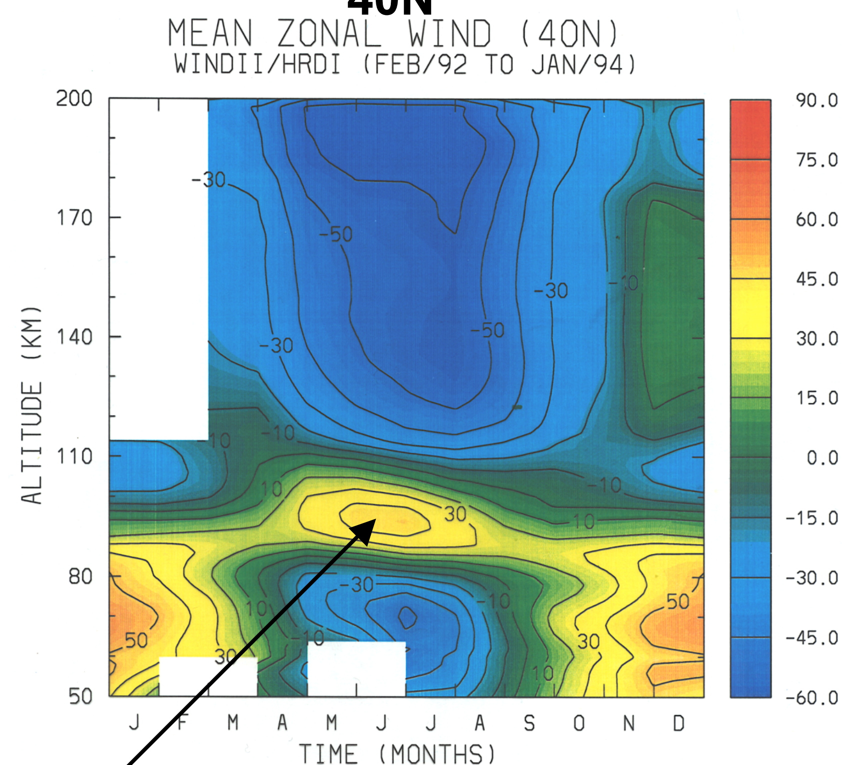
from Fels (1987)

Observed zonal mean zonal winds in mesosphere

40S



40N



Reversal of summer mesospheric winds

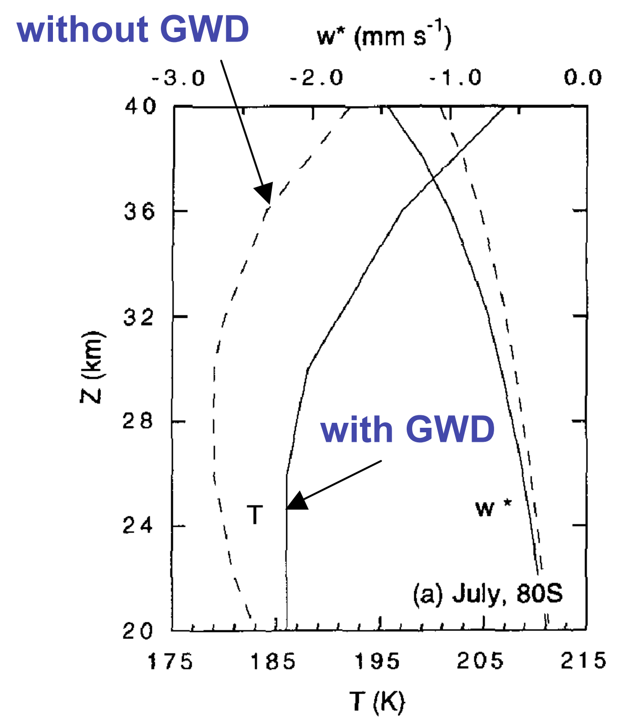
Alleviation of cold bias is SH winter polar stratosphere

• downwelling produced by mesospheric gravity wave drag extends into the stratosphere as a consequence of ‘downward control’ and the long radiative timescales in the cold winter pole.

• this downwelling produces adiabatic warming as a result of compression of air parcels.

• this effect is more important in the SH where the effects of other waves (planetary Rossby waves and topographically generated gravity waves) are weaker.

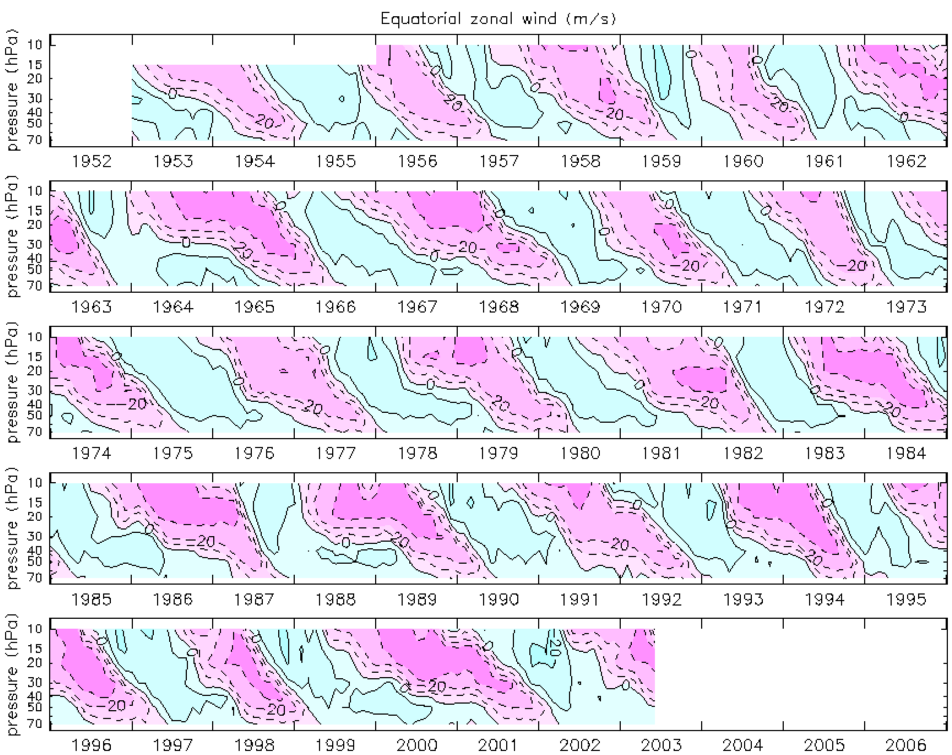
• GCMs typically have a cold bias in the SH winter polar stratosphere, which is also referred to as the ‘cold pole problem’.



(from Garcia & Boville, 1994)

Quasi-biennial oscillation

• an oscillation of the zonal mean zonal winds in the equatorial lower stratosphere with a period varying from 22 to 34 months.



(Radiosonde observations from 1953-2003)

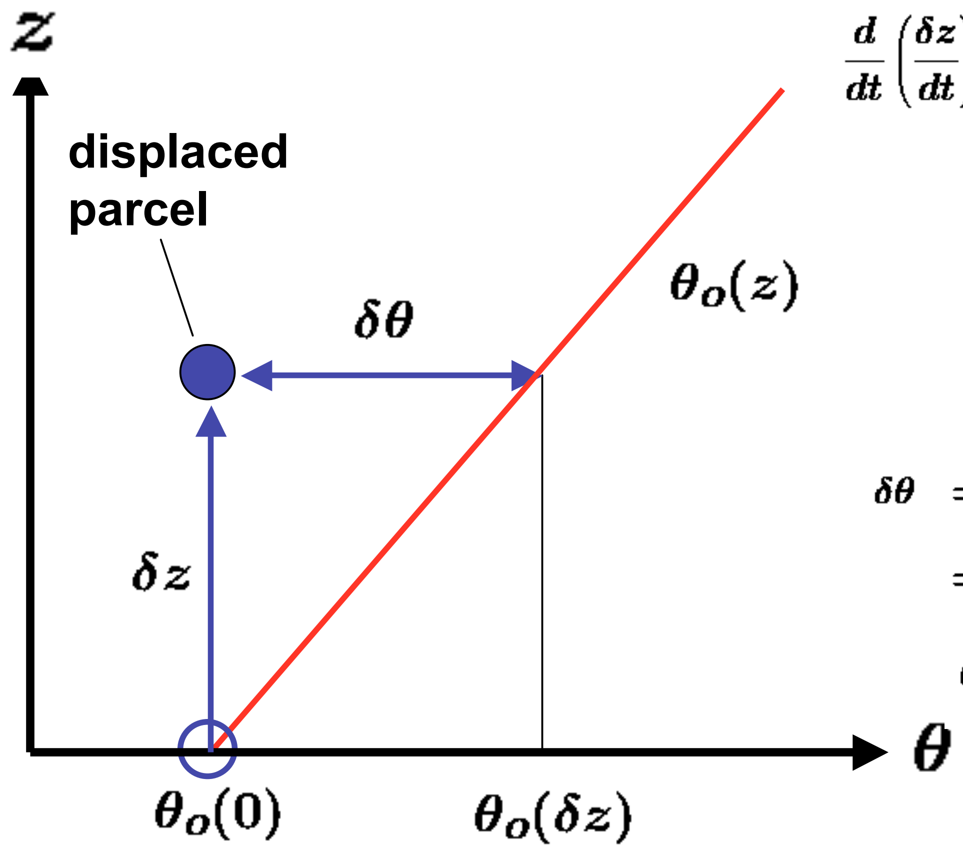
• the QBO is a wave-driven phenomenon.

• the original theory proposed by Holton and Lindzen (1972) assumes that equatorial planetary waves provide the wave driving.

• Dunkerton (1997) showed that in the presence of tropical upwelling planetary waves were not enough; he proposed that gravity waves provided the additional wave driving.

Part 2: Background Theory

Buoyancy oscillations



$$\frac{d}{dt}\left(\frac{\delta z}{dt}\right) = -g - \frac{1}{\rho}\frac{\partial p}{\partial z}$$ (Newton's 2nd Law)

$$= -g - \frac{1}{\rho}\frac{dp_o}{dz}$$ (hydrostatic basic state)

$$= g\left(\frac{\rho_o - \rho}{\rho}\right)$$

$$= g\left(\frac{\theta - \theta_o}{\theta_o}\right)$$ (Ideal gas law and definition of potential temperature)

$$\delta\theta = \theta(\delta z) - \theta_o(\delta z) = \theta_o(0) - \theta_o(\delta z)$$ (potential temperature of parcel is conserved)

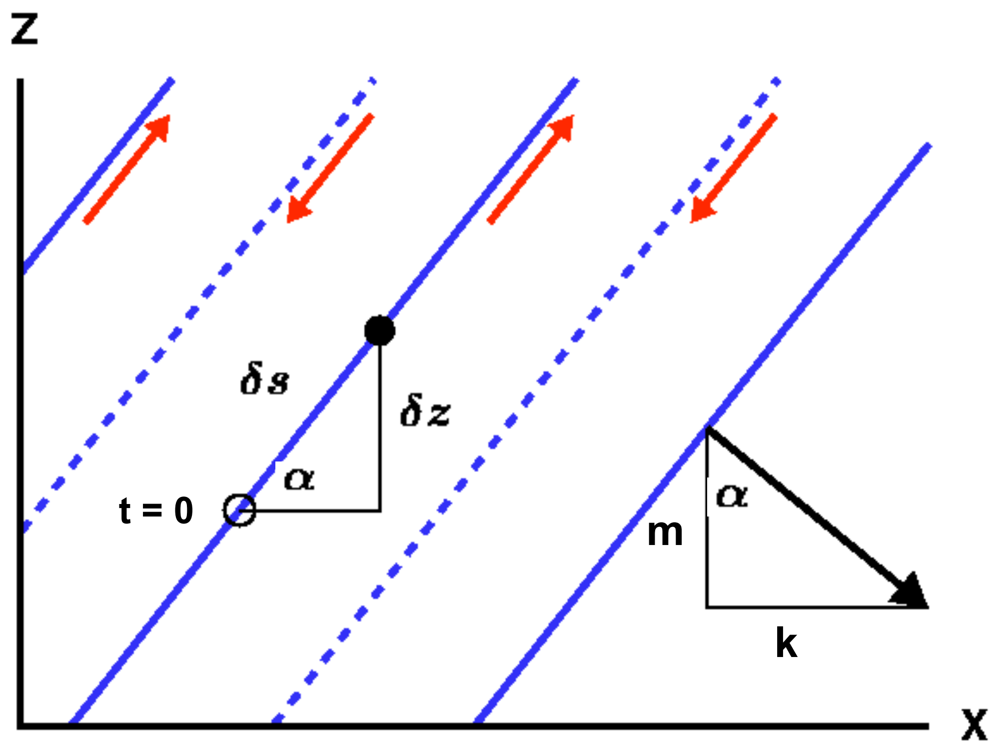
$$\theta_o(\delta z) \approx \theta_o(0) + \frac{d\theta_o}{dz}\delta z$$ (for small vertical displacement)

$$g\left(\frac{\theta - \theta_o}{\theta_o}\right) = -\left(\frac{g}{\theta_o}\frac{d\theta_o}{dz}\right)\delta z = -N^2\,\delta z$$ (buoyancy force)

$$\frac{d^2(\delta z)}{dt^2} + N^2(\delta z) = 0 \rightarrow \delta z = Re\left\{Ae^{iNt} + Be^{-iNt}\right\}$$

Parcel oscillates in the vertical with the buoyancy frequency N.

Internal gravity waves



$$\omega^2 = N^2 \sin^2\alpha$$
$$= N^2 \frac{k^2}{k^2+m^2} \approx \frac{N^2 k^2}{m^2}$$

which is the dispersion relation for a hydrostatic gravity wave in the Boussinesq approximation.

Now consider the parcel of air displaced upward along a sloping surface:

Acceleration of parcel up the slope is

$$\frac{d^2(\delta s)}{dt^2} = \frac{d^2}{dt^2}\left(\frac{\delta z}{\sin\alpha}\right)$$

Component of buoyancy force along the slope is

$$-N^2 \delta z \sin\alpha$$

Newton's 2nd law gives

$$\frac{d^2}{dt^2}\left(\frac{\delta z}{\sin\alpha}\right) + N^2 \sin\alpha\, \delta z = 0$$

$$\Rightarrow \frac{d^2}{dt^2}\delta z + \omega^2 \delta z = 0$$

Linear gravity wave equations

Equations governing hydrostatic gravity waves without rotation are:

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + \frac{\partial \Phi'}{\partial x} = 0$$

(zonal momentum equation)

$$\frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial z}\right) + \bar{u}\frac{\partial}{\partial x}\left(\frac{\partial \Phi'}{\partial z}\right) + N^2 w' = 0$$

(thermodynamic equation)

$$\frac{\partial u'}{\partial x} + \frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o w') = 0$$

(mass continuity equation)

Consider plane wave solutions:

$$f' = e^{z/2H}\text{Re}\left\{\tilde{f}\, e^{i(kx+mz-\omega t)}\right\}$$

Resulting **dispersion relation** is:

$$\hat{\omega}^2 = \frac{N^2 k^2}{m^2 + 1/(4H^2)}$$

$$\hat{\omega} = \omega - k\bar{u}$$ (intrinsic frequency)

Nonhydrostatic and rotational effects

• the previous example was for linear hydrostatic GWs without the effects of the Earth's rotation.

• a more general dispersion relation which accounts for nonhydrostatic and rotational effects is:

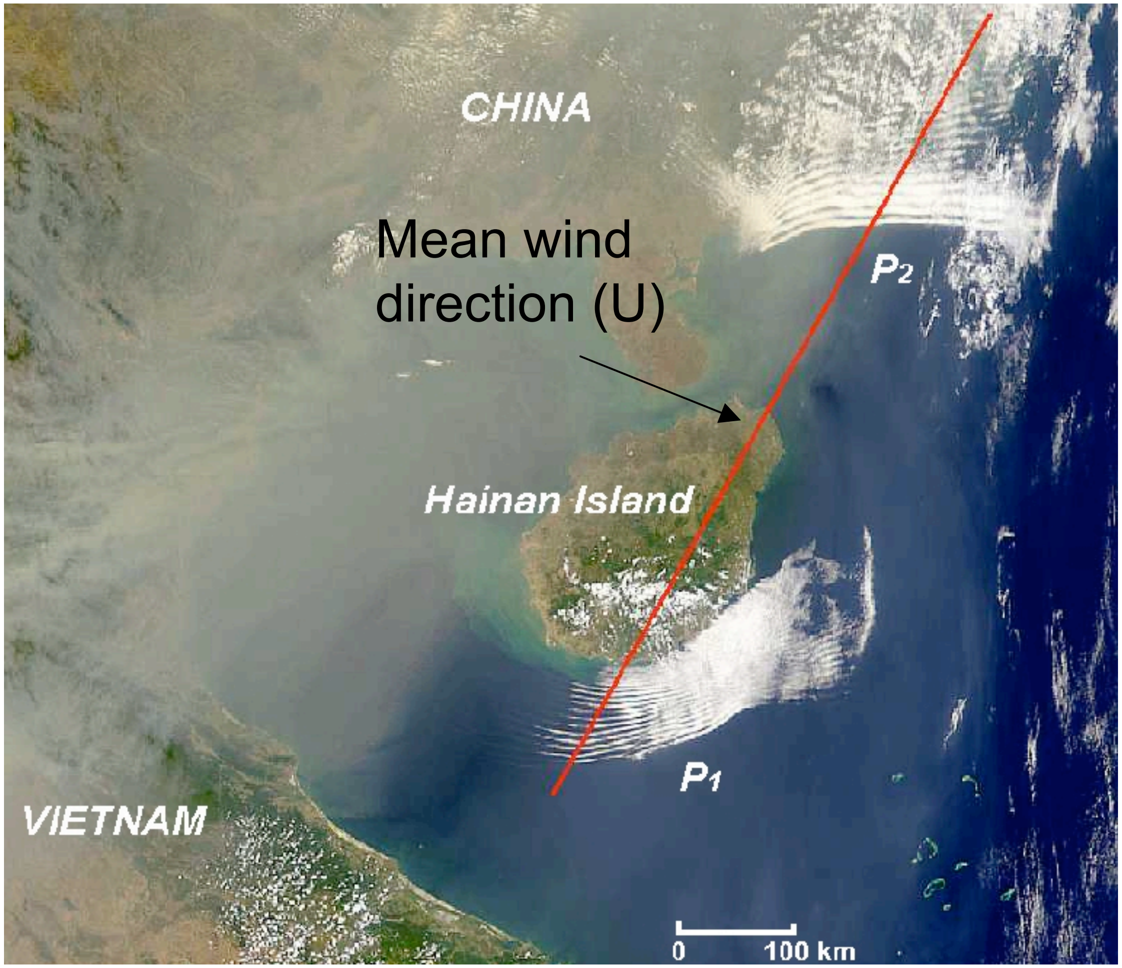
$$m^2 = \frac{(N^2 - \hat{\omega}^2)\, k^2}{\hat{\omega}^2 - f^2}$$

where *f* is the Coriolis parameter.

• a vertically propagating wave (*m* real) now requires that

$$|f| < |\hat{\omega}| < N$$

Vertically trapped mountain waves



- For a stationary mountain wave with

$$|\hat{\omega}| >> |f|$$

$$\hat{\omega} = -k\bar{u}$$

- Dispersion relation becomes

$$m^2 \approx \frac{(N^2 - \hat{\omega}^2)}{\hat{\omega}^2} k^2 = \frac{N^2}{\bar{u}^2} - k^2$$

- Short horizontal wavelengths (k > N/U) are vertically trapped.

Satellite image of vertically trapped mountain waves (courtesy of Sam Shen, U of Alta)

Momentum flux

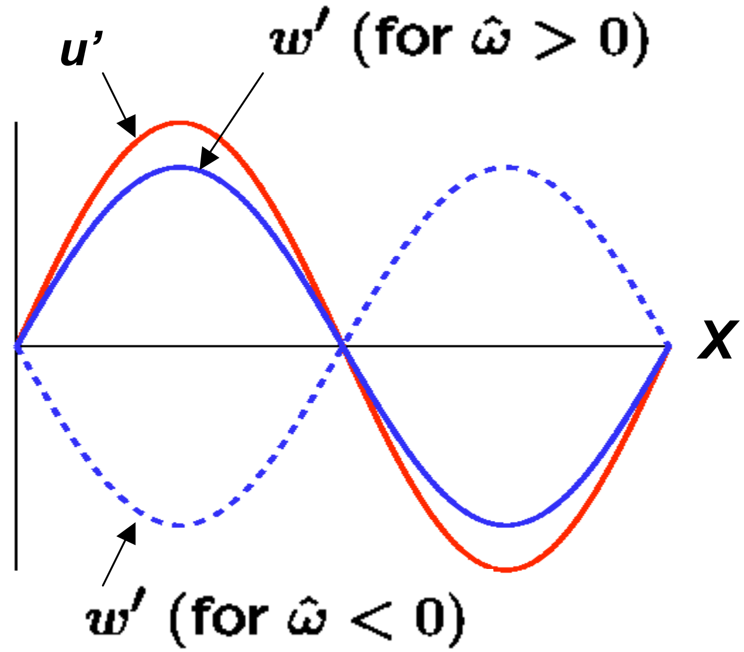
• vertically propagating gravity waves "transport" momentum over large height ranges.

• consider the continuity equation for the hydrostatic gravity in the case where *m* << 2*H*:

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \approx 0$$

$$\Rightarrow \quad \tilde{w} = -\frac{k}{m}\tilde{u}$$

$$\therefore \quad \rho_o \overline{u'w'} = \frac{1}{2}\,\rho_o \frac{\hat{\omega}}{N}\,|\tilde{u}|^2$$



• GWs with eastward (westward) intrinsic frequencies have positive (negative) momentum flux.

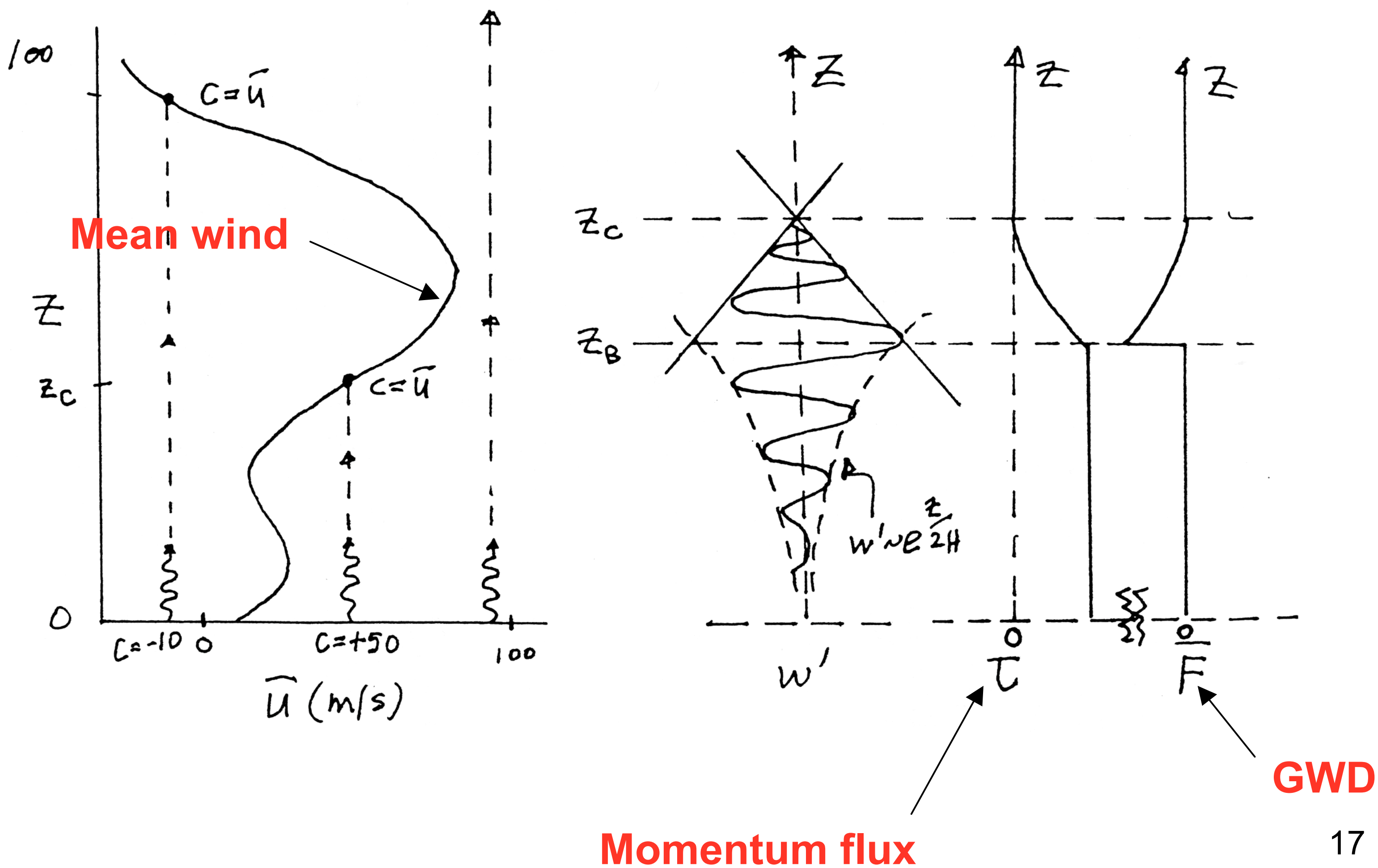
Momentum flux deposition

- gravity waves interact with the mean flow through the deposition of momentum:

$$\frac{\partial \bar{u}}{\partial t} + \ldots = \boxed{-\frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o \overline{u'w'})} \longleftarrow \textbf{GWD}$$

- for a steady, linear & undamped GW, the momentum flux is independent of height $\Longrightarrow$ GWD = 0.
- GWD arises when the momentum flux changes with height, which will occur if:
    1. the GW approaches a critical level (c = U)
    2. the GW ‘breaks’ and undergoes turbulent dissipation.

Cartoon depicting critical level 'filtering' and nonlinear breakdown of a monochromatic GW



Mesoscale model simulations of convectively generated GWs

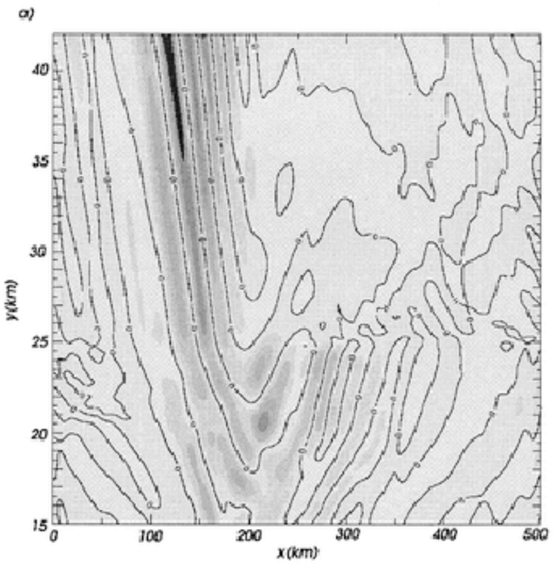
- to simulate small-scale GWs generated by convection a 2D or 3D mesoscale model must be used.
- compared to a GCM a mesoscale model employs a very fine horizontal resolution (1km) and a short timestep (5 s).
- here we will examine results from a 3D simulation of a tropical squall line.
- the model equations are nonlinear, compressible, nonhydrostatic and nonrotating; cloud microphysics parameterization is used.

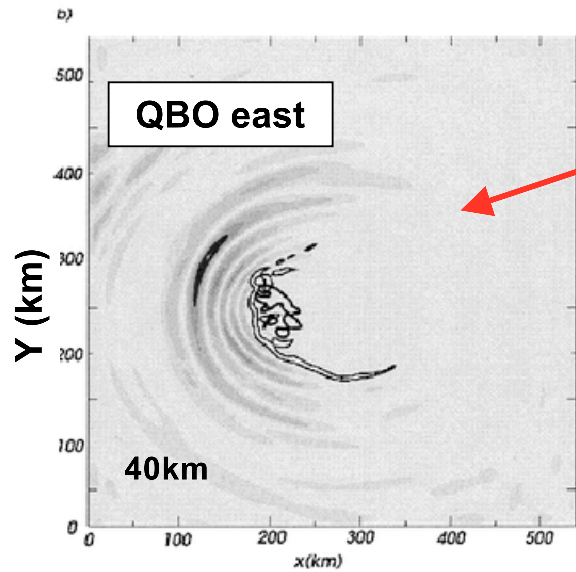
3D mesoscale model results

From Piani and Durran (JAS 2001)

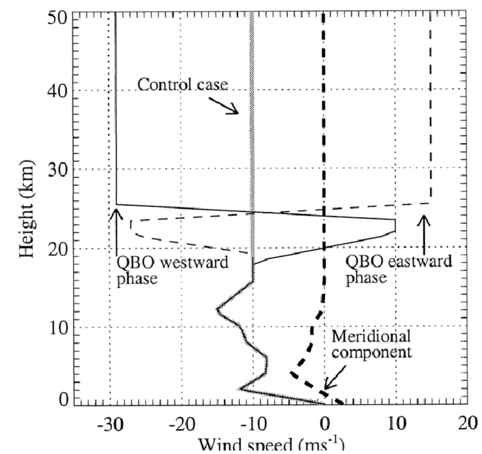
550 km

Height (km)

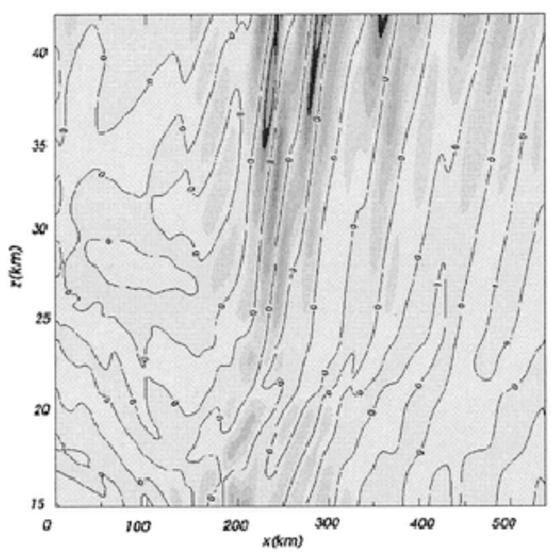


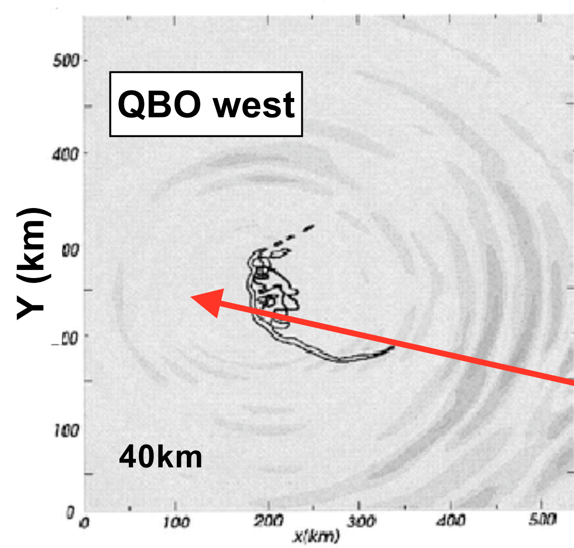


Eastward propagating GWs filtered out by eastward QBO winds



Height (km)





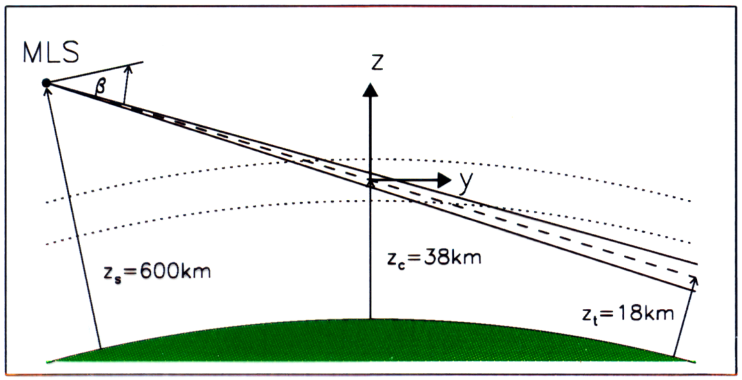
Westward propagating GWs filtered out by westward QBO winds

X (km)

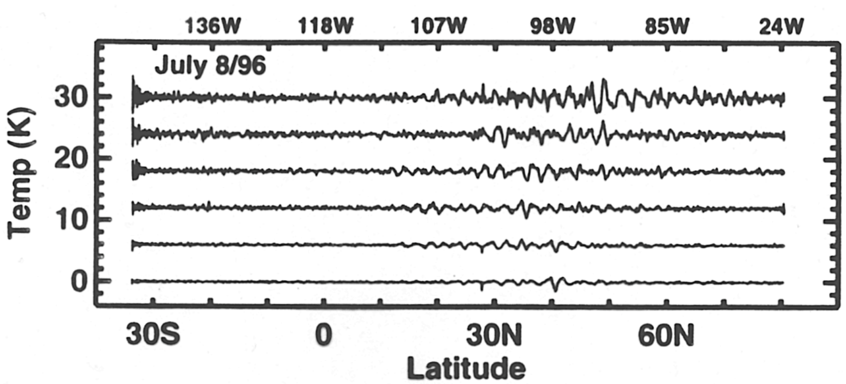
W X (km) E

Satellite observations of convectively generated small-scale gravity waves

Microwave Limb Sounder geometry

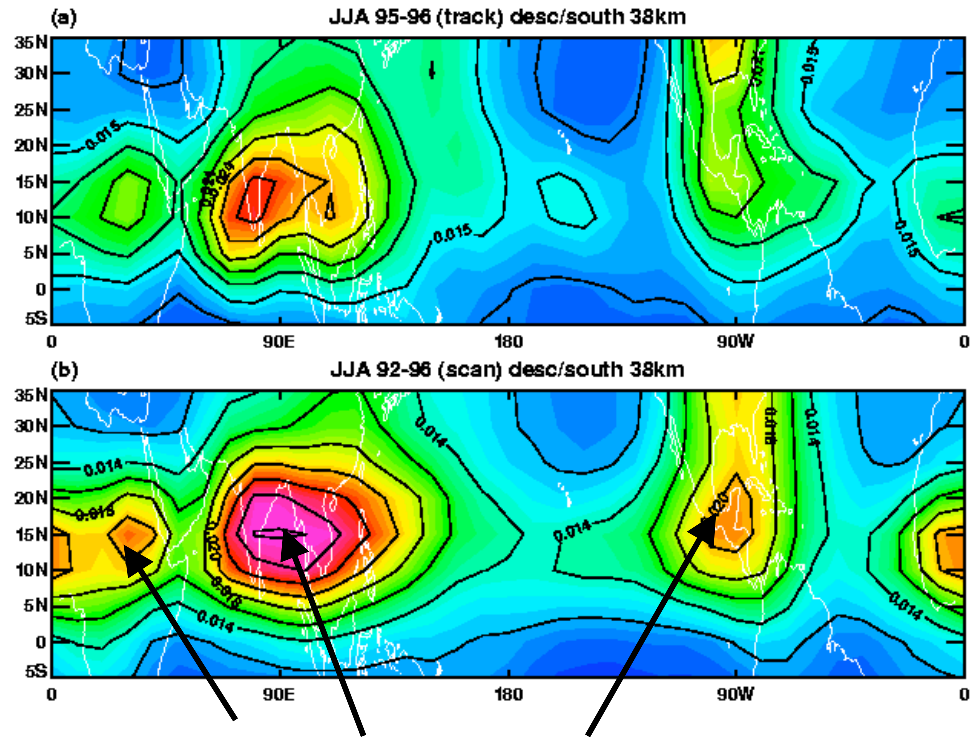


Single day of temperature data



McLandress et al (2000)

Seasonal average (NH summer)

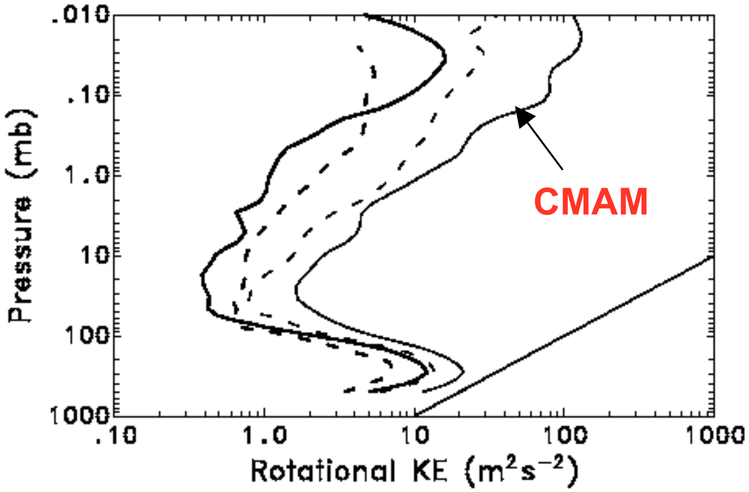


Temperature variance strongest over convective regions

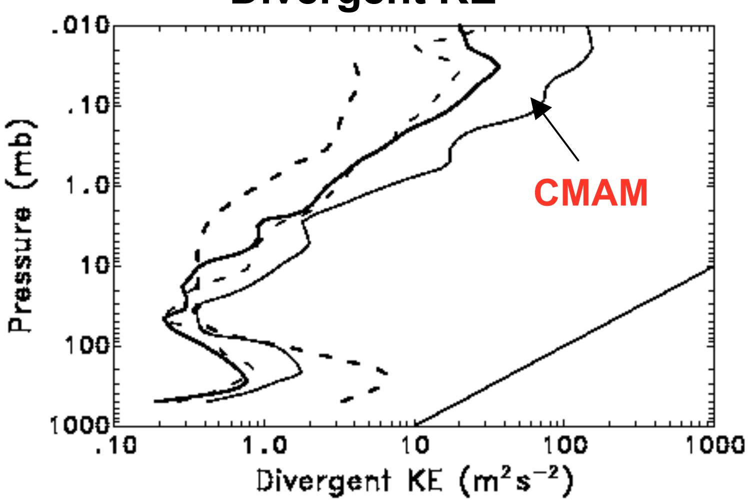
Part 3: Resolved Gravity Waves in GCMs

Kinetic energy spectra in middle atmosphere GCMs

Rotational KE

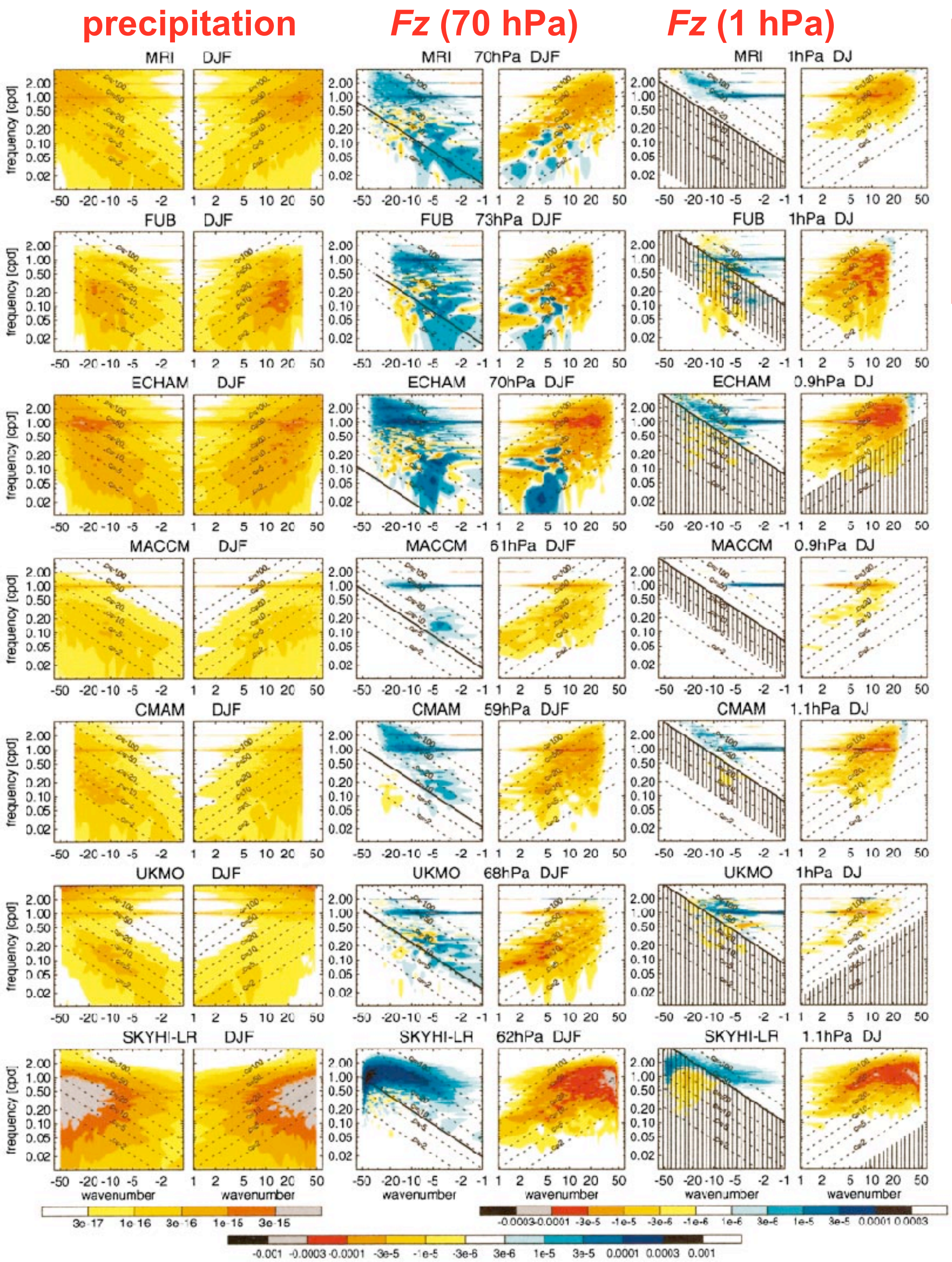


Divergent KE



- resolved GWs are included in the divergent part of the KE.
- all models show that divergent KE grows faster with height than the rotational KE.
- there are large differences in the amplitude of the spectra between models.

Koshyk et al (1999)

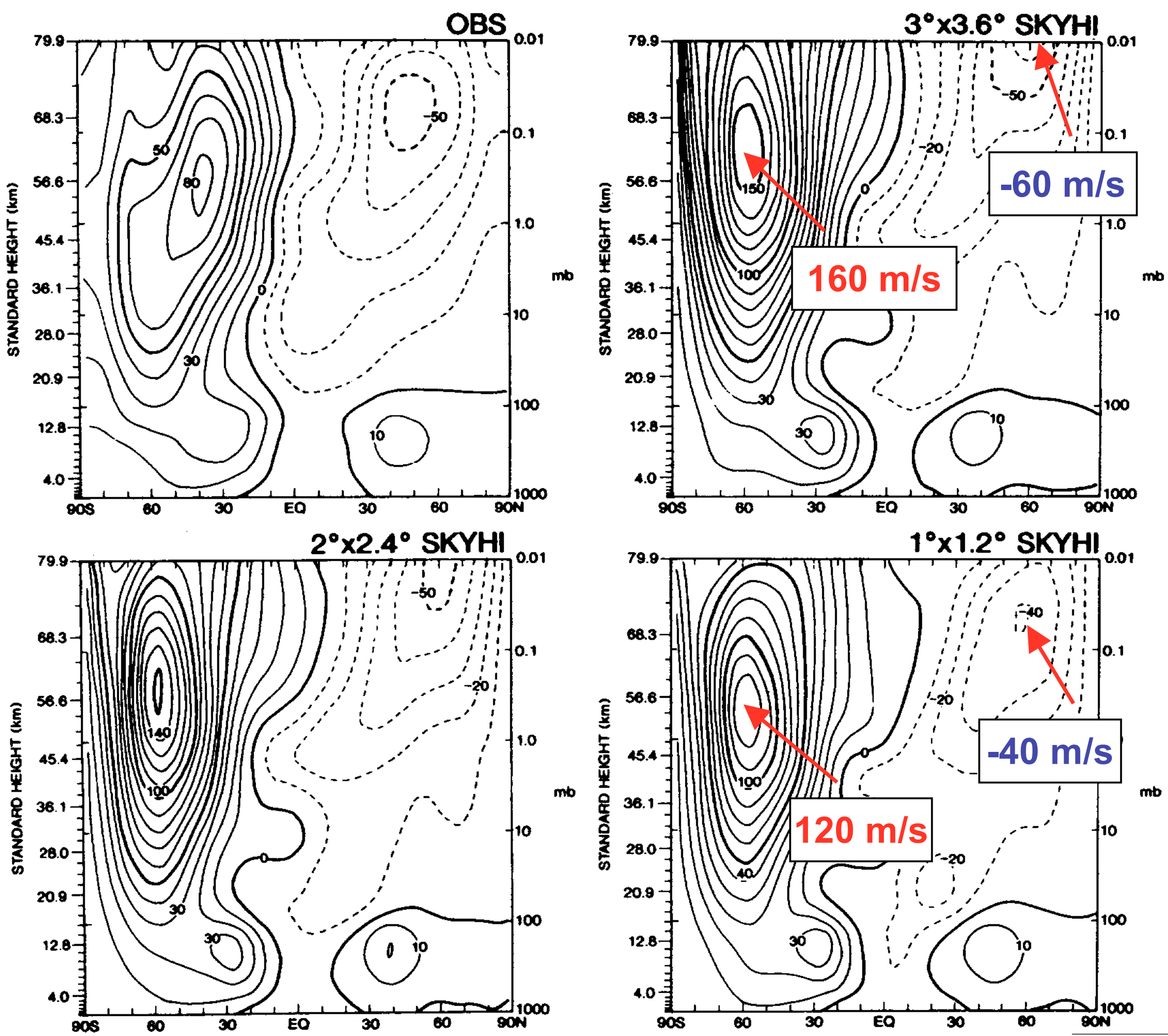


Wavenumber-frequency spectra of precipitation and the vertical component of the EP flux for different GCMs

- 2D spectra of precipitation and vertically propagating waves in the tropics is closely tied to the type of convective parameterization that is used in the GCM.

Horinouchi et al (JAS 2003)

Impact of model resolution on mesospheric mean winds



- zonal mean zonal winds for June-August from the SKYHI model for different horizontal resolutions.
- As the model resolution increases the speed of the mesospheric jets decrease:
    - higher resolution means more GWs are resolved.
    - more GWs mean more drag and weaker winds.

from Hamilton (JATP 1996)

Resolved mesospheric gravity wave drag

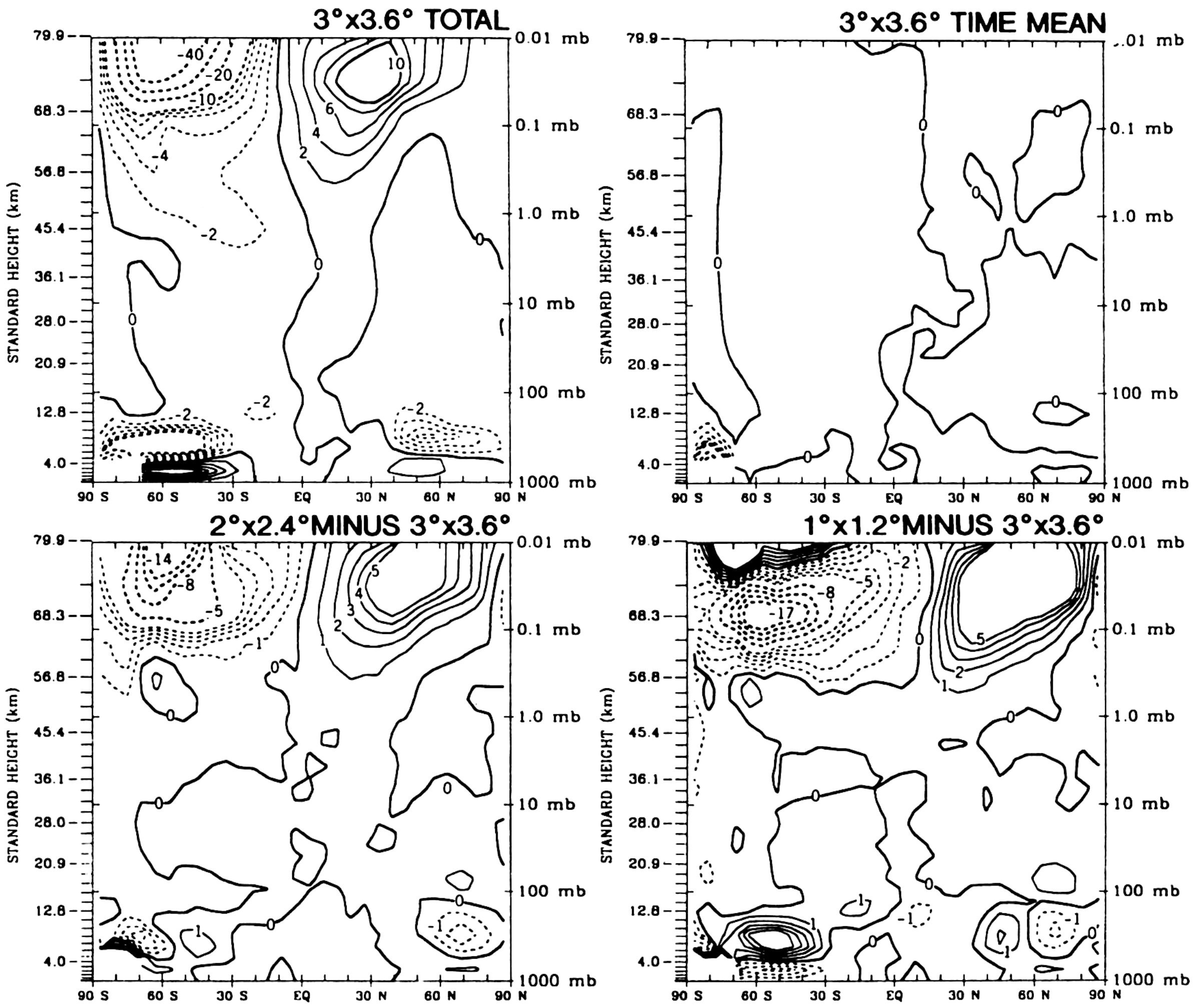


Fig. 12. As in Fig. 10. but for June–August.

- Eliassen-Palm flux divergence for June-August from the SKYHI model for different horizontal resolutions
- the magnitude of the EPFD increases as model resolution increases.

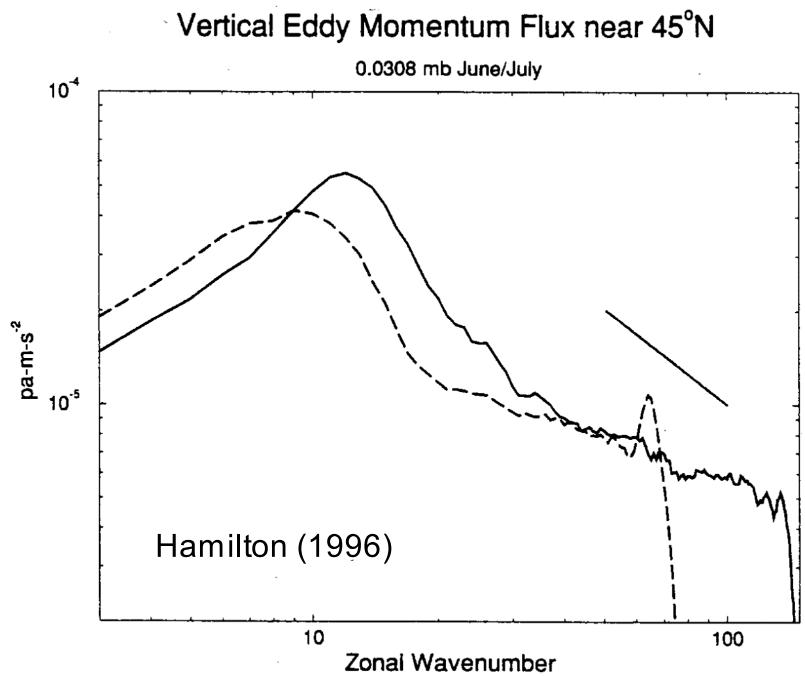
From Hamilton (JATP 1996)

Part 4: Parameterized Gravity Waves in GCMs

• the SKYHI model results demonstrated that a GCM with extremely high resolution is required to simulate the small-scale GWs that produce the required GWD in the mesosphere.

• this is further demonstrated by comparing the vertical flux of horizontal momentum from the low and high resolution simulations of SKYHI

- the shallow slope means that very high horizontal resolution is required.



• since such high resolution is currently not possible for a GCM the effects of these ‘missing’ gravity waves must be parameterized.

Overview of the GWD parameterization problem

- the basic idea is to include the effects of GWD on the mean circulation without actually solving for the gravity waves.
- a GWD parameterization is built on ideas from linear gravity wave theory in conjunction with a mechanism for the nonlinear breakdown (i.e., dissipation) of GWs.
- gross simplifications are required in order that the parameterization be computationally efficient.
- all GWD parameterizations currently used in GCMs consist of three components:

1. ‘Launch’ spectrum in the lower atmosphere
2. Linear propagation upward from the launch level
3. Nonlinear dissipation mechanism when waves attain large amplitudes.

1. Launch spectrum

• must specify a discrete or continuous spectrum of GWs at some level in the lower atmosphere which is assumed to lie above the actual gravity source regions.

• spectrum is a function of horizontal wavenumber and frequency.

• amplitude of waves in spectrum determine the amount of momentum flux they carry.

• great uncertainty in how to specify the launch spectrum since there are insufficient observations.

• a further complication is that waves with long vertical wavelengths (i.e., high intrinsic frequencies) are impossible to measure in the lower atmosphere:

  • BUT these are the waves that have the greatest impact on the mesosphere.

2. Linear propagation

- current GWD parameterizations assume that GW propagation is straight up (i.e., no propagation into adjacent grid boxes).
- a linear dispersion relation is used to relate the various wave parameters (e.g., the dispersion relation for hydrostatic nonrotating GWs is often employed).
- a GW is assumed to propagate linearly, conserving its momentum flux until either:

  1. a critical level is reached where c=U or
  2. its amplitude exceeds a specified threshold.

3. Nonlinear dissipation mechanisms

1. **Convective instability and saturation**:
   - Lindzen (1981)
   - amplitude of GW increases to point where the total temperature lapse rate is convectively unstable.
   - Small-scale turbulence damps the GW and maintains it at an amplitude of neutral stability.
2. **Convective instability and obliteration:**
   - Alexander and Dunkerton (1999)
   - Uses Lindzen's convective instability criterion but deposits all of the GWs momentum flux at the breaking height.

3. **Doppler spreading:**

   - Hines (1991, 1993, 1997)
   - a GW is dissipated as it reaches a critical level given by the background wind and the wind associated with the parameterized spectrum (i.e., c = U + u’).
   - the term ‘spreading’ is used since the Doppler shifting of the vertical wavenumber is defined only in a statistical sense.

4. **Nonlinear diffusion:**

   - Weinstock (1982); Medvedev and Klaassen (1995)
   - A complicated and virtually incomprehensible (to me at least) mechanism that we will discuss no more.

5. Empirical saturation condition:

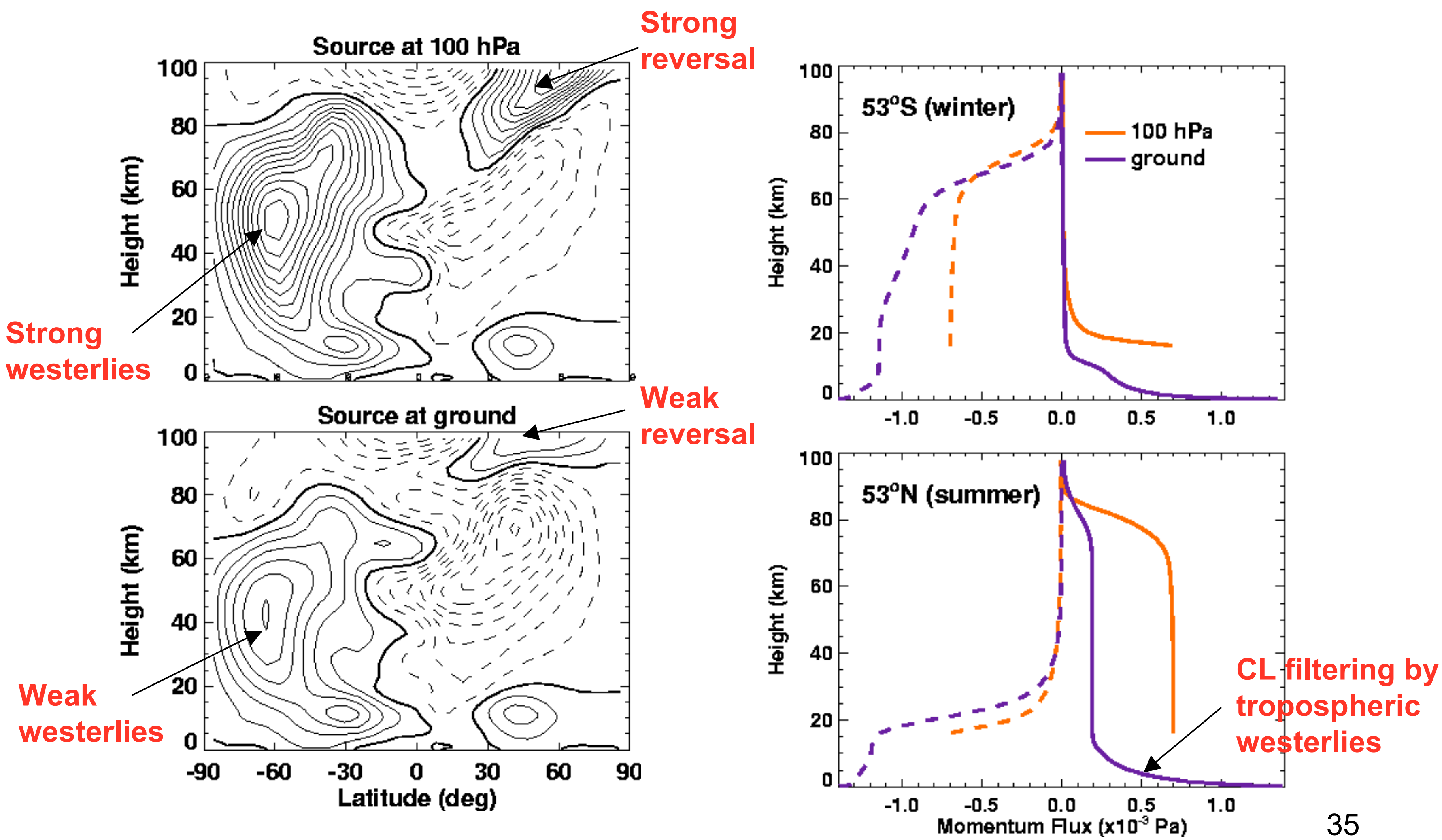
- Warner and McIntyre (1986)
- The observed m^{-3} form of the observed spectrum is used to limit wave amplitudes

Simulations using the Canadian Middle Atmosphere Model (CMAM)

- the CMAM is a middle atmosphere GCM that extends from the ground to 100 km.
- horizontal resolution is T32 (about 5-6 deg).
- comprehensive set of physical parameterizations relevant to the troposphere and middle atmosphere.
- a single GWD parameterization that contains different nonlinear dissipation mechanisms.
- experiments all use identical launch spectrum for the parameterized GWs.
- McLandress and Scinocca (JAS, in press).

Filtering of GW spectrum by tropospheric winds

- Simulations using Hines Doppler spread parameterization:



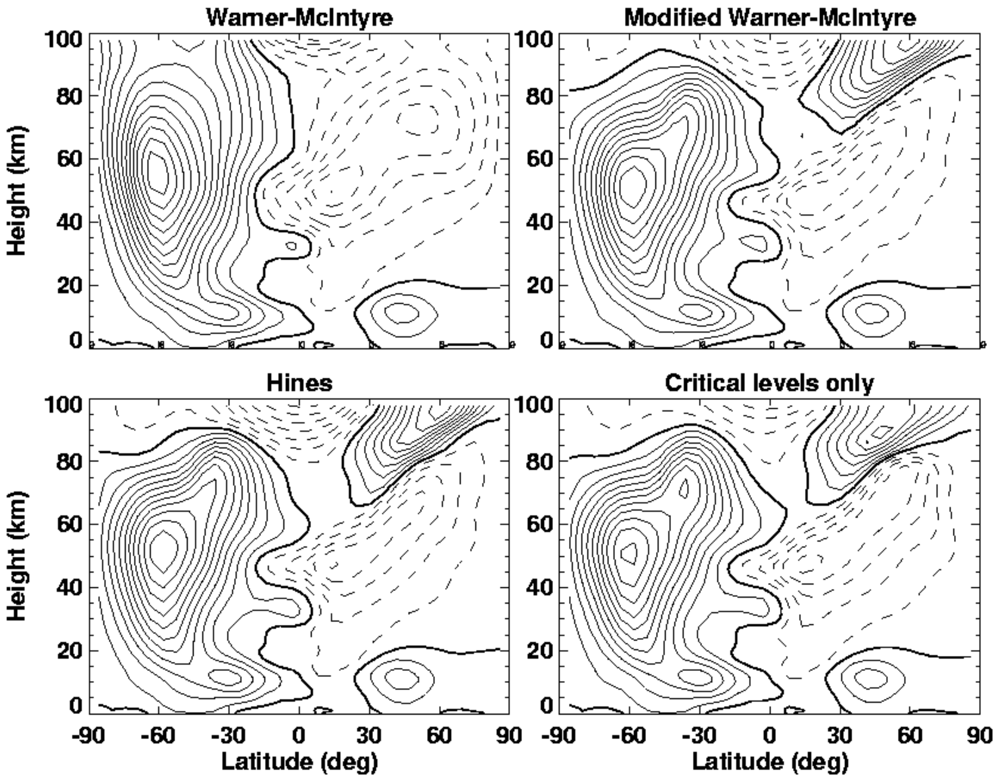
Simulations using different nonlinear dissipation mechanisms in the GWD parameterization

Warner & McIntyre (1996)

Modified Warner & McIntyre (1996)

Hines Doppler spread

With only critical level filtering



- important message here is that the different dissipation mechanisms can be made to produce nearly same GCM response by a simple scaling of the height at which momentum is deposited.
- this means that details of dissipation are not that important.

1D-QBO model results

$$\frac{\partial \bar{u}}{\partial t} + \bar{w}\frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho_o}\frac{\partial}{\partial z}\left(\rho_o \overline{u'w'}\right) + K_{zz}\frac{\partial^2 \bar{u}}{\partial z^2}$$

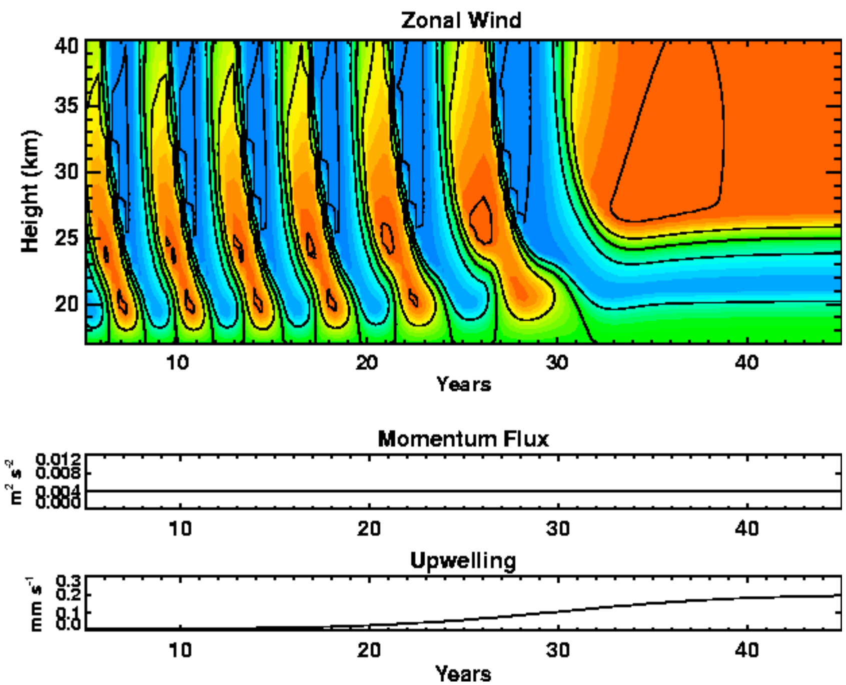
prescribed upwelling

wave momentum flux

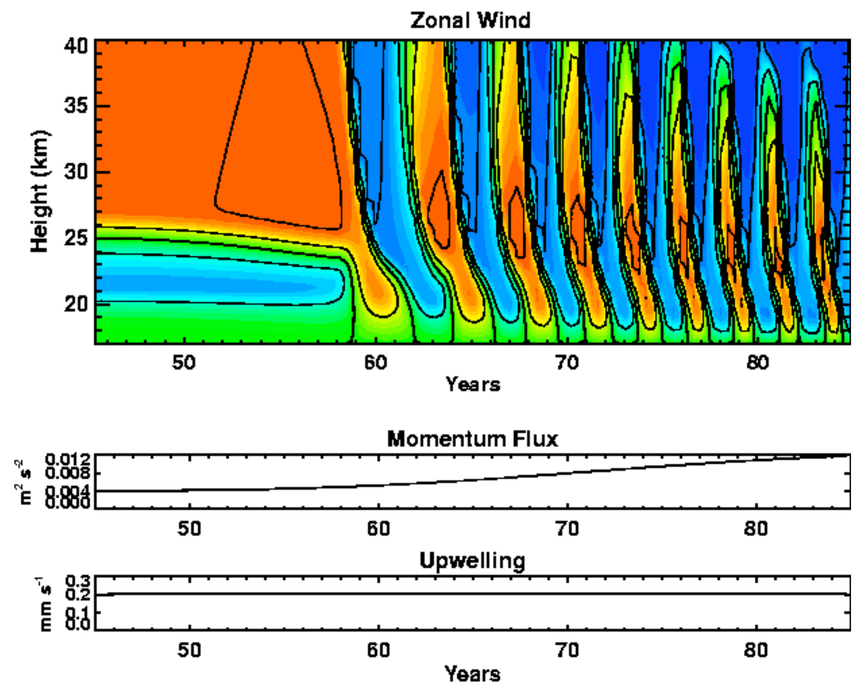
$$\overline{u'w'} = (\overline{u'w'})_{pw} + (\overline{u'w'})_{gw}$$

Planetary waves (Holton & Lindzen, 1972)

Small-scale gravity waves parameterized using Hines (1997)

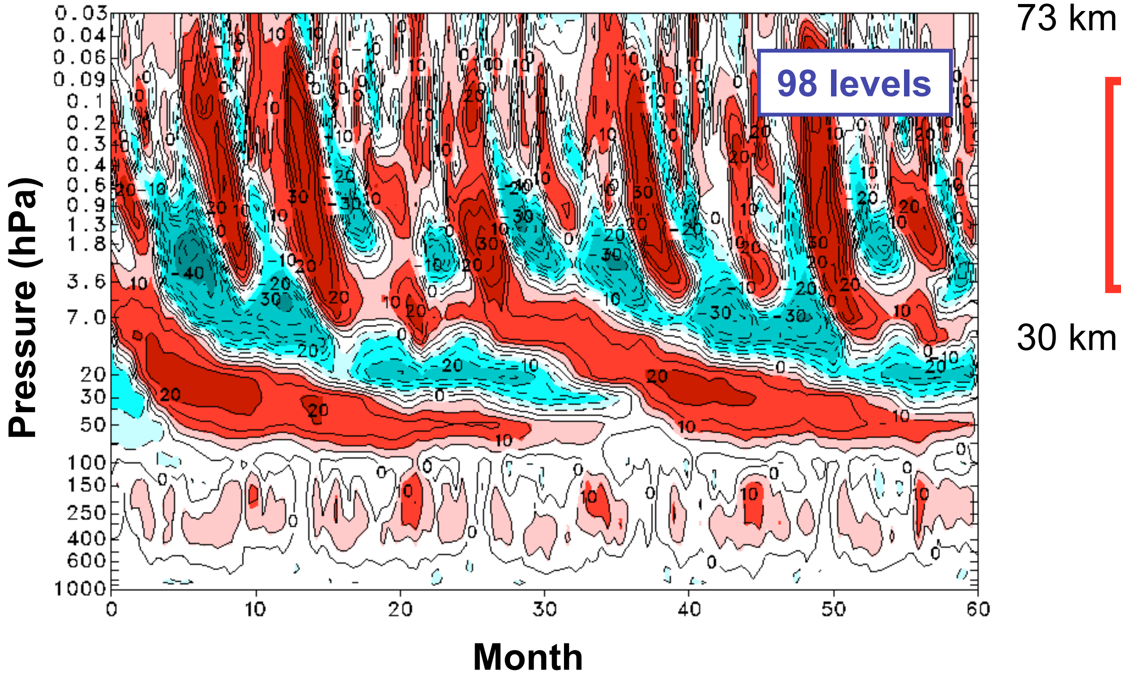


QBO is suppressed when upwelling is increased (only planetary waves).

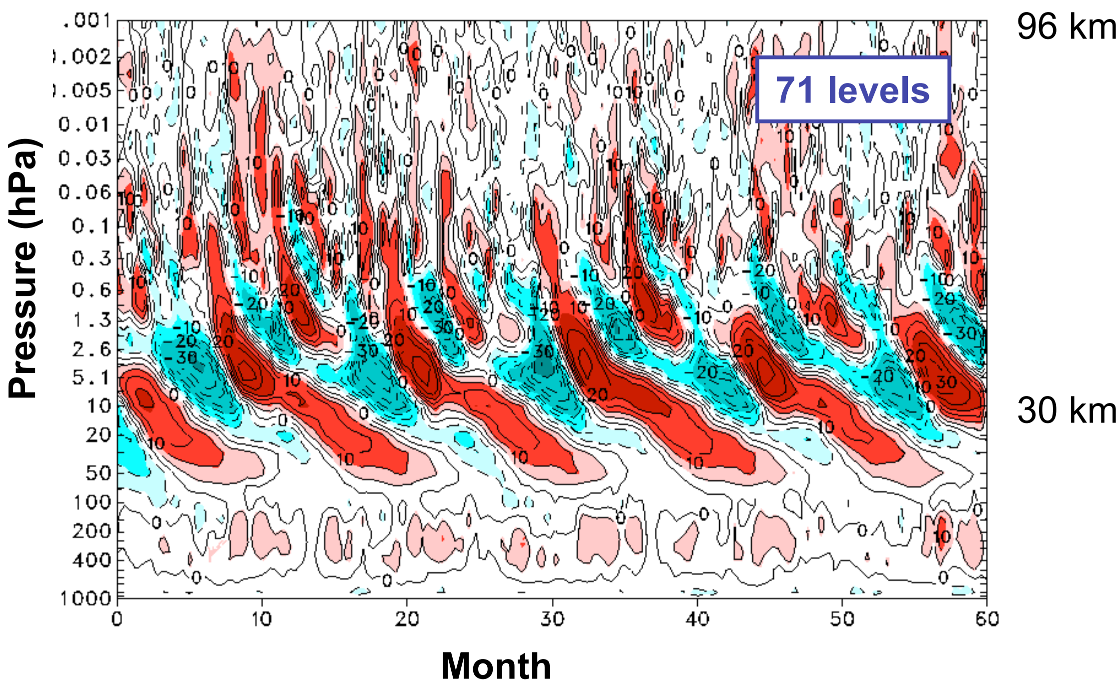


QBO reappears when gravity wave momentum flux is increased (both PWs and GWs).

Simulating the QBO in the CMAM



Zonal mean zonal winds at equator



- T47 horizontal resolution
- Warner and McIntyre GWD parameterization:
  - momentum flux enhanced by factor of four in tropics.

The End